

AP[®] Calculus BC Exam

SECTION I: Multiple Choice

2013

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time

1 hour, 45 minutes

Number of Questions

45

Percent of Total Score

50%

Writing Instrument

Pencil required

Part A

Number of Questions

28

Time

55 minutes

Electronic Device

None allowed

Part B

Number of Questions

17

Time

50 minutes

Electronic Device

Graphing calculator
required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question Sample Answer

Chicago is a

(A) ☒ (B) ☐ (C) ☐ (D) ☐ (E) ☐

(A) state

(B) city

(C) country

(D) continent

(E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

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Form I
Form Code 4JBP4-Q-S

68

PLACE SEAL HERE

DO NOT seal answer sheet inside

4. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$?

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\frac{7}{3}(x - 1)$

(C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$

(E) $y + 3 = \frac{11}{3}(x - 1)$

5. If $y = \frac{1}{2}x^{4/5} - \frac{3}{x^5}$, then $\frac{dy}{dx} =$

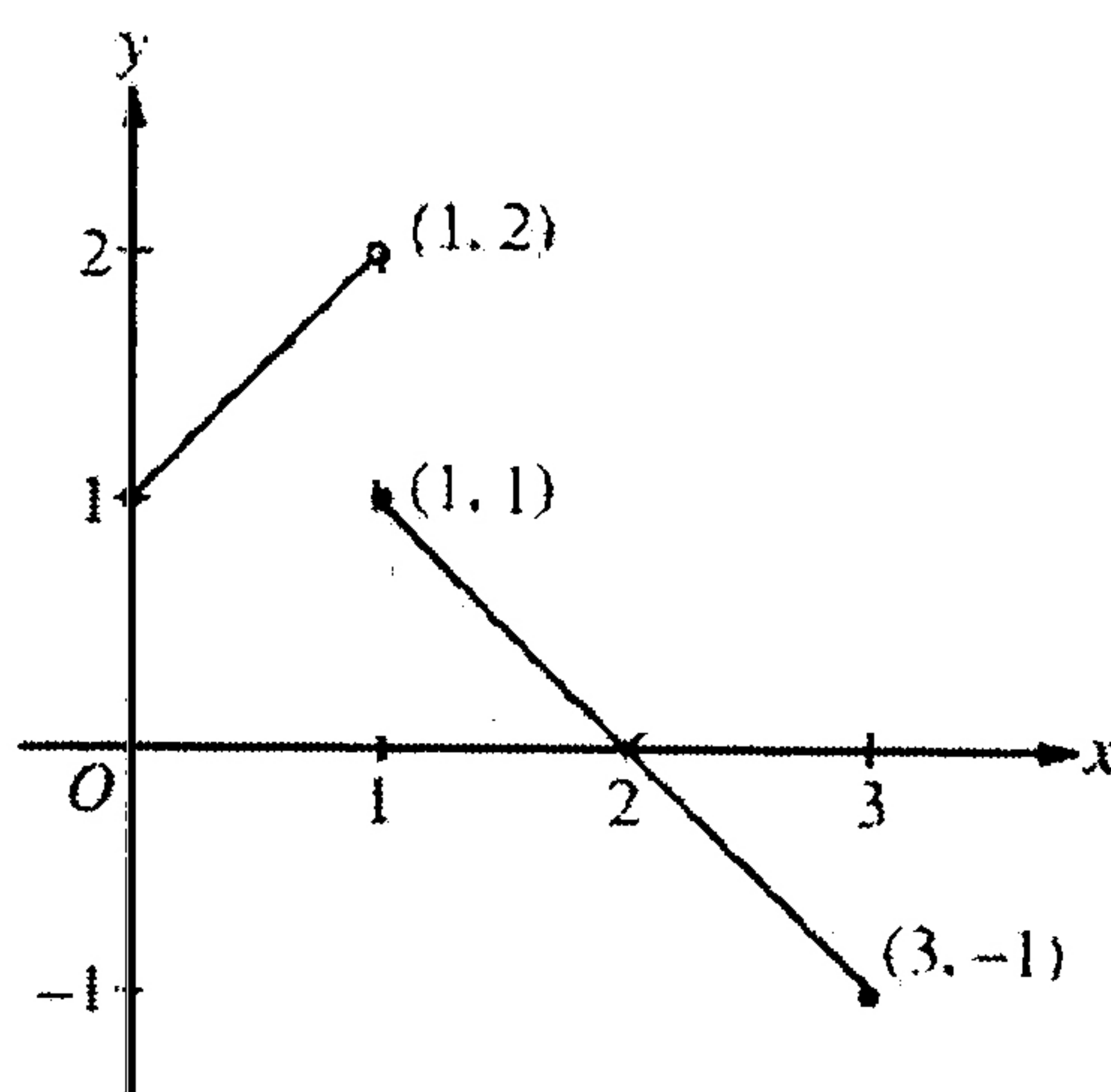
(A) $\frac{2}{5x^{1/5}} + \frac{15}{x^6}$

(B) $\frac{2}{5x^{1/5}} + \frac{15}{x^4}$

(C) $\frac{2}{5x^{1/5}} - \frac{3}{5x^4}$

(D) $\frac{2x^{1/5}}{5} + \frac{15}{x^6}$

(E) $\frac{2x^{1/5}}{5} - \frac{3}{5x^4}$



Graph of f

6. The graph of the function f consists of two line segments, as shown in the figure above. The value of $\int_0^3 |f(x)| dx$ is

(A) $-\frac{3}{2}$

(B) $\frac{1}{2}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

(E) nonexistent

7. A population y changes at a rate modeled by the differential equation $\frac{dy}{dt} = 0.2y(1000 - y)$, where t is measured in years. What are all values of y for which the population is increasing at a decreasing rate?
- (A) 500 only
 (B) $0 < y < 500$ only
 (C) $500 < y < 1000$ only
 (D) $0 < y < 1000$
 (E) $y > 1000$
8. Which of the following gives the length of the path described by the parametric equations $x(t) = 2 + 3t$ and $y(t) = 1 + t^2$ from $t = 0$ to $t = 1$?
- (A) $\int_0^1 \sqrt{1 + \frac{4t^2}{9}} dt$
 (B) $\int_0^1 \sqrt{1 + 4t^2} dt$
 (C) $\int_0^1 \sqrt{3 + 3t + t^2} dt$
 (D) $\int_0^1 \sqrt{9 + 4t^2} dt$
 (E) $\int_0^1 \sqrt{(2 + 3t)^2 + (1 + t^2)^2} dt$
9. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length, starting at $x = 1$?
- (A) 0 (B) 1 (C) 2.75 (D) 3 (E) 6
10. If $\int_0^k \frac{x}{x^2 + 4} dx = \frac{1}{2} \ln 4$, where $k > 0$, then $k =$
- (A) 0 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{12}$ (E) $\frac{1}{2} \tan(\ln \sqrt{2})$

11. The third-degree Taylor polynomial for a function f about $x = 4$ is $\frac{(x-4)^3}{512} - \frac{(x-4)^2}{64} + \frac{(x-4)}{4} + 2$. What is the value of $f'''(4)$?

- (A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$ (C) $\frac{1}{512}$ (D) $\frac{3}{256}$ (E) $\frac{81}{256}$

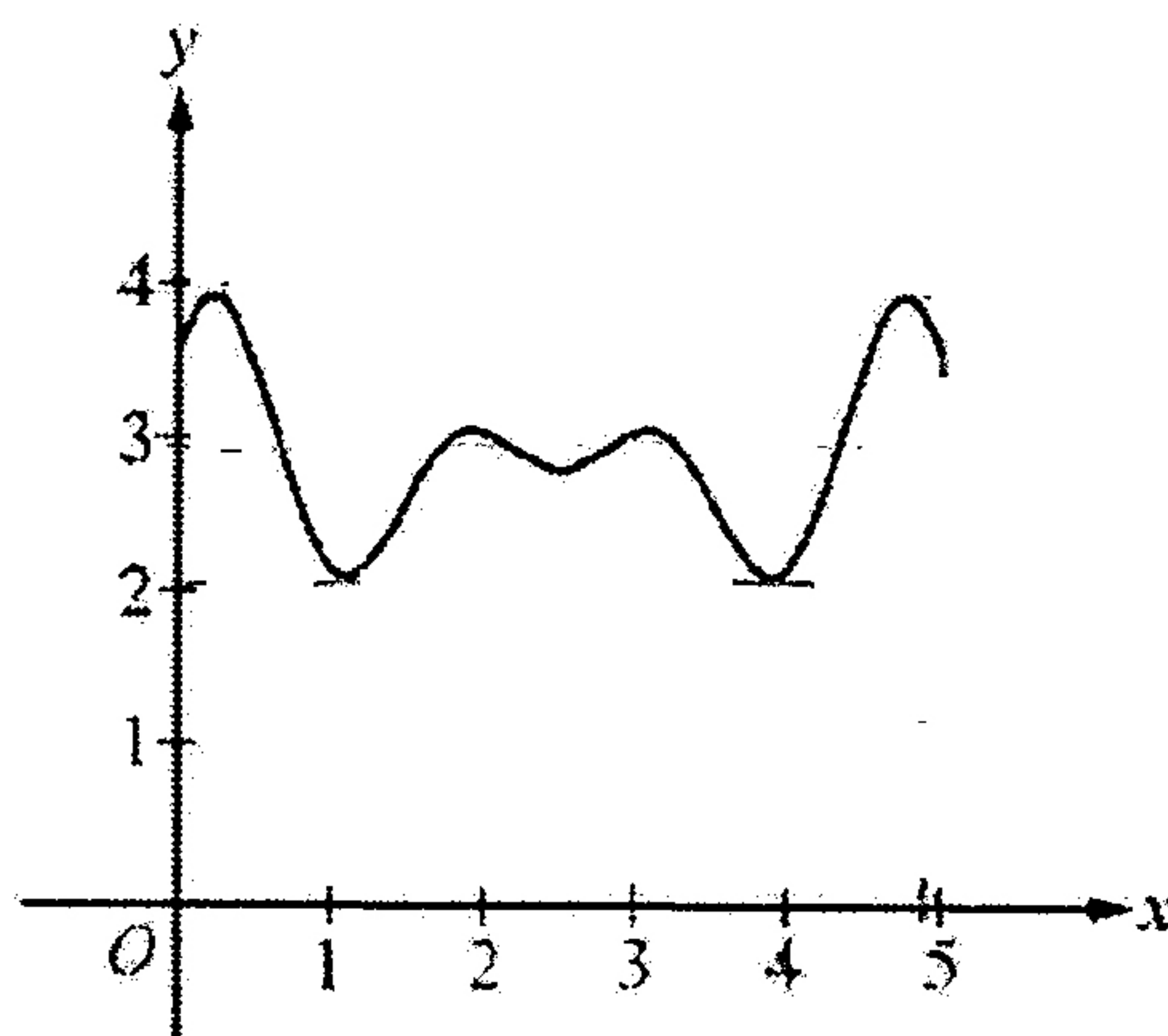
12. For which of the following does $\lim_{x \rightarrow \infty} f(x) = 0$?

