

The 2003 AP[®] Calculus AB and AP Calculus BC Released Exams

Contains:

- Multiple-Choice Questions, Answer Keys, and Diagnostic Guides
- Free-Response Questions with:
 - Scoring Commentary
 - Scoring Guidelines
 - Sample Student Responses
- Statistical Information About Student Performance on the 2003 Exams

Materials included in this Released Exam may not reflect the current *AP Course Description* and exam in this subject, and teachers are advised to take this into account as they use these materials to support their instruction of students. For up-to-date information about this AP course and exam, please download the official *AP Course Description* from the AP Central[®] Web site at apcentral.collegeboard.com.

CALCULUS BC

SECTION I, Part A

Time—55 minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

- (1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).

1. If $y = \sin(3x)$, then $\frac{dy}{dx} =$

- (A) $-3 \cos(3x)$ (B) $-\cos(3x)$ (C) $-\frac{1}{3} \cos(3x)$ (D) $\cos(3x)$ (E) $3 \cos(3x)$

2. $\lim_{x \rightarrow 0} \frac{e^x - \cos x - 2x}{x^2 - 2x}$ is

- (A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) nonexistent
-

3. $\int (3x + 1)^5 dx =$
- (A) $\frac{(3x + 1)^6}{18} + C$
- (B) $\frac{(3x + 1)^6}{6} + C$
- (C) $\frac{(3x + 1)^6}{2} + C$
- (D) $\frac{\left(\frac{3x^2}{2} + x\right)^6}{2} + C$
- (E) $\left(\frac{3x^2}{2} + x\right)^5 + C$

-
4. For $0 \leq t \leq 13$, an object travels along an elliptical path given by the parametric equations $x = 3 \cos t$ and $y = 4 \sin t$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

- (A) $-\frac{4}{3}$ (B) $-\frac{3}{4}$ (C) $-\frac{4 \tan 13}{3}$ (D) $-\frac{4}{3 \tan 13}$ (E) $-\frac{3}{4 \tan 13}$
-

5. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?
- (A) 3 (B) 5 (C) 6 (D) 10 (E) 12

-
6. What are all values of p for which $\int_1^{\infty} \frac{1}{x^{2p}} dx$ converges?
- (A) $p < -1$
(B) $p > 0$
(C) $p > \frac{1}{2}$
(D) $p > 1$
(E) There are no values of p for which this integral converges.
-

7. The position of a particle moving in the xy -plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?
- (A) -1 only (B) 0 only (C) 2 only (D) -1 and 2 only (E) -1 , 0 , and 2

8. $\int x^2 \cos(x^3) dx =$

- (A) $-\frac{1}{3} \sin(x^3) + C$
(B) $\frac{1}{3} \sin(x^3) + C$
(C) $-\frac{x^3}{3} \sin(x^3) + C$
(D) $\frac{x^3}{3} \sin(x^3) + C$
(E) $\frac{x^3}{3} \sin\left(\frac{x^4}{4}\right) + C$
-

9. If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0)$ is

- (A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

10. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

- (A) 1 (B) 2 (C) 4 (D) 6 (E) The series diverges.
-

11. The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$?

(A) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$

(B) $x^2 + x^3 + x^4 + x^5 + \cdots$

(C) $x^2 + 2x^3 + 3x^4 + 4x^5 + \cdots$

(D) $x^2 + x^4 + x^6 + x^8 + \cdots$

(E) $x^2 - x^4 + x^6 - x^8 + \cdots$

12. The rate of change of the volume, V , of water in a tank with respect to time, t , is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

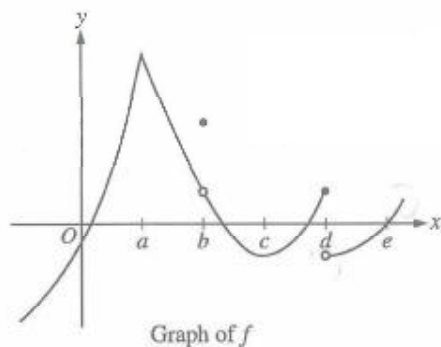
(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

