

2021 AP Calculus AB & BC International Exam Questions

Extrated from the AP Question Bank

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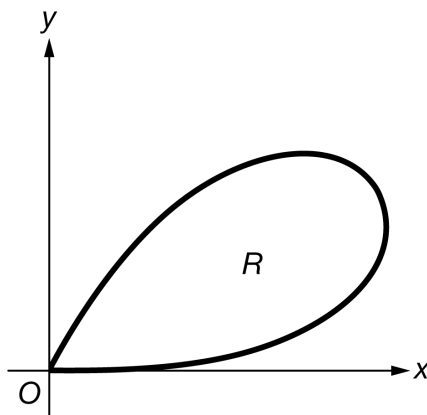
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1 Multiple Choice Questions

- (Calculator Active) An ellipse is defined parametrically by $x(t) = 1 + 2\cos t$ and $y(t) = 3\sin t$ for $0 \leq t \leq 2\pi$. What is the perimeter of the ellipse?
(A) 6.283
(B) 15.865
(C) 16.574
(D) 40.841
- (Calculator Active) For time $t \geq 0$, the velocity of a particle moving in the xy -plane is given by the vector $\langle t^5, (1 + \sqrt{t})^2 \rangle$. At time $t = 1$, the position of the particle is $(4, 3)$. What is the distance between the position of the particle at time $t = 1$ and the position of the particle at time $t = 2$?
(A) 11.603
(B) 12.129
(C) 28.142
(D) 76.011
- (Calculator Active) In a small town, water is being used at a rate of $r(t) = 12 + 9\sin(0.5t - 1.6)$ thousand gallons per hour, where t is measured in hours since midnight. To the nearest integer, how much water, in thousands of gallons, is used in the town during the 6-hour period from 3 A.M. ($t = 3$) to 9 A.M. ($t = 9$) ?
(A) 3
(B) 76
(C) 107
(D) 125
- (Calculator Active) The derivative of a function f is given by $f'(x) = -1 + \sqrt{x + e^{\frac{x}{3}}}$. It is known that $f(0) = 3$. In which of the following intervals is there a value of x such that $f(x) = 6$?
(A) $3 < x < 4$
(B) $5 < x < 6$
(C) $6 < x < 7$
(D) $10 < x < 11$
- (Calculator Active) A particle moves along the curve $y = \frac{15}{x^2 + 1.3^x}$ for $x > 0$. The x -coordinate of the particle changes at a constant rate of 3 units per second. At the instant when the y -coordinate of the particle is 2, what is the rate of change of the y -coordinate of the particle, in units per second?
(A) -0.466
(B) -0.787
(C) -1.397
(D) -4.190

6. (Calculator Active) The derivative of the function f is given by $f'(x) = 1 - 5\cos(3x^2 - 2)$. For how many values of x in the interval $0 \leq x \leq 2$ is the line tangent to the graph of $y = f(x)$ at the point $(x, f(x))$ parallel to the line $y = 1 + 4x$?
- (A) One
(B) Two
(C) Three
(D) Four
7. (Calculator Active) Let R be the region in the first quadrant bounded by the y -axis, the line $y = 2$, and the curve $y = 2\sqrt[3]{x}$. What is the volume of the solid formed by revolving R about the y -axis?
- (A) 0.500
(B) 0.898
(C) 1.571
(D) 23.937
8. (Calculator Active) The derivative of the function f is given by $f'(x) = 5(x - 1)^2 \sin(x^2)$. On the interval $(-1, 3)$, at which of the following values of x does f have a local minimum?
- (A) 1.558
(B) 1.772
(C) 2.248
(D) 2.507
9. (Calculator Active) For $x > -2$, at any point on the graph of the function f the slope is given by $\sqrt{x^3 + 8} - \sqrt{3x^2 + 7}$. How many points of inflection does the graph of f have for $-1 < x < 5$?
- (A) Zero
(B) One
(C) Two
(D) Three
10. (Calculator Active) The first derivative of the function f is given by $f'(x) = xe^{\sin x}$. How many points of inflection does the graph of f have on the interval $-3 < x < 6$?
- (A) One
(B) Two
(C) Three
(D) Four

11. Let f be a function with derivatives of all orders. If $P_3(x)$ is the third-degree Taylor polynomial for f about $x = 0$, which of the following statements about $P_3(x)$ must be true?
- (A) $P_3'''(0) = f'''(0)$
- (B) $P_3'''(0) = \frac{f'''(0)}{3!}$
- (C) One of the terms of $P_3(x)$ is $\frac{x^2}{2!}$.
- (D) One of the terms of $P_3(x)$ is $\frac{x^3}{3!}$.
12. The series $x + x^3 + \frac{x^5}{2!} + \frac{x^7}{3!} + \cdots + \frac{x^{2n+1}}{n!} + \cdots$ is the Maclaurin series for what function?
- (A) $\sin(x)$
- (B) $\sin(-x)$
- (C) xe^{x^2}
- (D) $x \cos x$
13. Information is spreading through a population. The number of people who have the information at time t hours is modeled by an increasing function $y(t)$ that satisfies a logistic differential equation. If the information is spreading most rapidly when 7500 people have the information, which of the following differential equations could represent this model?
- (A) $\frac{dy}{dt} = k(7500 - y)$
- (B) $\frac{dy}{dt} = k(15000 - y)$
- (C) $\frac{dy}{dt} = ky(7500 - y)$
- (D) $\frac{dy}{dt} = ky(15000 - y)$
14. If $x = t^4 - 1$ and $y = 4t + 1$, what $\frac{d^2y}{dx^2}$ is in terms of t ?
- (A) 0
- (B) $-\frac{3}{t^4}$
- (C) $-\frac{3}{4t^5}$
- (D) $-\frac{3}{4t^7}$



15. Let R be the region in the first quadrant that is bounded by the polar curve $r = \sqrt{6 \sin(3\theta)}$, as shown in the figure above. What is the area of R ?
- (A) 1
(B) 2
(C) 4
(D) 6
16. A particle moves in the xy -plane so that its position at time $t > 0$ is given by $x(t) = 2 \ln t$ and $y(t) = \frac{4}{t+1}$. What is the speed of the particle when $t = 1$?
- (A) 1
(B) $\sqrt{2}$
(C) 2
(D) $\sqrt{5}$
17. $\int \frac{1}{x(x+1)} dx =$
- (A) $\ln|x| + \ln|x+1| + C$
(B) $\ln|x| - \ln|x+1| + C$
(C) $\ln|x+1| - \ln|x| + C$
(D) $2 \ln|x| - \ln|x+1| + C$
18. Let f be a differentiable function such that $\int e^{-x} f(x) dx = -e^{-x} f(x) + \int 2xe^{-x} dx$. Which of the following could be $f(x)$?
- (A) $2x$
(B) x^2
(C) $-e^{-x}$
(D) e^{-x}

19. $\int_{10}^{\infty} \frac{1000}{x^2} dx$ is
- (A) $\frac{1}{3}$
 (B) 2
 (C) 100
 (D) divergent
20. If $\cos b = \frac{1}{3}$ for a constant b , then $\sum_{n=0}^{\infty} (\sin^2 b)^n =$
- (A) $\frac{1}{9}$
 (B) $\frac{8}{9}$
 (C) $\frac{9}{8}$
 (D) 9
21. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x(y+1)$ with initial condition $f(2) = 1$. What is the approximation for $f(3)$ obtained by using Euler's method, starting at $x = 2$ with a step size of 0.5?
- (A) -1
 (B) 5
 (C) 8
 (D) 20
22. If $\sum_{n=1}^{\infty} a_n = 5$ and $a_n > 0$ for all n , which of the following must be true?
- (A) $\lim_{n \rightarrow \infty} a_n = 0$
 (B) $\lim_{n \rightarrow \infty} a_n = 5$
 (C) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$
 (D) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$
23. What are all values of x for which the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n}$ converges?
- (A) $-1 < x < 1$ only
 (B) $-1 < x \leq 1$ only
 (C) $-1 \leq x \leq 1$ only
 (D) $-\infty < x < \infty$

24. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{10^n}{n!}$

II. $\sum_{n=1}^{\infty} e^{-n}$

III. $\sum_{n=1}^{\infty} \frac{2^n}{n(n+1)}$

- (A) I only
(B) II only
(C) I and II only
(D) I, II, and III

25. Consider the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} + \cdots$. Which of the following statements is true?