I $f(x) = \frac{\ln x}{x^{99}}$

II. $f(x) = \frac{e^x}{\ln x}$

III $f(x) = \frac{x^{99}}{e^x}$

- (A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only



Graph of f'

13. The graph of f' , the derivative of f , is shown in the figure above. If $f(0) = 20$, which of the following could be the value of $f(5)$?

- (A) 15 (B) 20 (C) 25 (D) 35 (E) 40

14. If a and b are positive constants, then $\lim_{x \rightarrow \infty} \frac{\ln(bx+1)}{\ln(ax^2+3)} =$

- (A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{2}ab$ (D) 2 (E) ∞

15. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(x + \frac{3}{2}\right)^n$ converges?

(A) $-\frac{5}{2} < x < -\frac{1}{2}$

(B) $-\frac{5}{2} < x \leq -\frac{1}{2}$

(C) $-\frac{5}{2} \leq x < -\frac{1}{2}$

(D) $-\frac{1}{2} < x < \frac{1}{2}$

(E) $x \leq -\frac{1}{2}$

16. For $0 < P < 100$, which of the following is an antiderivative of $\frac{1}{100P - P^2}$?

(A) $\frac{1}{100} \ln(P) - \frac{1}{100} \ln(100 - P)$

(B) $\frac{1}{100} \ln(P) + \frac{1}{100} \ln(100 - P)$

(C) $100 \ln(P) - 100 \ln(100 - P)$

(D) $\ln(100P - P^2)$

(E) $\frac{1}{50P^2 - \frac{P^3}{3}}$

17. If $\lim_{h \rightarrow 0} \frac{\arcsin(a + h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a ?

(A) $\frac{\sqrt{2}}{2}$

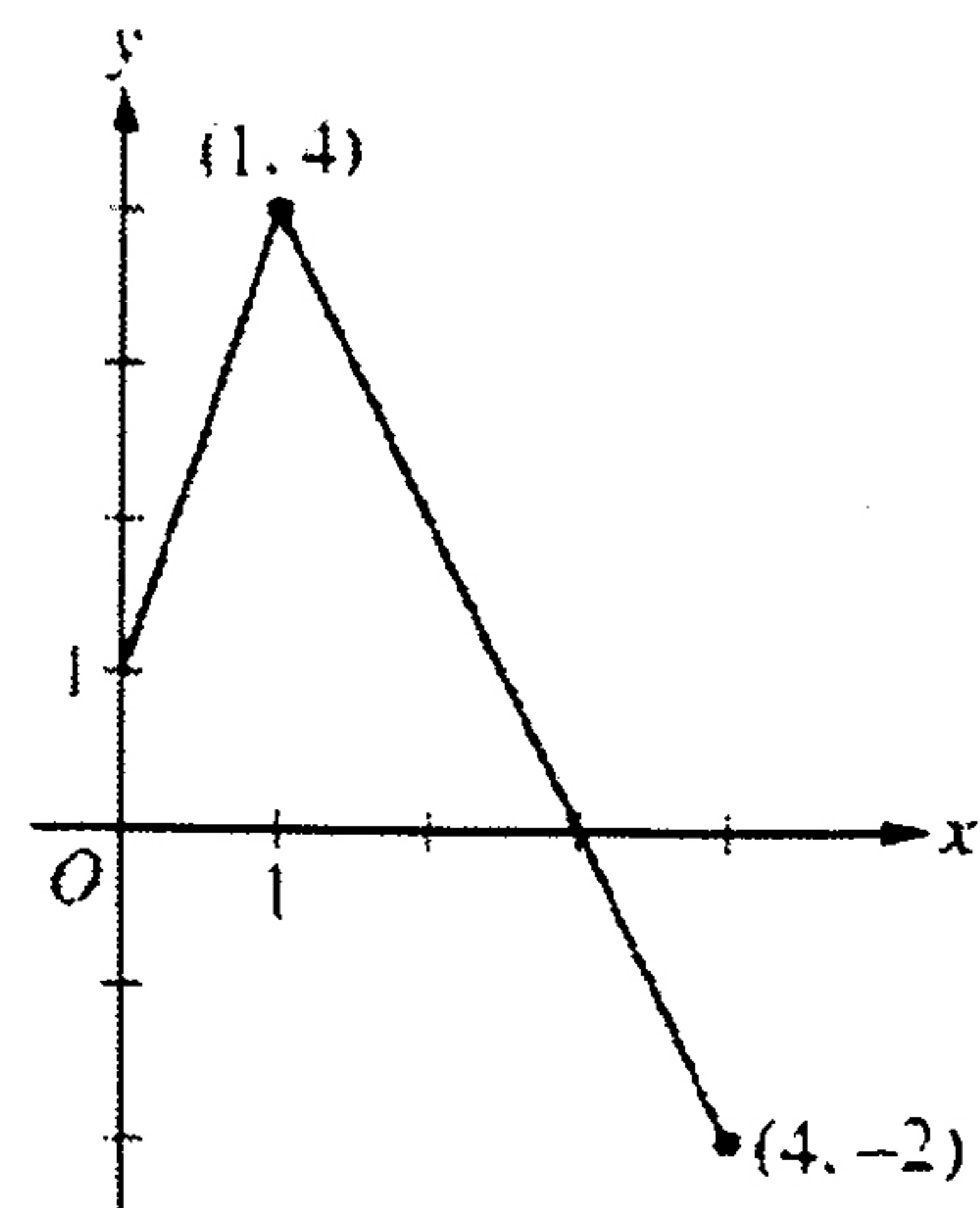
(B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$

(D) $\frac{1}{2}$

(E) 2

Graph of f



18. The graph of the function f , consisting of two line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + 1$, and let h be the function given by $h(x) = f(g(x))$. What is the value of $h'(1)$?