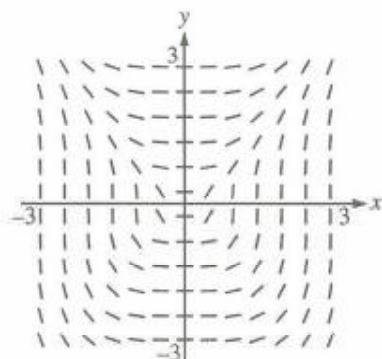
(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$



13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

- (A) a (B) b (C) c (D) d (E) e



14. Shown above is a slope field for which of the following differential equations?

- (A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = \frac{x^2}{y^2}$ (C) $\frac{dy}{dx} = \frac{x^3}{y}$ (D) $\frac{dy}{dx} = \frac{x^2}{y}$ (E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

15. The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1 + 9x^4} \, dx$. If the curve contains the point $(1, 6)$, which of the following could be an equation for this curve?
- (A) $y = 3 + 3x^2$
(B) $y = 5 + x^3$
(C) $y = 6 + x^3$
(D) $y = 6 - x^3$
(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$
-

16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is
- (A) -5 (B) 1 (C) 3 (D) 7 (E) undefined
-

17. A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of C at the point $(-3, 8)$?

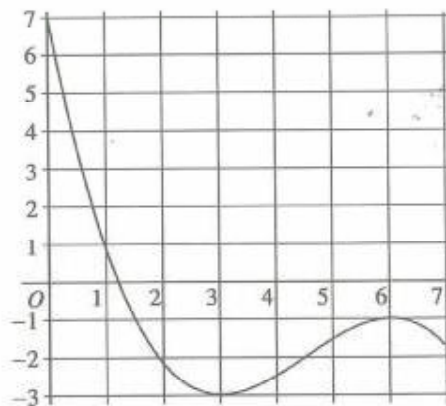
(A) $x = -3$

(B) $x = 2$

(C) $y = 8$

(D) $y = -\frac{27}{10}(x + 3) + 8$

(E) $y = 12(x + 3) + 8$

Graph of f

18. The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$.

If $g(x) = \int_0^{2x} f(t) dt$, what is the value of $g'(3)$?

- (A) 0 (B) -1 (C) -2 (D) -3 (E) -6

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

- (A) $y = 5x - 3$
(B) $y = x^2 + 1$
(C) $y = x^2 + 3x$
(D) $y = x^2 + 3x - 2$
(E) $y = 2x^2 + 3x - 3$

-
20. A function f has Maclaurin series given by $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots + \frac{x^{n+3}}{(n+1)!} + \cdots$. Which of the following is an expression for $f(x)$?

- (A) $-3x \sin x + 3x^2$
(B) $-\cos(x^2) + 1$
(C) $-x^2 \cos x + x^2$
(D) $x^2 e^x - x^3 - x^2$
(E) $e^{x^2} - x^2 - 1$
-

21. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t \rightarrow \infty} M(t)$?

(A) 50 (B) 200 (C) 500 (D) 1000 (E) 2000

22. What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p + 1}$ converges?

(A) $p > 0$ (B) $p \geq 1$ (C) $p > 1$ (D) $p \geq 2$ (E) $p > 2$

23. $\int x \sin(6x) dx =$

- (A) $-x \cos(6x) + \sin(6x) + C$
(B) $-\frac{x}{6} \cos(6x) + \frac{1}{36} \sin(6x) + C$
(C) $-\frac{x}{6} \cos(6x) + \frac{1}{6} \sin(6x) + C$
(D) $\frac{x}{6} \cos(6x) + \frac{1}{36} \sin(6x) + C$
(E) $6x \cos(6x) - \sin(6x) + C$

24. Which of the following series diverge?

I. $\sum_{n=0}^{\infty} \left(\frac{\sin 2}{\pi} \right)^n$

II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

III. $\sum_{n=1}^{\infty} \left(\frac{e^n}{e^n + 1} \right)$

- (A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

x	2	5	10	14
$f(x)$	12	28	34	30

25. The function f is continuous on the closed interval $[2, 14]$ and has values as shown in the table above. Using the subintervals $[2, 5]$, $[5, 10]$, and $[10, 14]$, what is the approximation of $\int_2^{14} f(x) dx$ found by using a right Riemann sum?