- (A) The series converges, and $\frac{1}{3} < \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} < \frac{2}{3}$.
(B) The series converges, and $\frac{2}{3} < \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} < 1$.
(C) The series converges, and $1 < \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.
(D) The series diverges.

26. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$. Which of the following statements is true?

- (A) Both series converge absolutely.
(B) Both series converge conditionally.
(C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges absolutely, and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges conditionally.
(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges conditionally, and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges absolutely.

x	2	3	4
$g(x)$	-2	1	-4

27. Selected values of the continuous function g are given in the table above. Which of the following statements must be true?

- (A) The minimum value of g on the interval $[2, 4]$ is -4 .
- (B) $g'(x) = -1$ has at least one solution in the interval $[2, 4]$.
- (C) $g'(x) = 0$ has at least two solutions in the interval $[2, 4]$.
- (D) $g(x) = 0$ has at least two solutions in the interval $[2, 4]$.

$$g(x) = \begin{cases} 3x^2 - 5x + 1 & \text{if } x \leq 0 \\ \frac{1}{x(x+1)} & \text{if } x > 0 \end{cases}$$

28. Let g be the function given above. Which of the following statements is true?

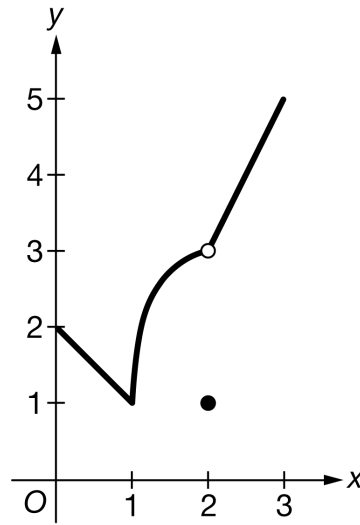
- (A) g is continuous at $x = 0$.
- (B) g has a jump discontinuity at $x = 0$.
- (C) g has a removable discontinuity at $x = 0$.
- (D) g has a discontinuity due to a vertical asymptote at $x = 0$.

29. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 5x + 6}$ is

- (A) -4
- (B) 0
- (C) 4
- (D) nonexistent

30. Let f and g be the functions defined by $f(x) = \frac{x^2 - 25}{x^2 - 29}$ and $g(x) = \frac{x + 5}{2x + 4}$. If the function h satisfies $f(x) \leq h(x) \leq g(x)$ for $1 \leq x \leq 5$, what is $\lim_{x \rightarrow 3} h(x)$?

- (A) $\frac{1}{2}$
- (B) $\frac{4}{5}$
- (C) 1
- (D) The limit cannot be determined from the given information.

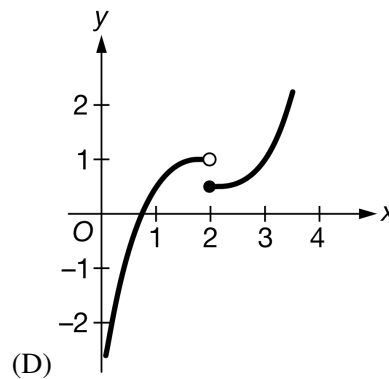
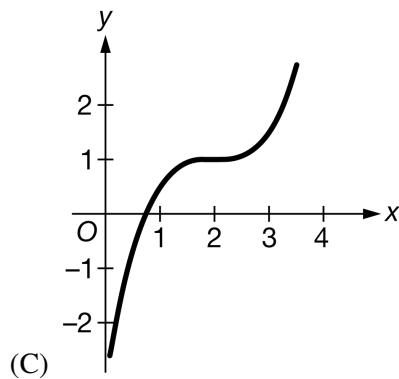
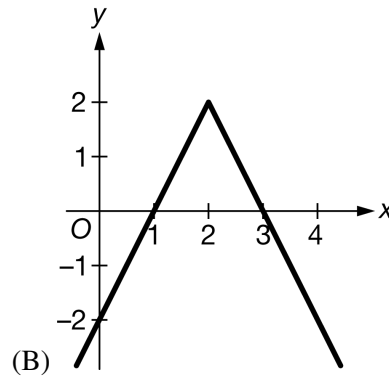
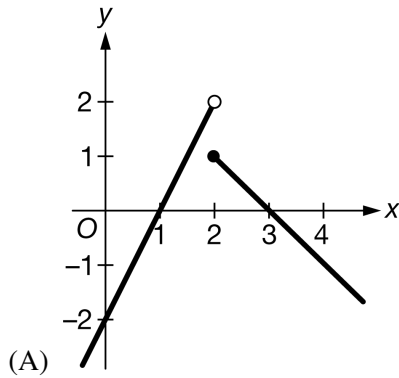


Graph of f

31. The graph of the function f is shown above. What is $\lim_{x \rightarrow 2} f(x)$?

- (A) 1
- (B) 2
- (C) 3
- (D) The limit does not exist.

32. Let g be a function such that $\lim_{x \rightarrow 2} g(x) = g(2)$ and $\lim_{x \rightarrow 2^-} g'(x) \neq \lim_{x \rightarrow 2^+} g'(x)$. Which of the following could be the graph of g ?



x	0	2	6	12	20
$f(x)$	-8	-2	0	2	18

33. The function f is differentiable and has values as shown in the table above. Of the following, on which interval is the average rate of change of f greatest?
- (A) $[0, 2]$
 (B) $[2, 6]$
 (C) $[6, 12]$
 (D) $[12, 20]$
34. If $f(x) = x^2 + 4x - 2$, $g(x) = 4x + 2$, and $h(x) = \frac{f(x)}{g(x)}$, then $h'(1) =$
- (A) $-\frac{2}{3}$
 (B) $\frac{2}{3}$
 (C) $\frac{4}{3}$
 (D) $\frac{3}{2}$
35. If $f'(x) = x^2$ and $g'(x) = 3e^x$ and if $h(x) = f(x)g(x)$, then $h'(x)$ could be
- (A) $3x^2e^x$
 (B) $x^3e^x + 3x^2e^x$
 (C) $x^3e^x + x^2e^x$
 (D) $3x^2e^x + 6xe^x$
36. If $y = \sqrt{x^9}$, then $\frac{dy}{dx} =$
- (A) $\frac{9\sqrt{x^7}}{2}$
 (B) $3\sqrt{x^8}$
 (C) $\frac{1}{2\sqrt{x^9}}$
 (D) $\frac{2\sqrt{x^{11}}}{11}$

37. Let f be the function defined by $f(x) = bx^2 + 3bx + b^2 + \frac{1}{x^2}$, where b is a nonzero constant. Which of the following is an expression for $f'(x)$, the derivative of $f(x)$?