(A) -4

(B) -2

(C) 4

(D) 6

(E) nonexistent

19. Which of the following is the Maclaurin series for $\frac{1}{(1-x)^2}$

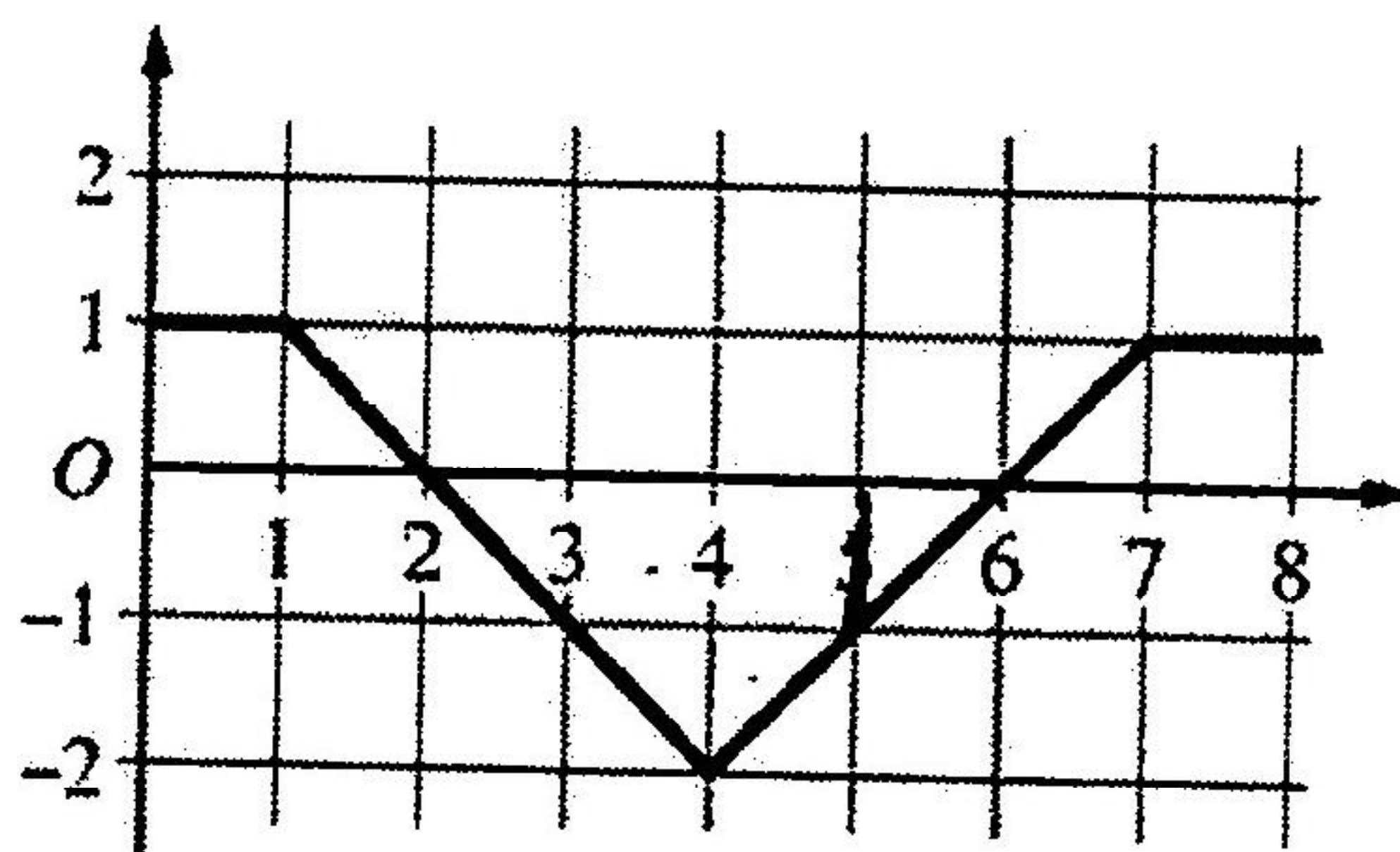
(A) $1 - x + x^2 - x^3 + \dots$

(B) $1 - 2x + 3x^2 - 4x^3 + \dots$

(C) $1 + 2x + 3x^2 + 4x^3 + \dots$

(D) $1 + x^2 + x^4 + x^6 + \dots$

(E) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$



Graph of f

20. The graph of the function f in the figure above consists of four line segments. Let g be the function defined by $g(x) = \int_0^x f(t) dt$. Which of the following is an equation of the line tangent to the graph of g at $x = 5$?

(A) $y + 1 = x - 5$

(B) $y - 2 = x - 5$

(C) $y - 2 = -1(x - 5)$

(D) $y + 2 = x - 5$

(E) $y + 2 = -1(x - 5)$

21. At time $t \geq 0$, a cube has volume $V(t)$ and edges of length $x(t)$. If the volume of the cube decreases at a rate proportional to its surface area, which of the following differential equations could describe the rate at which the volume of the cube decreases?

(A) $\frac{dV}{dt} = -1.2x^2$

(B) $\frac{dV}{dt} = -1.2x^3$

(C) $\frac{dV}{dt} = -1.2x^2t$

(D) $\frac{dV}{dt} = -1.2t^2$

(E) $\frac{dV}{dt} = -1.2V^2$

22. Which of the following is true about the curve $x^2 - xy + y^2 = 3$ at the point $(2, 1)$?

(A) $\frac{dy}{dx}$ exists at $(2, 1)$, but there is no tangent line at that point.

(B) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is horizontal.

(C) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is neither horizontal nor vertical.

(D) $\frac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is vertical.

(E) $\frac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is horizontal.

23. What is the coefficient of x^6 in the Taylor series for $\frac{e^{3x^2}}{2}$ about $x = 0$?

(A) $\frac{1}{1440}$ (B) $\frac{81}{160}$ (C) $\frac{9}{4}$ (D) $\frac{9}{2}$ (E) $\frac{27}{2}$

24. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7 (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

- (A) $y = -x - \ln 4$
- (B) $y = x - \ln 4$
- (C) $y = -\ln(-e^x + 5)$
- (D) $y = -\ln(e^x + 3)$
- (E) $y = \ln(e^x + 3)$

26. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$

II. $\sum_{n=1}^{\infty} e^{-n}$

III. $\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I and III only

27. If $\int_1^x f(t) dt = \frac{20x}{\sqrt{4x^2 + 21}} - 4$, then $\int_1^{\infty} f(t) dt$ is

- (A) 6
- (B) 1
- (C) -3
- (D) -4
- (E) divergent

28. If $x = t^2 - 1$ and $y = \ln t$, what is $\frac{d^2y}{dx^2}$ in terms of t ?

- (A) $-\frac{1}{2t^4}$
- (B) $\frac{1}{2t^4}$
- (C) $-\frac{1}{t^3}$
- (D) $-\frac{1}{2t^2}$
- (E) $\frac{1}{2t^2}$

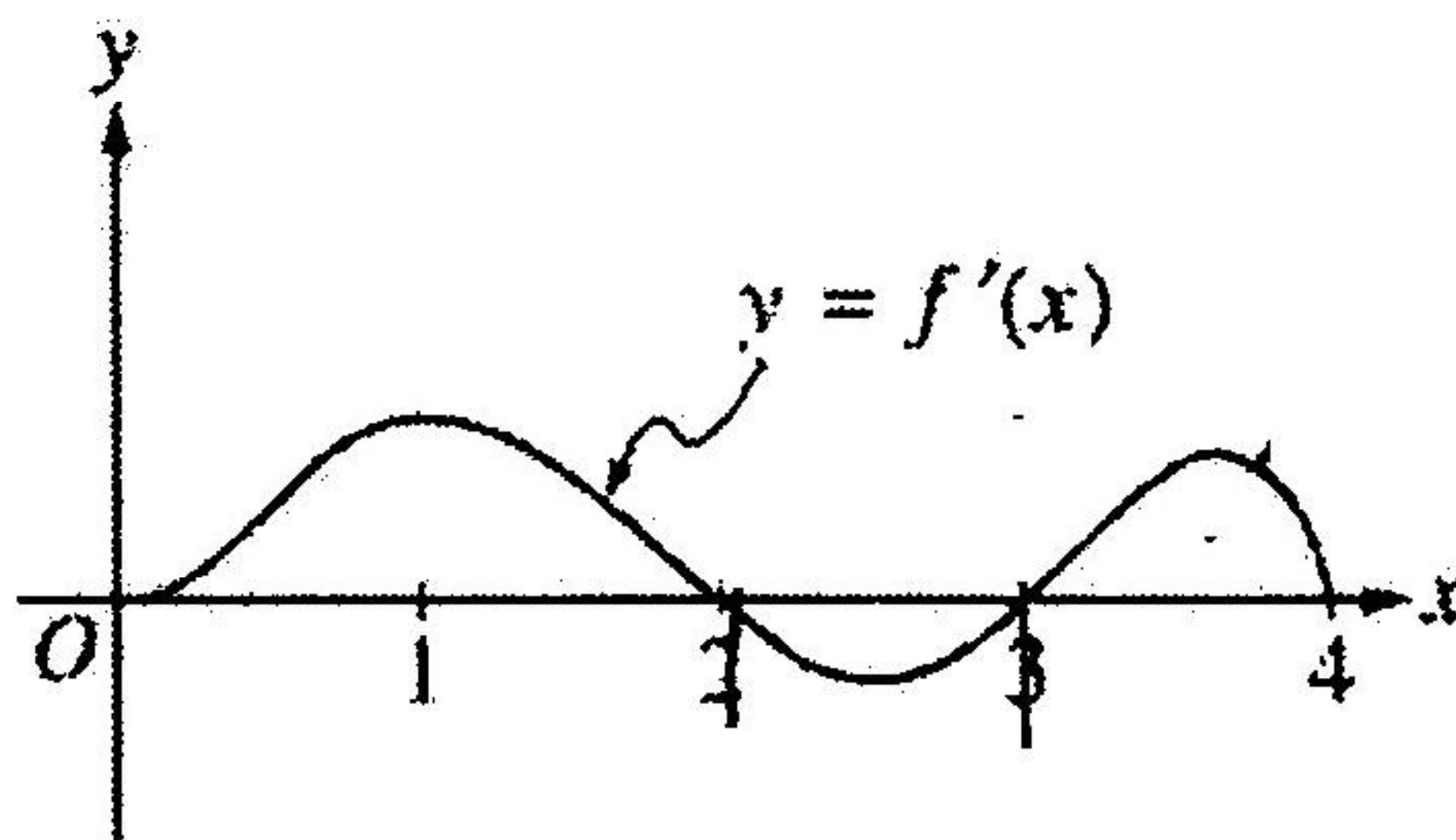
Calculator Is Required In This Section

76. Let f be a function whose derivative is given by $f'(x) = \ln(x^4 + 5x^3 + x^2 - 7x + 28)$. On the open interval $(-4, 1)$, at which of the following values of x does f attain a relative maximum?
- (A) -3.623 only
 (B) -0.871 only
 (C) -3.623 and -3.284
 (D) -3.459 and 0.581 only
 (E) -3.459 , -0.871 , and 0.581