(A) 296 (B) 312 (C) 343 (D) 374 (E) 390

26. $\int \frac{2x}{(x+2)(x+1)} dx =$

- (A) $\ln|x+2| + \ln|x+1| + C$
(B) $\ln|x+2| + \ln|x+1| - 3x + C$
(C) $-4\ln|x+2| + 2\ln|x+1| + C$
(D) $4\ln|x+2| - 2\ln|x+1| + C$
(E) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$
-

27. $\frac{d}{dx} \left(\int_0^{x^3} \ln(t^2 + 1) dt \right) =$
- (A) $\frac{2x^3}{x^6 + 1}$ (B) $\frac{3x^2}{x^6 + 1}$ (C) $\ln(x^6 + 1)$ (D) $2x^3 \ln(x^6 + 1)$ (E) $3x^2 \ln(x^6 + 1)$

-
28. What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x = 0$?
- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) 6
-

END OF PART A OF SECTION I

CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
THIS PART OF THE EXAMINATION.

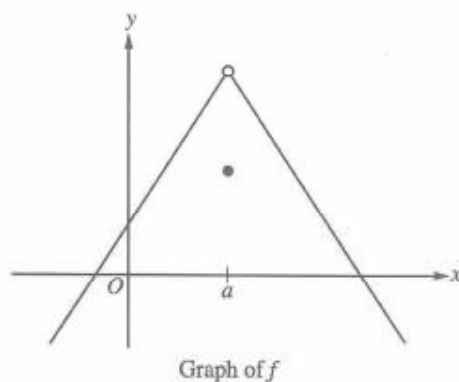
Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this test:

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



76. The graph of the function f is shown above. Which of the following statements must be false?

- (A) $f(a)$ exists.
- (B) $f(x)$ is defined for $0 < x < a$.
- (C) f is not continuous at $x = a$.
- (D) $\lim_{x \rightarrow a} f(x)$ exists.
- (E) $\lim_{x \rightarrow a} f'(x)$ exists.

77. Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

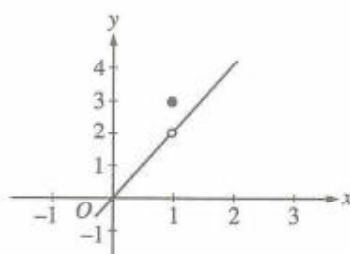
- (A) -30 (B) -15 (C) -5 (D) $-\frac{5}{6}$ (E) $-\frac{1}{6}$
-

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?
- (A) 0.04π m²/sec
(B) 0.4π m²/sec
(C) 4π m²/sec
(D) 20π m²/sec
(E) 100π m²/sec

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

79. The table above gives values of f , f' , g , and g' at selected values of x . If $h(x) = f(g(x))$, then $h'(1) =$
- (A) 5 (B) 6 (C) 9 (D) 10 (E) 12

80. Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2 - e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \leq t \leq 14$?
- (A) 125 (B) 100 (C) 88 (D) 50 (E) 12



Graph of f

81. The graph of the function f is shown in the figure above. The value of $\lim_{x \rightarrow 1} \sin(f(x))$ is
- (A) 0.909 (B) 0.841 (C) 0.141 (D) -0.416 (E) nonexistent

82. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \leq t \leq 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t) \, dt$