- (A) $2bx + 3b - \frac{2}{x}$
- (B) $2bx + 3b - \frac{2}{x^3}$
- (C) $2bx + 3b + b^2 - \frac{2}{x^3}$
- (D) $x^2 + 3x + 2b$

$$g(x) = \begin{cases} x^2 + 1 & \text{for } x < 3 \\ 13 - x & \text{for } x \geq 3 \end{cases}$$

38. Let g be the function given above. What is the average rate of change of g on the closed interval $[1, 5]$?

- (A) $\frac{1}{2}$
- (B) $\frac{3}{2}$
- (C) 5
- (D) Undefined

39. If $x = \frac{1}{y+1}$, then $\frac{dy}{dx} =$

- (A) $-(y+1)^2$
- (B) $-\frac{1}{(y+1)^2}$
- (C) $\frac{1}{(y+1)^2}$
- (D) $\frac{y}{(y+1)^2}$

40. If $y = -3\cos(2x)$, then $\frac{d^2y}{dx^2} =$

- (A) $-12\cos(2x)$
- (B) $12\cos(2x)$
- (C) $-3\cos(2x)$
- (D) $3\cos(2x)$

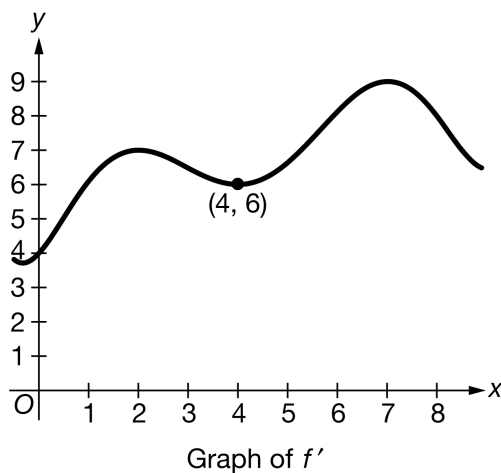
x	$f(x)$	$f'(x)$
1	2	3
2	3	4
3	5	2

41. The function f is increasing and differentiable. Selected values of f and its derivative f' are given in the table above. What is the value of $(f^{-1})'(3)$?

- (A) $\frac{1}{4}$
 (B) $\frac{1}{2}$
 (C) 1
 (D) 2

42. If $f(x) = \sqrt[3]{5x^2 - 7}$, then $f'(x) =$

- (A) $\frac{10x}{3}(5x^2 - 7)^{-\frac{2}{3}}$
 (B) $\frac{1}{3}(5x^2 - 7)^{-\frac{2}{3}}$
 (C) $\frac{3}{4}(5x^2 - 7)^{\frac{4}{3}}$
 (D) $\frac{1}{3}(10x)^{-\frac{2}{3}}$



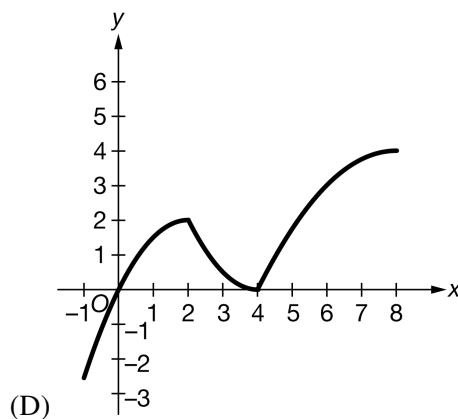
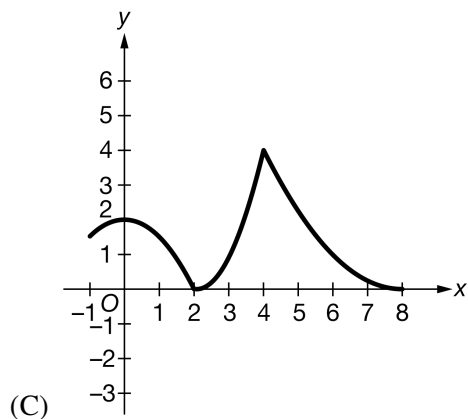
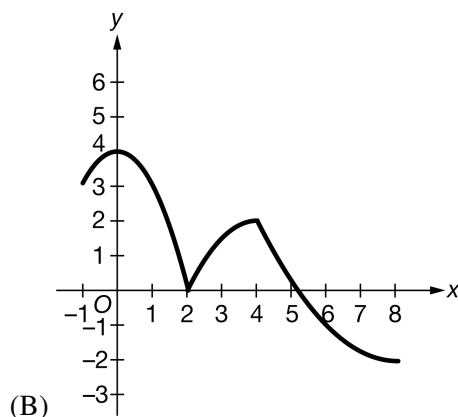
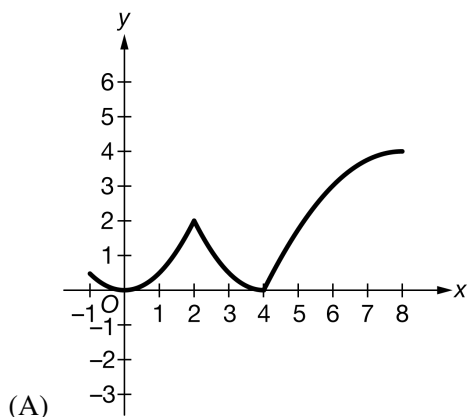
43. The graph of f' , the derivative of the function f , is shown above. If $f(4) = -1$, what is the approximation for $f(4.5)$ using the line tangent to the graph of f at $x = 4$?

- (A) -4
 (B) -1
 (C) 2
 (D) 6

44. For time $t > 0$, the position of a particle moving along the x -axis is given by $x(t) = t^4 - 4t^3$. What is the acceleration of the particle when the velocity of the particle is 0?
- (A) -16
 (B) 0
 (C) 36
 (D) 96
45. Paint spills onto a floor in a circular pattern. The radius of the spill increases at a constant rate of 2.5 inches per minute. How fast is the area of the spill increasing when the radius of the spill is 18 inches?
- (A) $5\pi \text{ in}^2/\text{min}$
 (B) $36\pi \text{ in}^2/\text{min}$
 (C) $45\pi \text{ in}^2/\text{min}$
 (D) $90\pi \text{ in}^2/\text{min}$
46. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = \frac{1}{2}$. Using the line tangent to the graph of f at $x = 2$ as a local linear approximation for f , what is the estimate for $f(1.8)$?
- (A) 2.5
 (B) 2.8
 (C) 2.9
 (D) 3.1
47. $\lim_{x \rightarrow 0} \frac{\int_0^x \cos(2t) dt}{5x}$ is
- (A) 0
 (B) $\frac{1}{5}$
 (C) $\frac{2}{5}$
 (D) nonexistent
48. $\lim_{x \rightarrow 0} \frac{\int_0^x \sin(3t^2) dt}{4x^3}$ is
- (A) $-\frac{1}{4}$
 (B) 0
 (C) $\frac{1}{4}$
 (D) nonexistent

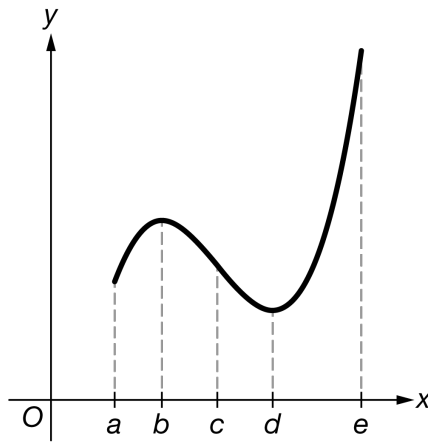
x	0	$0 < x < 2$	2	$2 < x < 4$	4	$4 < x < 8$
$f'(x)$	0	Negative	Does not exist	Positive	Does not exist	Negative
$f''(x)$	Negative	Negative	Does not exist	Negative	Does not exist	Positive

49. The first and second derivatives of a function f have the properties given in the table above. Which of the following could be the graph of f ?