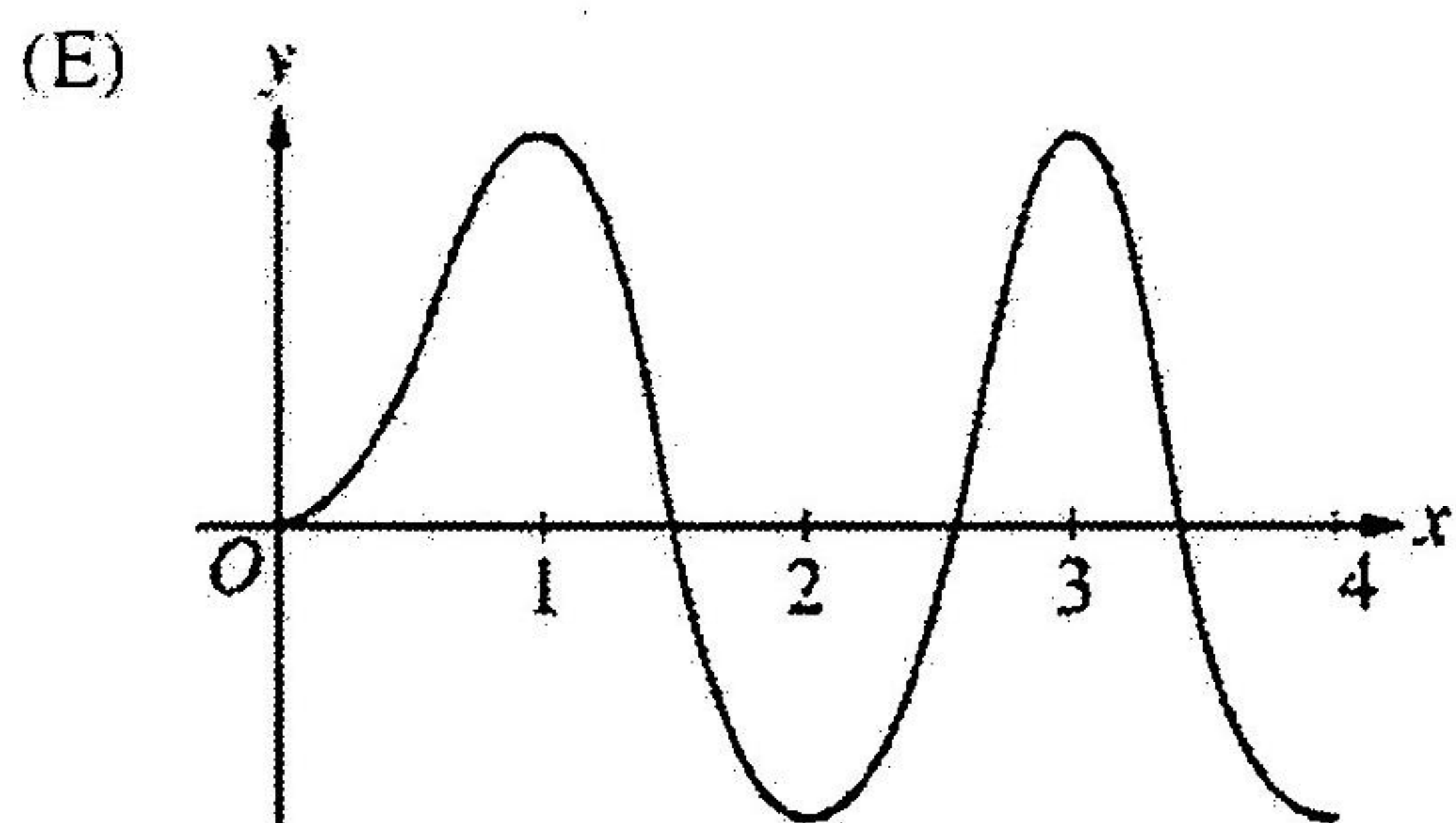
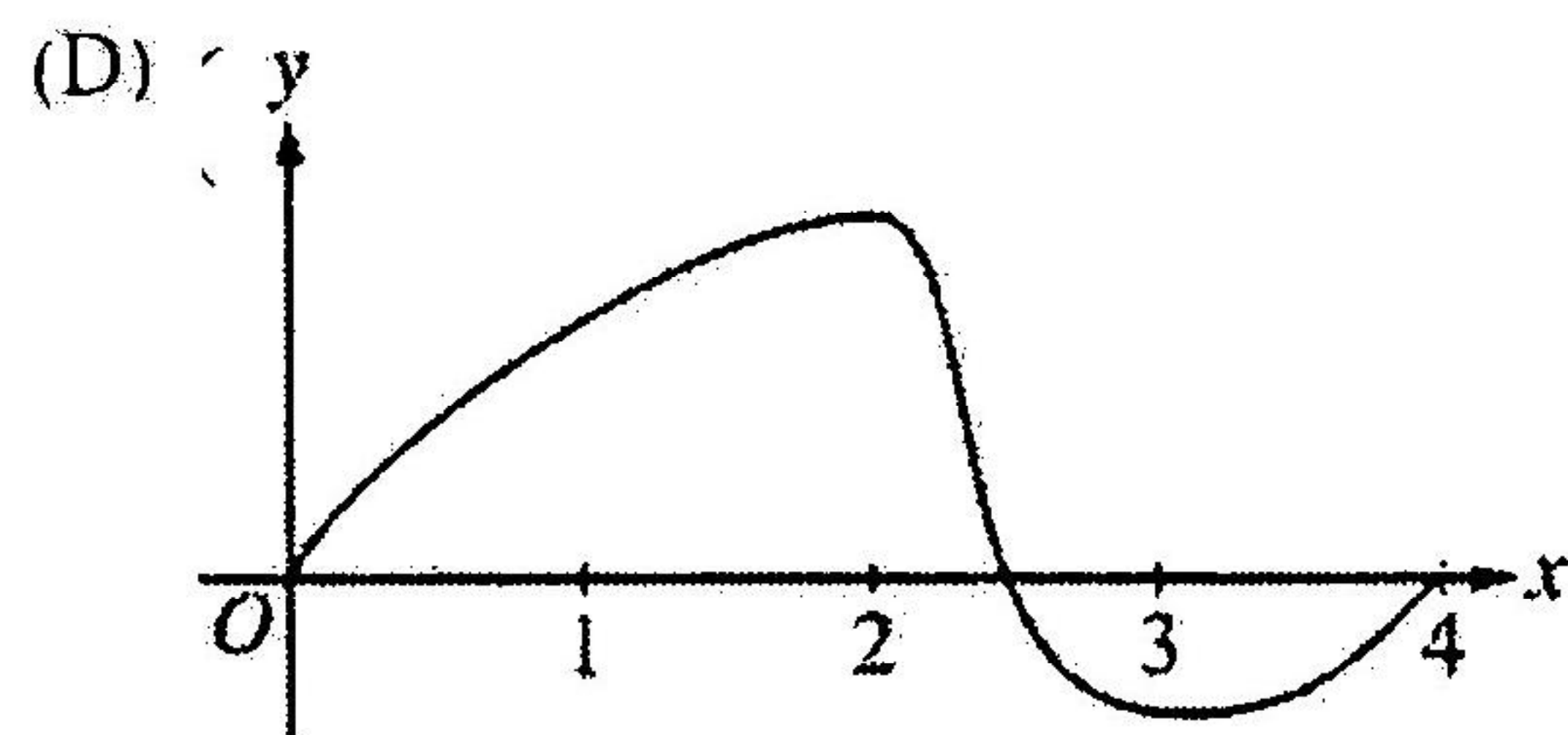
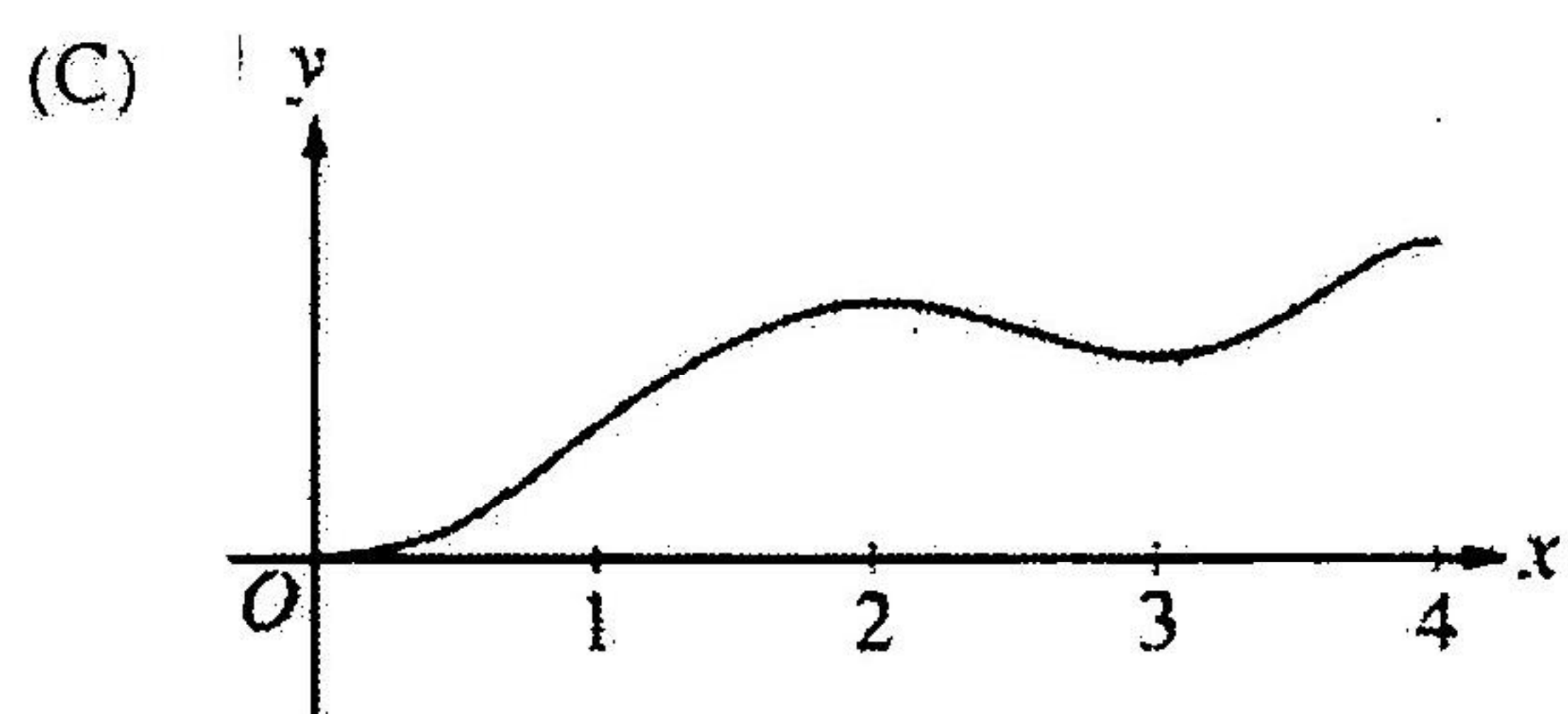
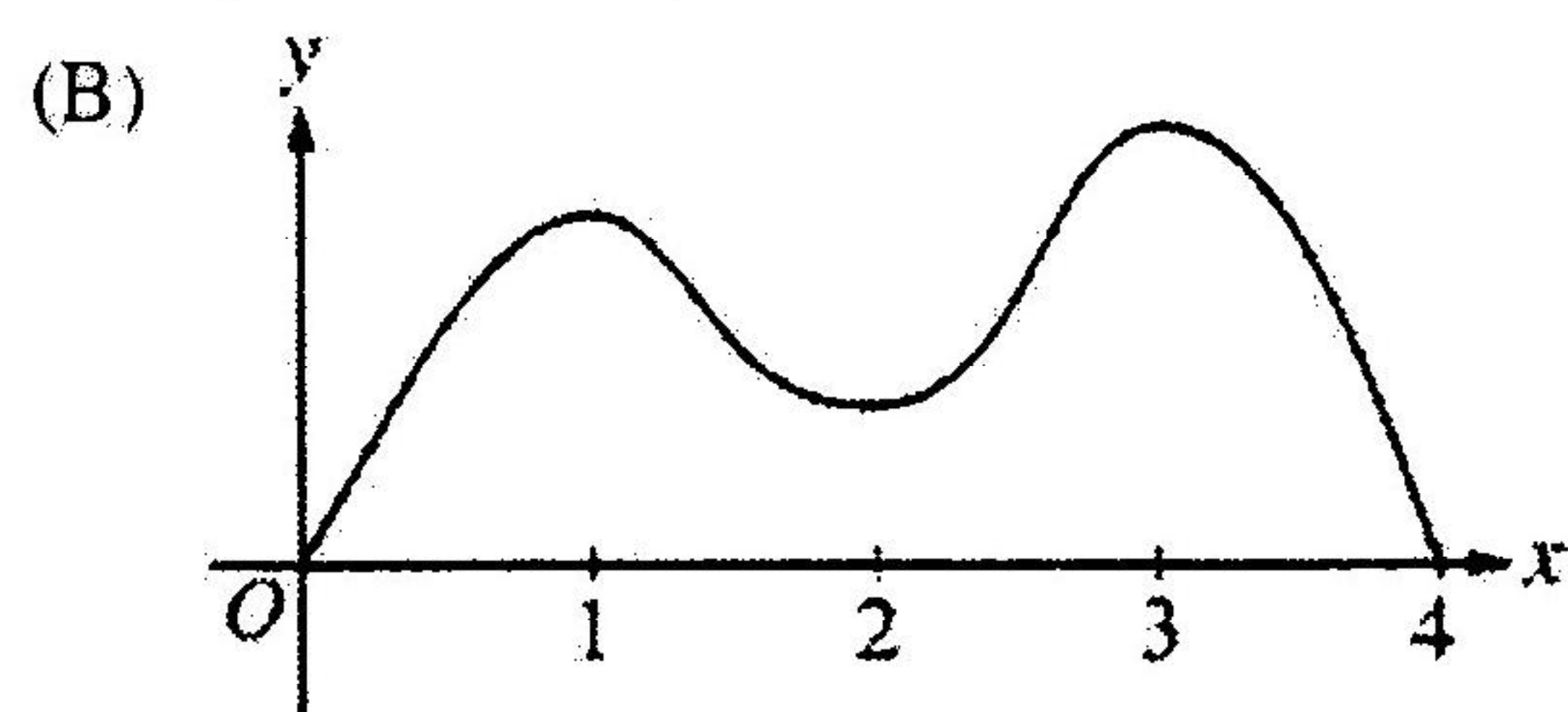