(B) $\int_0^8 r(t) \, dt$

(C) $\int_0^{2.667} r(t) \, dt$

(D) $\int_{1.572}^{3.514} r'(t) \, dt$

(E) $\int_0^{2.667} r'(t) \, dt$

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

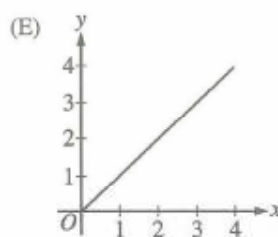
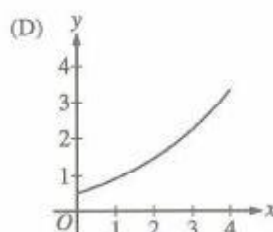
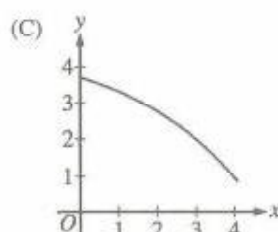
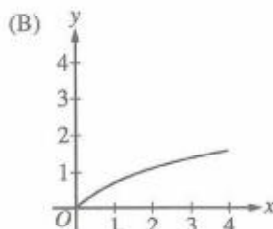
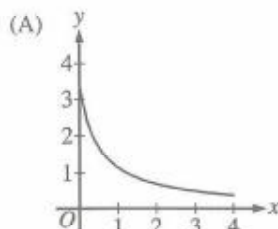
83. The function f is continuous and differentiable on the closed interval $[0, 4]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

- (A) The minimum value of f on $[0, 4]$ is 2.
- (B) The maximum value of f on $[0, 4]$ is 4.
- (C) $f(x) > 0$ for $0 < x < 4$
- (D) $f'(x) < 0$ for $2 < x < 4$
- (E) There exists c , with $0 < c < 4$, for which $f'(c) = 0$.

84. A particle moves in the xy -plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

- (A) 2.909 (B) 3.062 (C) 6.884 (D) 9.016 (E) 47.393

85. If a trapezoidal sum overapproximates $\int_0^4 f(x) dx$, and a right Riemann sum underapproximates $\int_0^4 f(x) dx$, which of the following could be the graph of $y = f(x)$?



86. Let f be the function with derivative defined by $f'(x) = \sin(x^3)$ on the interval $-1.8 < x < 1.8$. How many points of inflection does the graph of f have on this interval?

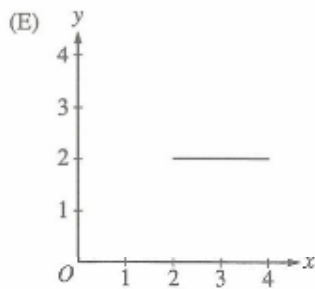
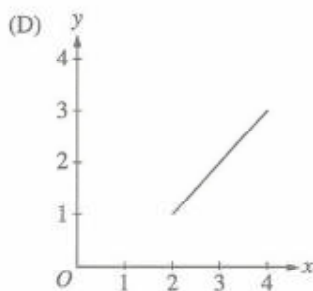
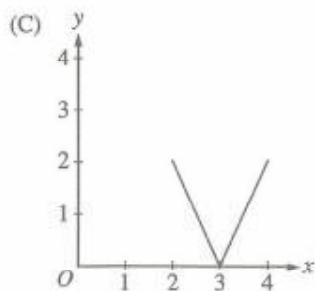
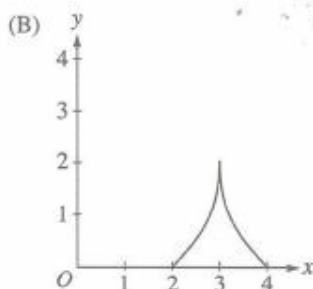
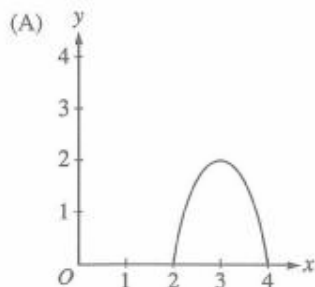
(A) Two (B) Three (C) Four (D) Five (E) Six

-
87. A particle moves along the x -axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos(2 - t^2)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