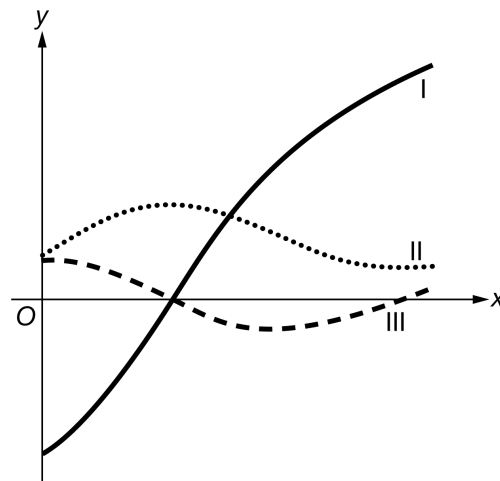
50. Let f be the function given by $f(x) = e^x \sin x$ on the closed interval $[0, 2\pi]$. On which of the following intervals is f increasing?

- (A) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$
 (B) $\left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$
 (C) $\left[0, \frac{\pi}{2}\right]$ and $\left[\frac{3\pi}{2}, 2\pi\right]$
 (D) $\left[0, \frac{3\pi}{4}\right]$ and $\left[\frac{7\pi}{4}, 2\pi\right]$



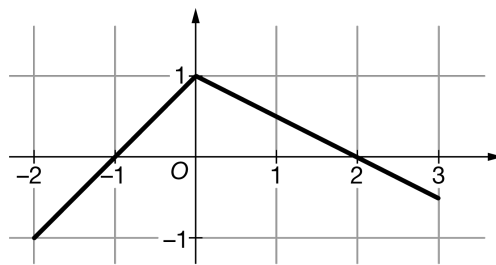
Graph of f'

51. The graph of f' , the derivative of the function f , is shown in the figure above. On what open intervals is the graph of f concave up?
- (A) $b < x < d$
 (B) $c < x < e$
 (C) $d < x < e$ only
 (D) $a < x < b$ and $d < x < e$



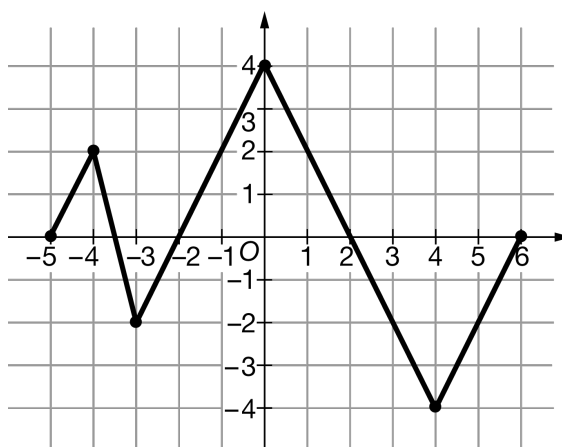
52. The figure above shows three graphs, labeled I, II, and III. One is the graph of f , one is the graph of f' , and one is the graph of f'' . Which of the following correctly identifies each of the three graphs?
- (A) The graph of f is I, the graph of f' is II, and the graph of f'' is III.
 (B) The graph of f is I, the graph of f' is III, and the graph of f'' is II.
 (C) The graph of f is II, the graph of f' is I, and the graph of f'' is III.
 (D) The graph of f is II, the graph of f' is III, and the graph of f'' is I.

53. A rectangle has its upper two vertices on the graph of $y = 12 - \frac{x^2}{3}$ and its lower two vertices on the x -axis. What is the maximum possible area of the rectangle?
- (A) $216 - 72\sqrt{5}$
 (B) $32\sqrt{3}$
 (C) 96
 (D) 144
54. Let f be the function given by $f(x) = x^2 \ln x$ for $x > 0$. On which of the following intervals is the graph of f concave down?
- (A) $(0, e^{-\frac{3}{2}})$ only
 (B) $(0, e^{-\frac{1}{2}})$
 (C) $(e^{-\frac{3}{2}}, \infty)$
 (D) $(e^{-\frac{1}{2}}, \infty)$ only



Graph of f

55. The graph of the function f consists of two line segments, as shown in the figure above. Let g be the function defined by $g(x) = \int_{-2}^x f(t) dt$. At what value of x does g attain its global maximum on the closed interval $[-2, 3]$?
- (A) -2
 (B) -1
 (C) 0
 (D) 2



Graph of f

56. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-5}^x f(t)dt$, how many zeros does g have on the closed interval $[-5, 6]$?

(A) One
 (B) Three
 (C) Four
 (D) Five

t	0	2	3	6
$f(t)$	12	36	48	42

57. The function f is continuous on the closed interval $[0, 6]$ and has values given in the table above. What is the approximation to $\int_0^6 f(t)dt$ obtained by using a right Riemann sum with the subintervals $[0, 2]$, $[2, 3]$, and $[3, 6]$?

(A) 204
 (B) 225
 (C) 246
 (D) 252

58. Let f be a continuous function such that $f(x) = f(-x)$ for all x . If $\int_{-2}^0 f(x)dx = 4$ and $\int_1^2 f(x)dx = 1$, what is the value of $\int_0^1 f(x)dx$?

(A) -5
 (B) 2
 (C) 3
 (D) 5

59. The function f is continuous, and g is a function such that $g'(x) = f(x)$. Which of the following is equal to $\int_6^{10} f\left(\frac{1}{2}x\right) dx$ in terms of g ?
- (A) $\frac{1}{2}g(5) - \frac{1}{2}g(3)$
 (B) $\frac{1}{2}g(10) - \frac{1}{2}g(6)$
 (C) $2g(5) - 2g(3)$
 (D) $2g(10) - 2g(6)$
60. $\int \frac{x-1}{x+1} dx =$
- (A) $\frac{2}{(x+1)^2} + C$
 (B) $x - 2 \ln|x+1| + C$
 (C) $\frac{x^2 - 2x}{x^2 + 2x} + C$
 (D) $\left(\frac{1}{2}x^2 - x\right) \ln|x+1| + C$
61. Let f be the function defined by $f(x) = 4 + \int_1^x \frac{1}{1+t} dt$. Which of the following is an equation for the line tangent to the graph of f at $x = 1$?
- (A) $y = 4$
 (B) $y - 4 = \frac{1}{2}(x - 1)$
 (C) $y - 4 = \frac{1}{4}(x - 1)$
 (D) $y - 4 = \frac{9}{2}(x - 1)$
62. $\int_0^{\frac{\pi}{4}} \sin x dx =$
- (A) $-\frac{\sqrt{2}}{2}$
 (B) $1 - \frac{\sqrt{2}}{2}$
 (C) $\frac{\sqrt{2}}{2} - 1$
 (D) $\frac{\sqrt{2}}{2}$