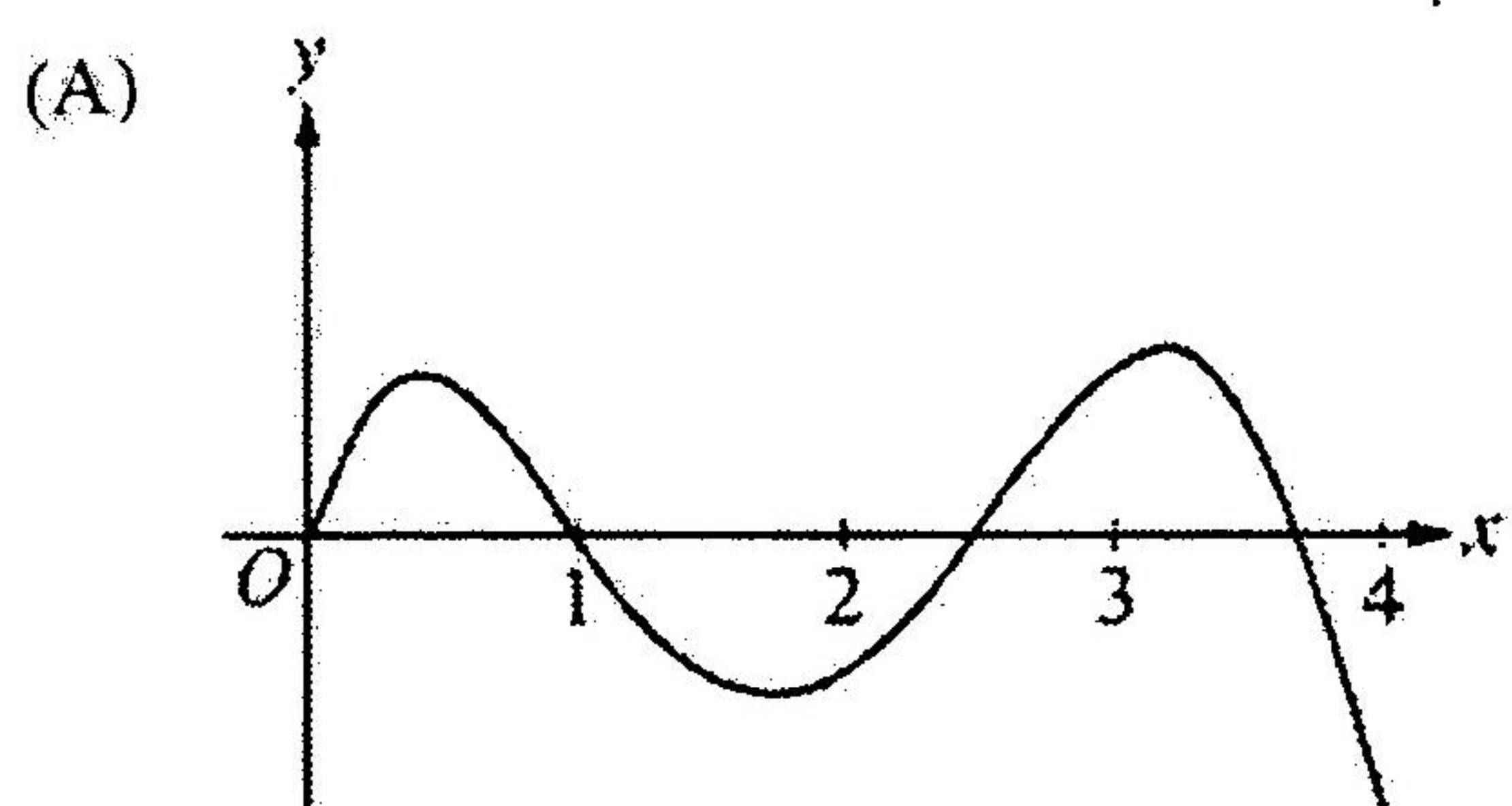
a	$\lim_{x \rightarrow a^-} f(x)$	$\lim_{x \rightarrow a^+} f(x)$	$f(a)$
-1	4	6	4
0	-3	-3	5
1	2	2	2