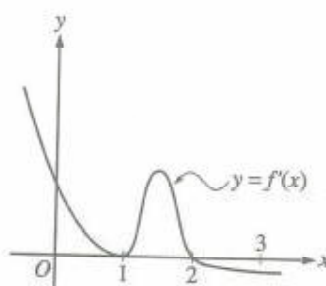
(A) 0.411 (B) 1.310 (C) 2.816 (D) 3.091 (E) 3.411

88. On the closed interval $[2, 4]$, which of the following could be the graph of a function f with the property that

$$\frac{1}{4-2} \int_2^4 f(t) \, dt = 1?$$



89. The region bounded by the graph of $y = 2x - x^2$ and the x -axis is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an equilateral triangle. What is the volume of the solid?
- (A) 1.333 (B) 1.067 (C) 0.577 (D) 0.462 (E) 0.267



90. The graph of f' , the derivative of the function f , is shown above. If $f(0) = 0$, which of the following must be true?
- I. $f(0) > f(1)$
 - II. $f(2) > f(1)$
 - III. $f(1) > f(3)$
- (A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

91. The height h , in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?
- (A) 2.545 meters
(B) 10.263 meters
(C) 34.125 meters
(D) 54.889 meters
(E) 89.005 meters
-

92. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over $[1, 4]$?
- (A) 0.456 (B) 1.244 (C) 2.164 (D) 2.342 (E) 2.452
-

END OF SECTION I

AFTER TIME HAS BEEN CALLED, TURN TO THE NEXT PAGE AND
ANSWER QUESTIONS 93-96.

93. Which graphing calculator did you use during the examination?
- (A) Casio 6300, Casio 7300, Casio 7400, Casio 7700, TI-73, TI-80, or TI-81
 - (B) Casio 9700, Casio 9800, Sharp 9200, Sharp 9300, TI-82, or TI-85
 - (C) Casio 9850, Casio FX 1.0, Sharp 9600, Sharp 9900, TI-83/TI-83 Plus, or TI-86
 - (D) Casio 9970, Casio Algebra FX 2.0, HP 38G, HP 39G, HP 40G, HP 48 series, HP 49 series, or TI-89
 - (E) Some other graphing calculator
94. During your Calculus BC course, which of the following best describes your calculator use?
- (A) I used my own graphing calculator.
 - (B) I used a graphing calculator furnished by my school, both in class and at home.
 - (C) I used a graphing calculator furnished by my school only in class.
 - (D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
 - (E) I did not use a graphing calculator.
95. During your Calculus BC course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?
- (A) Almost every class
 - (B) About three-quarters of the classes
 - (C) About one-half of the classes
 - (D) About one-quarter of the classes
 - (E) Seldom or never
96. During your Calculus BC course, which of the following describes the portion of testing time you were allowed to use a graphing calculator?
- (A) All or almost all of the time
 - (B) About three-quarters of the time
 - (C) About one-half of the time
 - (D) About one-quarter of the time
 - (E) Seldom or never



AP[®] Calculus BC 2003 Free-Response Questions

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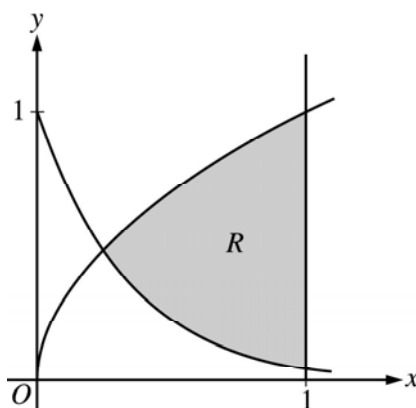
2003 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A

Time—45 minutes

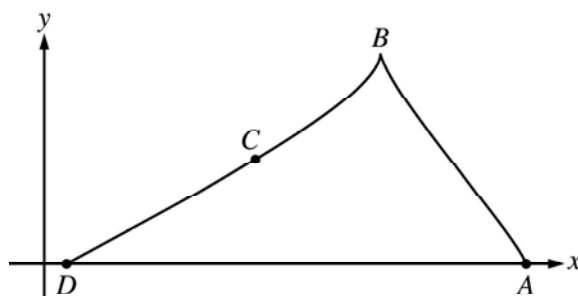
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