63. If $f(x) = \int_0^{3x} e^{t^2} dt$, then $f'(x) =$

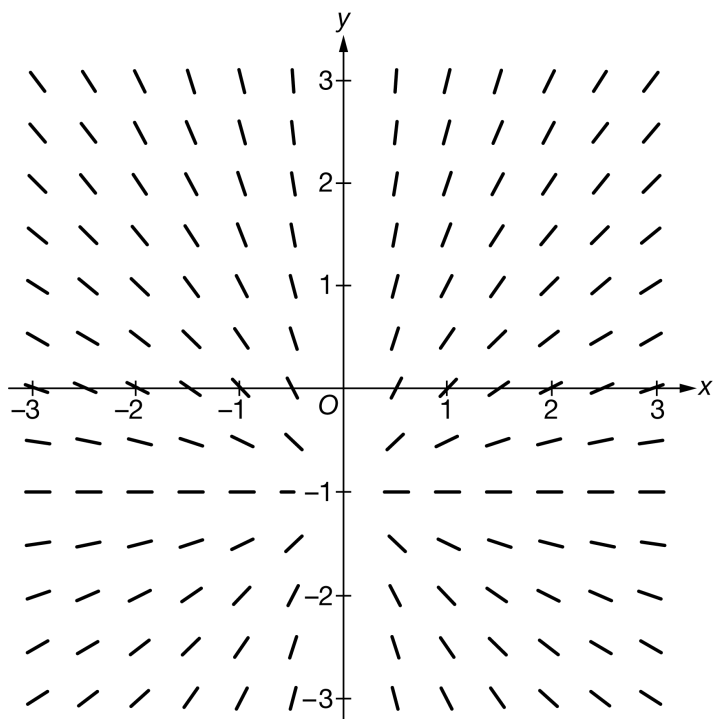
- (A) e^{x^2}
- (B) e^{9x^2}
- (C) $3e^{9x^2}$
- (D) $18xe^{9x^2}$

64. $\int_1^3 (4x^3 - 6x + 5) dx =$

- (A) 66
- (B) 92
- (C) 96
- (D) 282

65. For which of the following integrals is $\sum_{k=1}^n \ln\left(2 + \frac{5}{n}k\right) \frac{5}{n}$ the right Riemann sum approximation with n subintervals of equal length?

- (A) $\int_2^7 \ln x dx$
- (B) $\int_2^7 \ln(2+x) dx$
- (C) $\int_0^5 \ln(2+5x) dx$
- (D) $\int_2^7 \ln(2+5x) dx$



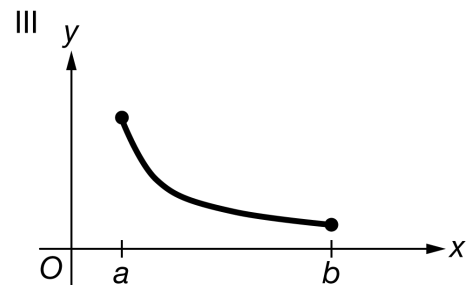
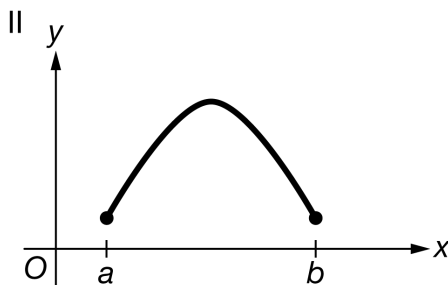
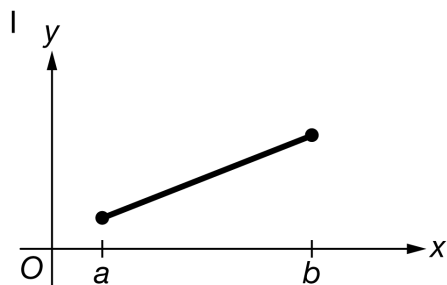
66. Shown above is a slope field for which of the following differential equations?

- (A) $\frac{dy}{dx} = \frac{y+1}{x}$
- (B) $\frac{dy}{dx} = -\frac{y+1}{x}$
- (C) $\frac{dy}{dx} = \frac{y-1}{x}$
- (D) $\frac{dy}{dx} = -\frac{y-1}{x}$

67. A rectangle has length x , width y , and area A . The instantaneous rate of change of the area with respect to the width is equal to the sum of the length and twice the width. Which of the following equations describes this relationship?

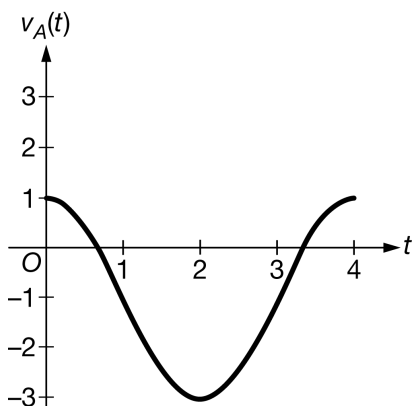
- (A) $A = x + 2y$
- (B) $\frac{dy}{dx} = x + 2y$
- (C) $\frac{dA}{dx} = x + 2y$
- (D) $\frac{dA}{dy} = x + 2y$

68. Which of the following is the solution to the differential equation $\frac{dy}{dx} = y \cos x$ with the initial condition $y\left(\frac{\pi}{2}\right) = e$?
- (A) $y = \cos x + e$
 (B) $y = e^{\cos x + 1}$
 (C) $y = e \cdot \sin x$
 (D) $y = e^{\sin x}$
69. If $\frac{dy}{dx} = 16 \sin^3 x \cos x$ and $y = 16$ when $x = \frac{\pi}{2}$, what is the value of y when $x = \frac{\pi}{6}$?
- (A) $\frac{49}{4}$
 (B) $\frac{1}{4}$
 (C) 10
 (D) 13
70. Let f be a continuous function that satisfies the condition $\frac{1}{b-a} \int_a^b f(x) dx = f(c)$ for exactly one value of c , where $a < c < b$. Which of the graphs above could be the graph of $y = f(x)$?

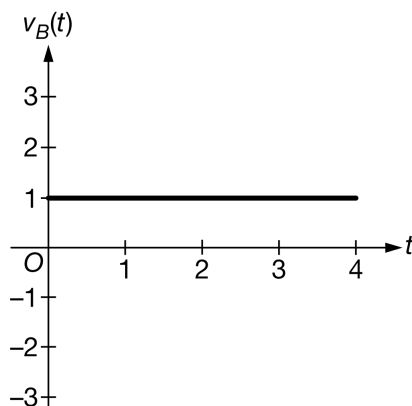


- (A) I only
 (B) II only
 (C) I and III only
 (D) II and III only

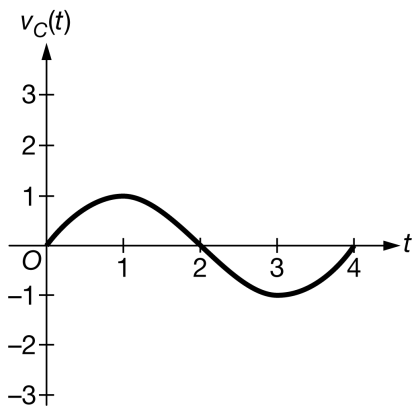
71. For time $0 \leq t \leq 4$, four particles— A , B , C , and D —move along parallel horizontal lines in the xy -plane. At time $t = 0$, the x -coordinate of the position of each particle is $x = 0$. The graphs of the velocity of each particle, in meters per second, are shown below. Which is the graph of the velocity of the particle that is farthest to the right at time $t = 4$?