77. The function f has the properties indicated in the table above. Which of the following must be true?
- (A) f is continuous at $x = -1$
 (B) f is continuous at $x = 0$
 (C) f is continuous at $x = 1$
 (D) f is differentiable at $x = 0$
 (E) f is differentiable at $x = 1$.

78. What is the area of the region in the first quadrant enclosed by the graphs of $y = \sin(2x)$ and $y = x$
- (A) 0.208 (B) 0.210 (C) 0.266 (D) 0.660 (E) 0.835



79. The figure above shows the graph of f' , the derivative of the function f . If $f(0) = 0$, which of the following could be the graph of f ?



80. The volume of a certain cone for which the sum of its radius, r , and height is constant is given by $V = \frac{1}{3}\pi r^2(10 - r)$. The rate of change of the radius of the cone with respect to time is 6. In terms of r , what is the rate of change of the volume of the cone with respect to time?

- (A) $-24\pi r$ (B) $6\pi r$ (C) $\frac{20}{3}\pi r - \pi r^2$ (D) $16\pi r - \frac{4}{3}\pi r^2$ (E) $40\pi r - 6\pi r^2$

81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit ($^{\circ}\text{F}$). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

82. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

(A) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(C) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(D) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $b_n \leq a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.

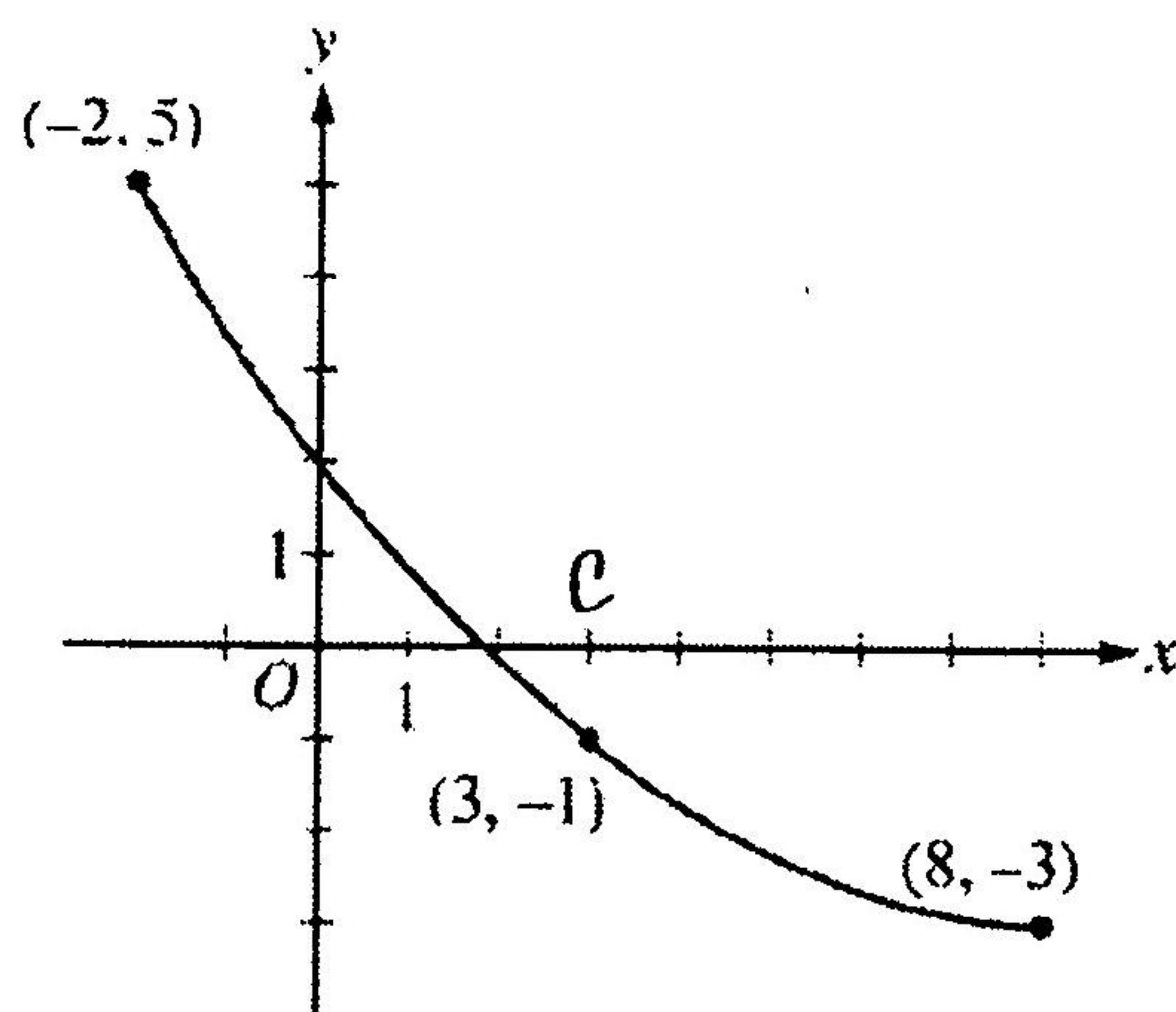
x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

83. The table above gives selected values for a continuous function f . If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x) dx$?

(A) 50 (B) 62 (C) 77 (D) 100 (E) 154

84. Let f be a function with derivative given by $f'(x) = x^3 - 5x^2 + e^x$. On which of the following intervals is the graph of f concave down?

- (A) $(-\infty, 0.117)$ only
 (B) $(-\infty, 1.144)$
 (C) $(0.116, 2.062)$
 (D) $(0.673, 2.863)$
 (E) $(2.863, \infty)$



Graph of f

85. A portion of the graph of a differentiable function f is shown above. If the value $c = 3$ satisfies the conclusion of the Mean Value Theorem applied to f on the open interval $-2 < x < 8$, what is the slope of the line tangent to the graph of f at $x = 3$?

- (A) $-\frac{7}{5}$ (B) $-\frac{5}{4}$ (C) $-\frac{4}{5}$ (D) $-\frac{5}{7}$ (E) $-\frac{1}{5}$

86. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x , which of the following could be a table of values for f ?

(A)

x	$f(x)$
-1	4
0	3
1	1

(B)

x	$f(x)$
-1	4
0	4
1	4

(C)

x	$f(x)$
-1	4
0	5
1	6

(D)

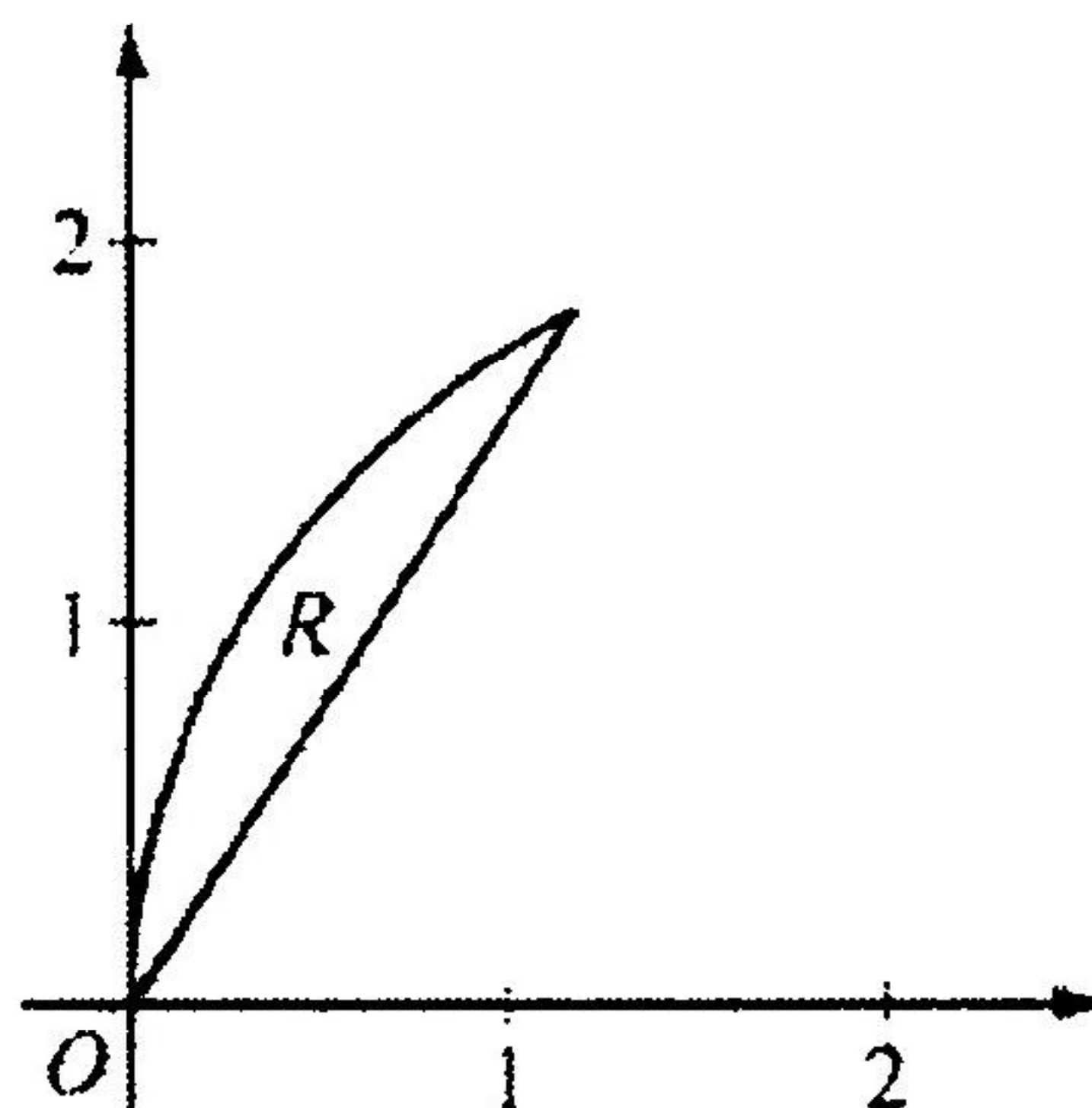
x	$f(x)$
-1	4
0	5
1	7

(E)

x	$f(x)$
-1	4
0	6
1	7

87. The position of a particle moving in the xy -plane is given by the parametric functions $x(t)$ and $y(t)$ for which $x'(t) = t \sin t$ and $y'(t) = 5e^{-3t} + 2$. What is the slope of the line tangent to the path of the particle at the point at which $t = 2$?

(A) 0.904 (B) 1.107 (C) 1.819 (D) 2.012 (E) 3.660



88. Let R be the region in the first quadrant that is bounded above by the polar curve $r = 4\cos \theta$ and below by the line $\theta = 1$, as shown in the figure above. What is the area of R ?