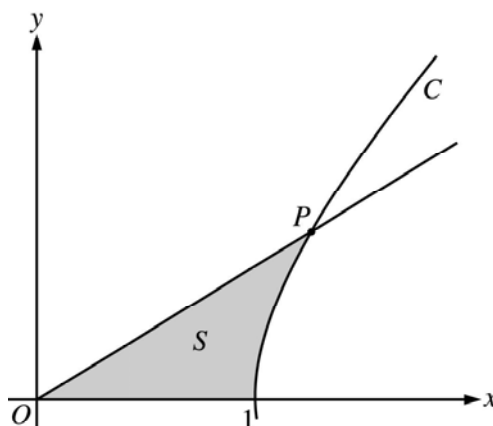
1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.
 - (a) Find the area of R .
 - (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
 - (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a rectangle whose height is 5 times the length of its base in region R . Find the volume of this solid.

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2. A particle starts at point A on the positive x -axis at time $t = 0$ and travels along the curve from A to B to C to D , as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of t , where $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \frac{dy}{dt}$ is not explicitly given. At time $t = 9$, the particle reaches its final position at point D on the positive x -axis.
- At point C , is $\frac{dy}{dt}$ positive? At point C , is $\frac{dx}{dt}$ positive? Give a reason for each answer.
 - The slope of the curve is undefined at point B . At what time t is the particle at point B ?
 - The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.
 - How far apart are points A and D , the initial and final positions, respectively, of the particle?
-

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3. The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x -axis. The line and the curve intersect at point P .
- Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P .
 - Set up and evaluate an integral expression with respect to y that gives the area of S .
 - Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2 \theta - \sin^2 \theta}$.
 - Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ that represents the area of S .
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END OF PART A OF SECTION II

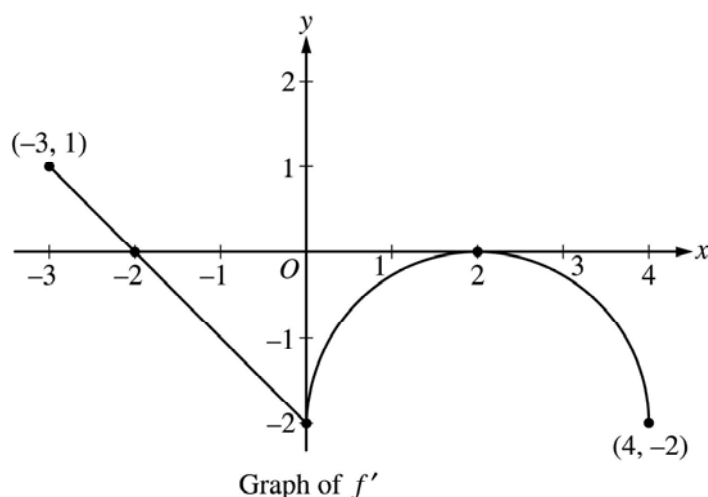
2003 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part B

Time—45 minutes

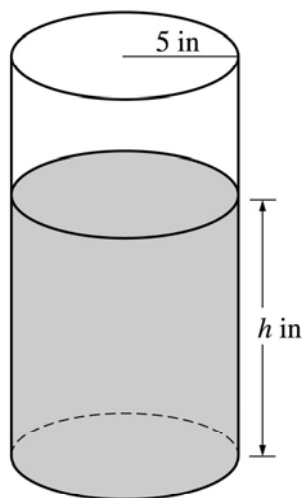
Number of problems—3

No calculator is allowed for these problems.



4. Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f' , the derivative of f , consists of one line segment and a semicircle, as shown above.
- (a) On what intervals, if any, is f increasing? Justify your answer.
 - (b) Find the x -coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.
 - (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.
 - (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.
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5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t , measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

- (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t .

- (c) At what time t is the coffeepot empty?

6. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

for all real numbers x .

- (a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

- (b) Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.

- (c) Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.

END OF EXAMINATION