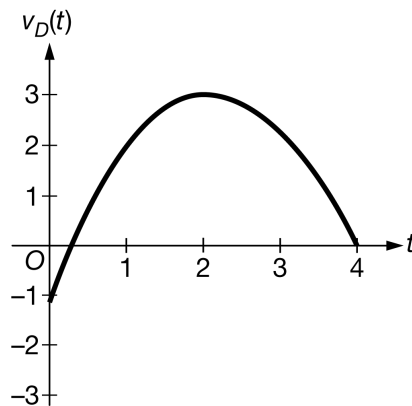
(A) Velocity of Particle A



(B) Velocity of Particle B



(C) Velocity of Particle C



(D) Velocity of Particle D

72. Let R be the region in the first quadrant bounded by the graph of $y = e^{-x}$, the x -axis, the y -axis, and the line $x = b$. Region R is the base of a solid whose cross sections perpendicular to the x -axis are squares. If the volume of the solid is $\frac{4}{9}$, what is the value of b ?

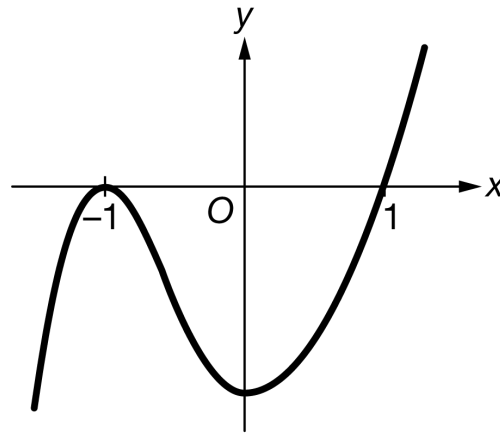
(A) $\ln\left(\frac{3}{\sqrt{17}}\right)$

(B) $\ln\left(\frac{3}{\sqrt{7}}\right)$

(C) $\ln\left(\frac{9}{5}\right)$

(D) $\frac{1}{2}\ln 9$

73. Let f be a continuous function defined on the closed interval $2 \leq x \leq k$, where $k > 2$. If the average value of f on this interval is 6 and $\int_2^k f(x)dx = 42$, then $k =$
- (A) 9
(B) 8
(C) 7
(D) 5
74. What is the area of the region in the first quadrant that is bounded by the graph of $y = 2^x$, the horizontal line $y = 4$, and the y -axis?
- (A) $8 - \frac{3}{\ln 2}$
(B) $\frac{3}{\ln 2}$
(C) $\frac{22}{3}$
(D) $\frac{15}{\ln 2}$
75. What is the area of the region in the first quadrant bounded by the lines $x = 1$ and $x = 3$ and the graphs of $y = e^x$ and $y = x - 1$?
- (A) $e^3 - e - 6$
(B) $e^3 - e - 2$
(C) $e^3 - 3$
(D) $e^3 - e$



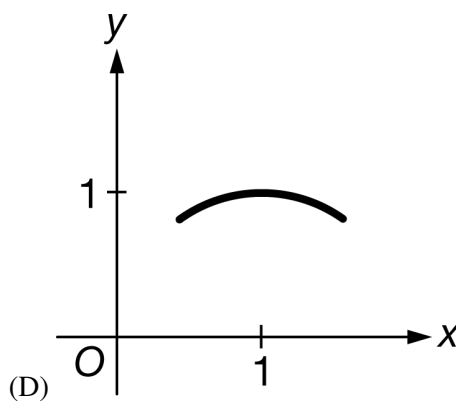
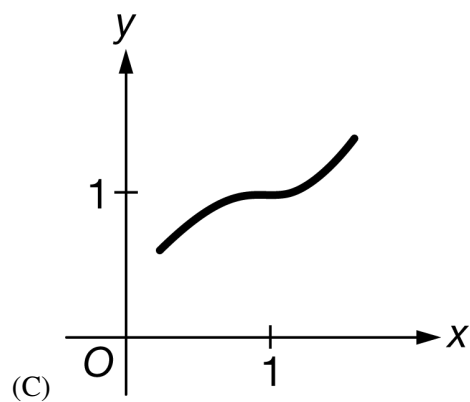
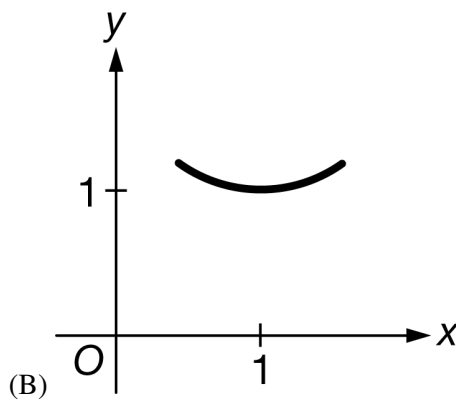
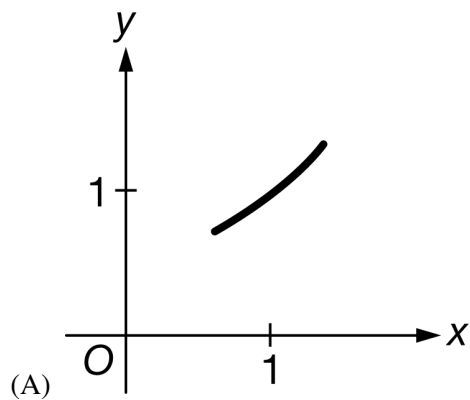
Graph of f

76. The figure above shows the graph of a differentiable function f . If $g(x) = f(x^2)$, which of the following statements is true?
- (A) $g'(-1) < 0$
 - (B) $g'(-1) > 0$
 - (C) $g'(1) < 0$
 - (D) $g'(1) = 0$
77. $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + 27}}{\sqrt{x^2 + 27x + 3}}$ is
- (A) 2
 - (B) 3
 - (C) 4
 - (D) infinite
78. For a car driven 100 kilometers at a constant speed, the amount of fuel used as a function of the speed is modeled by a differentiable function F . Fuel is measured in liters, and speed is measured in kilometers per hour (km/hr). In this context, which of the following is a correct interpretation of the statement $F'(50) > F'(80)$?
- (A) Driving at 50 km/hr uses more fuel than driving at 80 km/hr.
 - (B) The rate at which 50 liters of fuel is used is greater than the rate at which 80 liters of fuel is used.
 - (C) The rate of change of the speed is greater when driving at 50 km/hr than when driving at 80 km/hr.
 - (D) The rate of change of liters of fuel used with respect to the speed is greater when driving at 50 km/hr than when driving at 80 km/hr.

79. The function f is given by x^3 . The application of the Mean Value Theorem to f on the interval $[1, 3]$ guarantees a point in the interval $(1, 3)$ at which the slope of the line tangent to the graph of f is equal to which of the following?