(A) 0.317 (B) 0.465 (C) 0.929 (D) 2.618 (E) 5.819

89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y -axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137

90. Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \frac{(-1)^n}{\sqrt{n} + (-1)^n}$?

I. The series is alternating.

II. $|a_{n+1}| \leq |a_n|$ for all $n \geq 2$

III. $\lim_{n \rightarrow \infty} a_n = 0$

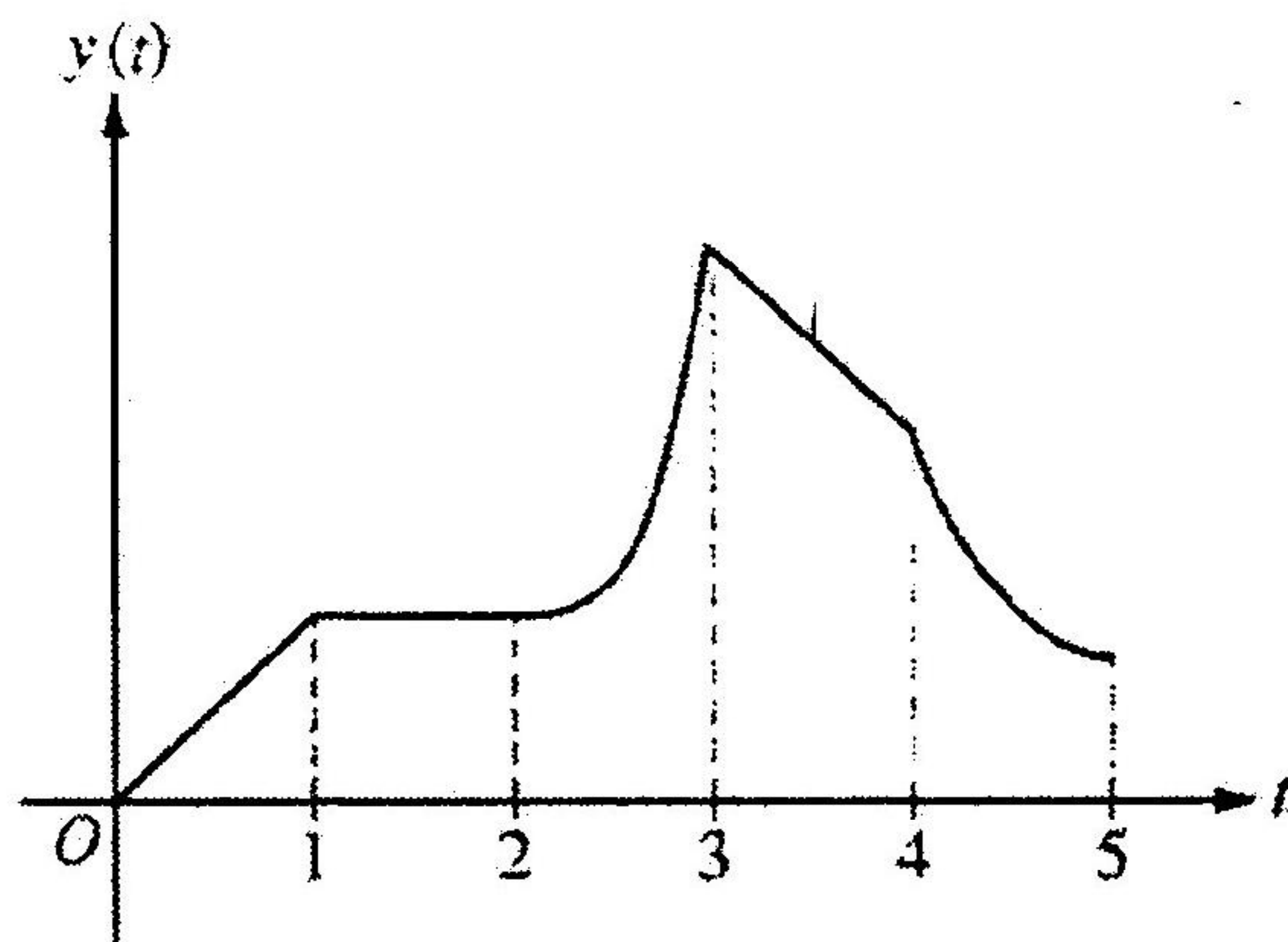
(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III



91. A particle moves along the y -axis. The graph of the particle's position $y(t)$ at time t is shown above for $0 \leq t \leq 5$. For what values of t is the velocity of the particle negative and the acceleration positive?

- (A) $0 < t < 1$ (B) $1 < t < 2$ (C) $2 < t < 3$ (D) $3 < t < 4$ (E) $4 < t < 5$

92. If f is a function such that $f'(x) = -f(x)$, then $\int x f(x) dx =$

- (A) $f(x)(x+1) + C$
 (B) $-f(x)(x+1) + C$
 (C) $\frac{x^2}{2} f(x) + C$
 (D) $-\frac{x^2}{2} f(x) + C$
 (E) $-\frac{x^2}{2} f(x) \left(1 + \frac{x}{3}\right) + C$

2013年BC全套真题



AP[®] Calculus BC 2013 Free-Response Questions

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2013年BC全套真题
2013 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A

Time—30 minutes

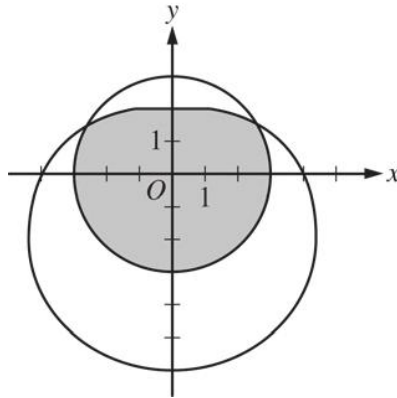
Number of problems—2

A graphing calculator is required for these problems.

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \leq t \leq 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \leq t \leq 8$, the plant processes gravel at a constant rate of 100 tons per hour.
- (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.
 - (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
 - (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.
 - (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.
-

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2013 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



2. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin \theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.
- (a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin \theta$. Find the area of S .
- (b) A particle moves along the polar curve $r = 4 - 2\sin \theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \leq t \leq 2$ for which the x -coordinate of the particle's position is -1 .
- (c) For the particle described in part (b), find the position vector in terms of t . Find the velocity vector at time $t = 1.5$.
-

END OF PART A OF SECTION II

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CALCULUS BC
SECTION II, Part B

Time—60 minutes

Number of problems—4

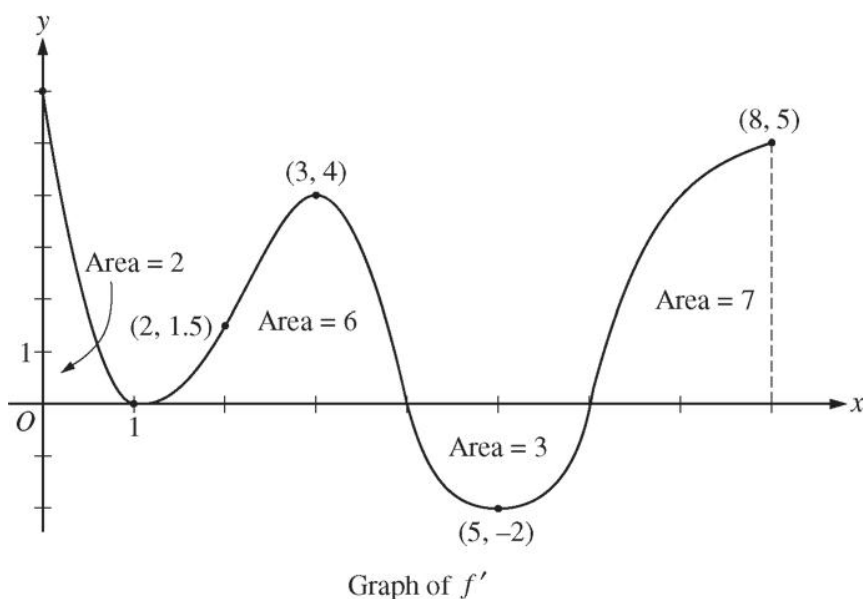
No calculator is allowed for these problems.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.
-

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4. The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the closed interval $0 \leq x \leq 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x -axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.
- Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.
 - Determine the absolute minimum value of f on the closed interval $0 \leq x \leq 8$. Justify your answer.
 - On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.
 - The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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5. Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.
- (a) Find $\lim_{x \rightarrow 0} \frac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.
- (b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.
- (c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.
-

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6. A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the n th-degree Taylor polynomial for f about $x = 0$.
- (a) It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$.
- (b) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.
- (c) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.
-

STOP

END OF EXAM

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Answer Key for AP Calculus BC Practice Exam, Section I

Multiple-Choice Questions	
Question #	Key
1	C
2	C
3	D
4	E
5	A
6	D
7	C
8	D
9	D
10	D
11	D
12	E
13	D
14	B
15	B
16	A
17	B
18	A
19	C
20	E
21	A
22	D

23	C
24	A
25	C
26	D
27	A
28	A
76	A
77	C
78	B
79	C
80	E
81	A
82	C
83	B
84	C
85	C
86	E
87	B
88	B
89	E
90	D
91	E
92	B

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Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \leq t \leq 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \leq t \leq 8$, the plant processes gravel at a constant rate of 100 tons per hour.

- Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.
- Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
- Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.
- What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

(a) $G'(5) = -24.588$ (or -24.587)

The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time $t = 5$ hours.

2 : $\begin{cases} 1 : G'(5) \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^8 G(t) dt = 825.551$ tons

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $G(5) = 98.140764 < 100$

At time $t = 5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed.
Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t = 5$.