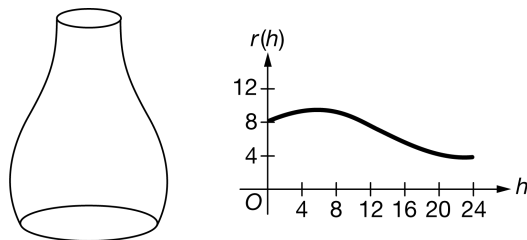
- (A) 0
- (B) 12
- (C) 13
- (D) 24

80. Which of the following could be the graph of a solution of the differential equation $\frac{dy}{dx} = (x - 1)y^2$ near the point $(1, 1)$?



2 Free-Response Questions

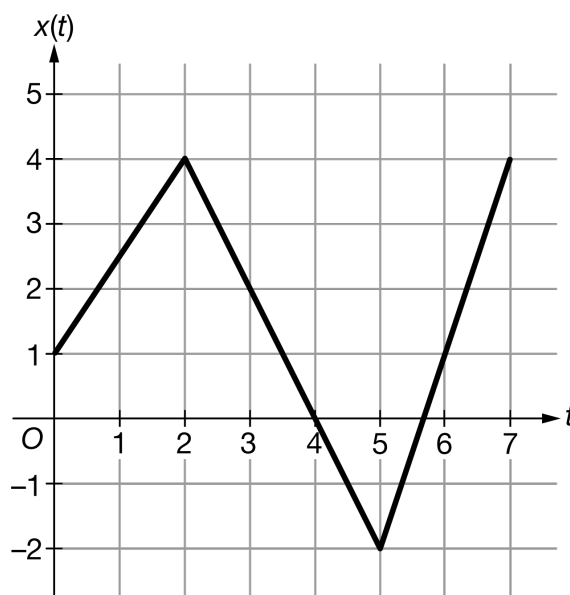
Directions: You should use a calculator to finish questions 1-3.



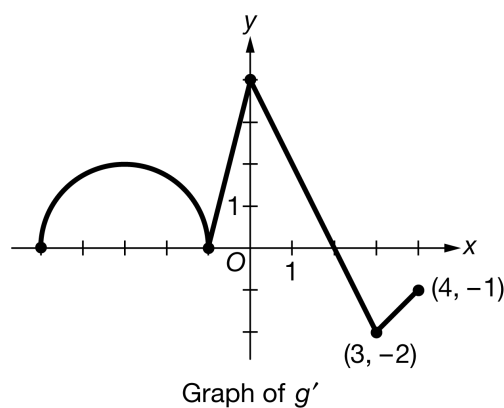
1. A 24-centimeter tall vase has cross sections parallel to the base that are circles. Each circular cross section h centimeters above the base has a radius of centimeters for $r(h) = 8 - 0.1h + 2\sin(0.3h^{0.9})$ centimeters for $0 \leq h \leq 24$. A sketch of the vase and the graph of r are shown above.
 - (a) Find the area of the region between the graph of r and the h -axis from $h = 0$ to $h = 24$.
 - (b) Find the volume of the vase.
 - (c) Evaluate $\int_4^{20} r'(h)dh$. Using correct units, interpret the meaning of $\int_4^{20} r'(h)dh$ in the context of the problem.
 - (d) Water is poured into the vase. At the instant when the depth of the water in the vase is 10 centimeters, the depth of the water is increasing at a rate of 0.6 centimeter per second. At that instant, what is the rate of change of the circumference of the surface of the water, in centimeters per second?

t (years)	0	10	22	30	40
$A(t)$ (hundred pounds per year)	163	142	124	118	113

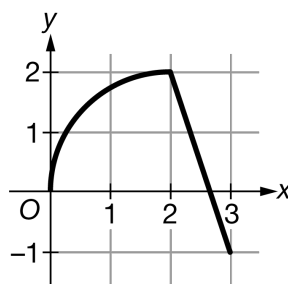
2. The amount of nitrogen in a particular river is affected by two sources. Nitrogen from the first source enters the river at a rate modeled by the decreasing, differentiable function A . Selected values of $A(t)$ are shown in the table above. Nitrogen from the second source enters the river at a rate modeled by the function B defined by $B(t) = 20 + \left(\frac{95}{1 + \sqrt[3]{t}}\right)^{1.25}$. Both $A(t)$ and $B(t)$ are measured in hundreds of pounds of nitrogen per year, and t is measured in years for $0 \leq t \leq 40$.
 - (a) Using a right Riemann sum with the four subintervals indicated by the data in the table, approximate the total amount of nitrogen from the first source that enters the river over the time interval $0 \leq t \leq 40$. Is this approximation an overestimate or an underestimate? Give a reason for your answer.
 - (b) Find $B'(39)$. Using correct units, interpret the meaning of $B'(39)$ in the context of the problem.
 - (c) Based on model B , find the average rate, in hundreds of pounds per year, at which nitrogen from the second source enters the river during the time interval $5 \leq t \leq 35$.
 - (d) Using model B and your approximation from part (a), find an approximation for the total combined amount of nitrogen from the two sources that enters the river over the time interval $0 \leq t \leq 40$. Give your answer to the nearest hundred pounds.



3. For time $0 \leq t \leq 7$, a drone flies in an xy -plane at a constant altitude over level ground. The position of the drone in the xy -plane is modeled by the parametric functions $(x(t), y(t))$, where $\frac{dy}{dt} = \sqrt{t + \cos t}$. Both $x(t)$ and $y(t)$ are measured in meters, and t is measured in seconds. The graph of $x(t)$, consisting of three line segments, is shown in the figure above. At time $t = 0$, the drone is at position $(1, 5)$.
- Find the rate of change of $\frac{dy}{dt}$ at time $t = 3$.
 - Find the position of the drone in the xy -plane at time $t = 2$.
 - Is there a time t , for $0 < t < 7$, at which the drone's y -direction changes? Give a reason for your answer.
 - For $0 < t < 7$, find the time t at which the line tangent to the path of the drone in the xy -plane has slope -1 .



4. Let g be a function that is defined on the closed interval $-5 \leq x \leq 4$ with $g(0) = 5$. The graph of g' , the derivative of g , consists of a semicircle and three line segments, as shown above.
- On what intervals, if any, is g decreasing? Give a reason for your answer.
 - Find the average rate of change of g on the interval $[-3, 3]$. Is there a value c , $-3 < c < 3$, for which $g'(c)$ is equal to the average rate of change of g on the interval $[-3, 3]$? Justify your answer.
 - Find the x -coordinate of each point of inflection of the graph of g on the open interval $(-5, 4)$. Justify your answer.
 - Let $H(x) = e^{3x}g(x)$. Find the value of $H'(0)$.