2 : $\begin{cases} 1 : \text{compares } G(5) \text{ to } 100 \\ 1 : \text{conclusion} \end{cases}$

- (d) The amount of unprocessed gravel at time t is given by

$$A(t) = 500 + \int_0^t (G(s) - 100) ds.$$

$$A'(t) = G(t) - 100 = 0 \Rightarrow t = 4.923480$$

t	$A(t)$
0	500
4.92348	635.376123
8	525.551089

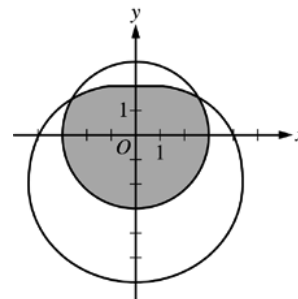
The maximum amount of unprocessed gravel at the plant during this workday is 635.376 tons.

3 : $\begin{cases} 1 : \text{considers } A'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

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Question 2

The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin \theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.



- (a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin \theta$. Find the area of S .
- (b) A particle moves along the polar curve $r = 4 - 2\sin \theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \leq t \leq 2$ for which the x -coordinate of the particle's position is -1 .
- (c) For the particle described in part (b), find the position vector in terms of t . Find the velocity vector at time $t = 1.5$.

(a) $\text{Area} = 6\pi + \frac{1}{2} \int_{\pi/6}^{5\pi/6} (4 - 2\sin \theta)^2 d\theta = 24.709$ (or 24.708)

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$

(b) $x = r \cos \theta \Rightarrow x(\theta) = (4 - 2\sin \theta) \cos \theta$
 $x(t) = (4 - 2\sin(t^2)) \cos(t^2)$
 $x(t) = -1$ when $t = 1.428$ (or 1.427)

3 : $\begin{cases} 1 : x(\theta) \text{ or } x(t) \\ 1 : x(\theta) = -1 \text{ or } x(t) = -1 \\ 1 : \text{answer} \end{cases}$

(c) $y = r \sin \theta \Rightarrow y(\theta) = (4 - 2\sin \theta) \sin \theta$
 $y(t) = (4 - 2\sin(t^2)) \sin(t^2)$

3 : $\begin{cases} 2 : \text{position vector} \\ 1 : \text{velocity vector} \end{cases}$

Position vector = $\langle x(t), y(t) \rangle$
 $= \langle (4 - 2\sin(t^2)) \cos(t^2), (4 - 2\sin(t^2)) \sin(t^2) \rangle$

$v(1.5) = \langle x'(1.5), y'(1.5) \rangle$
 $= \langle -8.072, -1.673 \rangle$ (or $\langle -8.072, -1.672 \rangle$)

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Question 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

- (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) \approx \frac{C(4) - C(3)}{4 - 3} = \frac{12.8 - 11.2}{1} = 1.6$ ounces/min

2 : $\begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

(b) C is differentiable $\Rightarrow C$ is continuous (on the closed interval)
 $\frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = 2$

Therefore, by the Mean Value Theorem, there is at least one time t , $2 < t < 4$, for which $C'(t) = 2$.

2 : $\begin{cases} 1 : \frac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c) $\frac{1}{6} \int_0^6 C(t) dt \approx \frac{1}{6} [2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$
 $= \frac{1}{6} (2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$
 $= \frac{1}{6} (60.6) = 10.1$ ounces

$\frac{1}{6} \int_0^6 C(t) dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \leq t \leq 6$ minutes.

3 : $\begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$

(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$
 $B'(5) = 6.4e^{-0.4(5)} = \frac{6.4}{e^2}$ ounces/min

2 : $\begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$

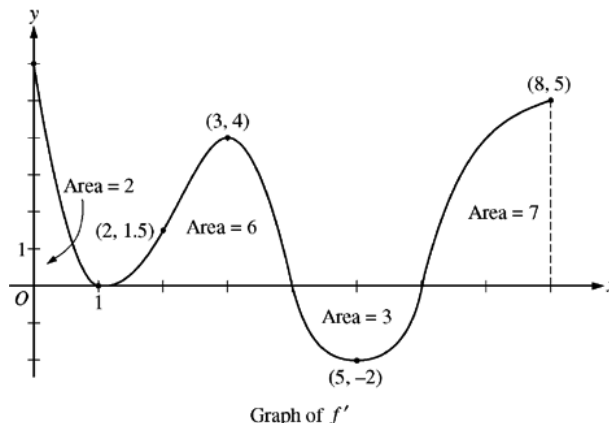
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Question 4

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the closed interval $0 \leq x \leq 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x -axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



- Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.
- Determine the absolute minimum value of f on the closed interval $0 \leq x \leq 8$. Justify your answer.
- On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.
- The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

- (a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.

- (b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint.

$$\begin{aligned} f(0) &= f(8) + \int_8^0 f'(x) \, dx \\ &= f(8) - \int_0^8 f'(x) \, dx = 4 - 12 = -8 \end{aligned}$$

$$\begin{aligned} f(6) &= f(8) + \int_8^6 f'(x) \, dx \\ &= f(8) - \int_6^8 f'(x) \, dx = 4 - 7 = -3 \end{aligned}$$

$$f(8) = 4$$

The absolute minimum value of f on the closed interval $[0, 8]$ is -8 .

- (c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.

- (d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$

$$g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$$

1 : answer with justification

3 : $\begin{cases} 1 : \text{considers } x = 0 \text{ and } x = 6 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

3 : $\begin{cases} 2 : g'(x) \\ 1 : \text{answer} \end{cases}$

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Question 5

Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

- (a) Find $\lim_{x \rightarrow 0} \frac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.
- (b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.
- (c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) $\lim_{x \rightarrow 0} (f(x) + 1) = -1 + 1 = 0$ and $\lim_{x \rightarrow 0} \sin x = 0$

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 0} \frac{f(x) + 1}{\sin x} = \lim_{x \rightarrow 0} \frac{f'(x)}{\cos x} = \frac{f'(0)}{\cos 0} = \frac{(-1)^2 \cdot 2}{1} = 2$$

$$2 : \begin{cases} 1 : \text{L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$$

(b) $f\left(\frac{1}{4}\right) \approx f(0) + f'(0)\left(\frac{1}{4}\right)$
 $= -1 + (2)\left(\frac{1}{4}\right) = -\frac{1}{2}$

$$2 : \begin{cases} 1 : \text{Euler's method} \\ 1 : \text{answer} \end{cases}$$

$$f\left(\frac{1}{2}\right) \approx f\left(\frac{1}{4}\right) + f'\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$$

$$= -\frac{1}{2} + \left(-\frac{1}{2}\right)^2 \left(2 \cdot \frac{1}{4} + 2\right)\left(\frac{1}{4}\right) = -\frac{11}{32}$$

(c) $\frac{dy}{dx} = y^2(2x + 2)$
 $\frac{dy}{y^2} = (2x + 2) dx$

$$\int \frac{dy}{y^2} = \int (2x + 2) dx$$

$$-\frac{1}{y} = x^2 + 2x + C$$

$$-\frac{1}{-1} = 0^2 + 2 \cdot 0 + C \Rightarrow C = 1$$

$$-\frac{1}{y} = x^2 + 2x + 1$$

$$y = -\frac{1}{x^2 + 2x + 1} = -\frac{1}{(x + 1)^2}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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Question 5

Note: This solution is valid for $x > -1$.

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Question 6

A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the n th-degree Taylor polynomial for f about $x = 0$.

- (a) It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$.
- (b) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.
- (c) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.

(a) $P_1(x) = f(0) + f'(0)x = -4 + f'(0)x$

$$P_1\left(\frac{1}{2}\right) = -4 + f'(0) \cdot \frac{1}{2} = -3$$

$$f'(0) \cdot \frac{1}{2} = 1$$

$$f'(0) = 2$$

$$2 : \begin{cases} 1 : \text{uses } P_1(x) \\ 1 : \text{verifies } f'(0) = 2 \end{cases}$$

(b) $P_3(x) = -4 + 2x + \left(-\frac{2}{3}\right) \cdot \frac{x^2}{2!} + \frac{1}{3} \cdot \frac{x^3}{3!}$

$$= -4 + 2x - \frac{1}{3}x^2 + \frac{1}{18}x^3$$

$$3 : \begin{cases} 1 : \text{first two terms} \\ 1 : \text{third term} \\ 1 : \text{fourth term} \end{cases}$$

- (c) Let $Q_n(x)$ denote the Taylor polynomial of degree n for h about $x = 0$.

$$h'(x) = f(2x) \Rightarrow Q_3'(x) = -4 + 2(2x) - \frac{1}{3}(2x)^2$$

$$Q_3(x) = -4x + 4 \cdot \frac{x^2}{2} - \frac{4}{3} \cdot \frac{x^3}{3} + C; \quad C = Q_3(0) = h(0) = 7$$

$$Q_3(x) = 7 - 4x + 2x^2 - \frac{4}{9}x^3$$

OR

$$h'(x) = f(2x), \quad h''(x) = 2f'(2x), \quad h'''(x) = 4f''(2x)$$

$$h'(0) = f(0) = -4, \quad h''(0) = 2f'(0) = 4, \quad h'''(0) = 4f''(0) = -\frac{8}{3}$$

$$Q_3(x) = 7 - 4x + 4 \cdot \frac{x^2}{2!} - \frac{8}{3} \cdot \frac{x^3}{3!} = 7 - 4x + 2x^2 - \frac{4}{9}x^3$$

$$4 : \begin{cases} 2 : \text{applies } h'(x) = f(2x) \\ 1 : \text{constant term} \\ 1 : \text{remaining terms} \end{cases}$$