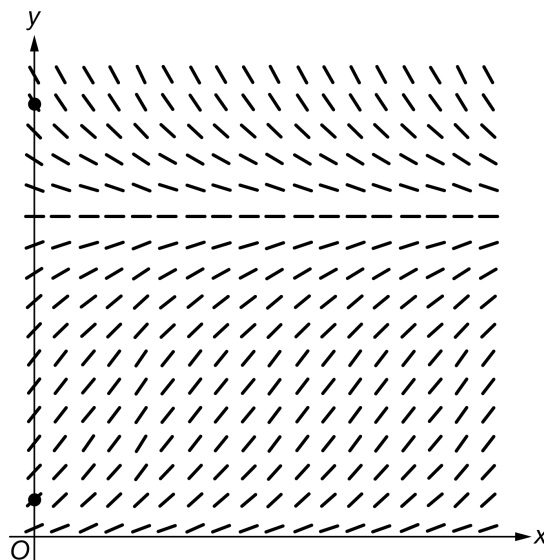


Graph of f

5. The function f is continuous on the interval $0 \leq x \leq 3$. The graph of f consists of a line segment and a quarter circle, as shown in the figure above. On the interval $0 \leq x \leq 2$, $(f(x))^2 = 4x - x^2$. The function g is defined by $g(x) = \int_2^x f(t) dt$.
- Find $g(0)$ and $g(3)$.
 - Find $\lim_{x \rightarrow 2} \frac{g(x)}{\sin(x-2)}$. Justify your answer.
 - Find $\int_1^2 \frac{1}{(f(x))^2} dx$
 - Let h be a differentiable function on the interval $[0, 3]$ with $h(0) = 0$, $h(3) = -1$, and $h(x) < f(x)$ for all x in the interval $(0, 3)$. Let R be the region enclosed by the graphs of f and h on the interval $[0, 3]$. Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of region R .
6. Consider the curve defined by the equation $y^2 = 3xy - 3x$.
- Show that $\frac{dy}{dx} = \frac{3y-3}{2y-3x}$.
 - Find $\frac{d^2y}{dx^2}$ in terms of x , y , and $\frac{dy}{dx}$.
 - Is there a point on the curve at which the line tangent to the curve is horizontal? Explain your reasoning.
 - A particle travels along the curve. The line tangent to the path of the particle is vertical at the point P . What are all possible coordinates (x, y) for point P ?
7. The function f is defined by $f(x) = \int_0^x (t^2 - 3t - 10) dt$.
- Write an equation for the line tangent to the graph of f at $x = 6$.
 - Find all values of x at which f has a critical point. Determine whether f has a relative minimum, a relative maximum, or neither at each of these values. Justify your answers.
 - On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.
 - Let g be the function defined by $g(x) = f(\tan x)$. Find $g'(\frac{\pi}{4})$.

8. Consider the differential equation $\frac{dy}{dx} = -\frac{y}{4} \ln\left(\frac{y}{50}\right)$.

- (a) A portion of the slope field for the differential equation is given below. Sketch the solution curves through the two indicated points for $x \geq 0$.



- (b) Find $\lim_{y \rightarrow 50} \frac{dy}{dx}$.
- (c) Let f be the particular solution to the differential equation with initial condition $f(0) = 20$. Find $f'(0)$, and explain why f is increasing for all x .
- (d) For $0 < y < 40$, at what value of y does $\frac{dy}{dx}$ attain its maximum value? Justify your answer.
9. The Maclaurin series for the function f is given by $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n(n+1)}$ and converges to $f(x)$ for all x in the interval of convergence.
- (a) What is the value of $f^{(20)}(0)$, the derivative of f evaluated at $x = 0$?
- (b) Determine the interval of convergence of the Maclaurin series for f .
- (c) The Maclaurin series for f can be used to represent $f(1)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1)$.
- (d) Show that the approximation found in part (c) is within $\frac{1}{100}$ of the exact value of $f(1)$. Justify your answer.

3 Answers to Multiple Choice Questions

Question	Answer	Question	Answer	Question	Answer	Question	Answer	Question	Answer
1	B	17	B	33	A	49	B	65	A
2	A	18	B	34	B	50	D	66	A
3	C	19	C	35	B	51	D	67	D
4	A	20	D	36	A	52	A	68	D
5	D	21	C	37	B	53	B	69	A
6	C	22	A	38	B	54	A	70	C
7	B	23	C	39	A	55	D	71	D
8	D	24	C	40	B	56	B	72	D
9	C	25	B	41	A	57	C	73	A
10	B	26	D	42	A	58	C	74	A
11	A	27	D	43	C	59	C	75	B
12	C	28	D	44	C	60	B	76	A
13	D	29	A	45	D	61	B	77	A
14	D	30	B	46	C	62	B	78	D
15	B	31	C	47	B	63	C	79	C
16	D	32	B	48	C	64	A	80	B

4 Answers to Free-Response Questions

4.1 Question 1

$$(a) \int_0^{24} r(h)dh = 165.780506$$

The area of the region is 165.781 (or 165.780).

$$(b) \int_0^{24} \pi(r(h))^2 dh = 3934.492$$

The volume of the vase is 3934.492 centimeters.

$$(c) \int_4^{20} r'(h)dh = -5.259388$$

The radius of the vase at $h = 20$ centimeters is 5.259 centimeters less than the radius of the vase at $h = 4$ centimeters.

$$(d) C = 2\pi r \Rightarrow \frac{dC}{dt} = \frac{d}{dt}(2\pi r) = 2\pi \left(\frac{dr}{dh} \cdot \frac{dh}{dt} \right)$$

$$\left. \frac{dC}{dt} \right|_{h=10} = 2\pi \left(\left. \frac{dr}{dh} \right|_{h=10} \cdot \left. \frac{dh}{dt} \right|_{h=10} \right) = -1.550641$$

The rate of change of the circumference of the surface of the water is -1.551 (or -1.550) centimeters per second.

4.2 Question 2

$$(a) \int_0^{40} A(t)dt \approx (10-0)A(10) + (22-10)A(22) + (30-22)A(30) + (40-30)A(40) = 4982$$

Approximately 4982 hundred pounds of nitrogen from the first source enters the river over the time interval $0 \leq t \leq 40$. The approximation is an underestimate because a right Riemann sum is used and A is decreasing.

$$(b) B'(39) = -0.384963$$

At time $t = 39$ years, the rate at which nitrogen from the second source enters the river was decreasing at a rate of 0.385 (or 0.384) hundred pounds per year per year.

$$(c) \frac{1}{35-5} \int_5^{35} B(t)dt = 80.063385$$

During the time interval $0 \leq t \leq 35$, the average rate at which nitrogen from the second source enters the river was 80.063 hundred pounds per year.

$$(d) \text{Total} = \int_0^{40} A(t)dt + \int_0^{40} B(t)dt \approx 8368$$

The total combined amount of nitrogen from the two sources that enters the river over the time interval $0 \leq t \leq 40$ is 8368 approximately hundred pounds.

4.3 Question 3

(a) $\frac{d^2y}{dt^2}\bigg|_{t=3} = 0.302903$

The rate of change of $\frac{dy}{dt}$ at time $t = 3$ is 0.303 (or 0.302).

(b) $x(2) = 4$

$$y(2) = y(0) + \int_0^2 \sqrt{t + \cos t} dt = 7.408276$$

At time the position of the drone in the xy -plane is $(4, 7.408)$

(c) No, there is no time t at which the drone's y -direction changes because $\frac{dy}{dt} = \sqrt{t + \cos t}$ does not change sign for $t > 0$.

(d) Let $t = t_0$ be the time when the drone's path in the xy -plane has slope -1 .

$$\text{At time } t = t_0, -1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \Rightarrow \frac{dy}{dt} = -\frac{dx}{dt}.$$

Because $\frac{dy}{dt} = \sqrt{t + \cos t}$ is positive for $t \geq 0$, $\frac{dx}{dt} > 0$ must be true at t_0 .

$$\frac{dx}{dt} < 0 \Rightarrow \frac{dx}{dt} = -2 \text{ and } 2 < t < 5. \text{ Therefore, } \frac{dy}{dt}\bigg|_{t=t_0} = 2 \Rightarrow t_0 = 4.3523297.$$

The line tangent to the path of the drone in the xy -plane has slope -1 at time $t_0 = 4.352$.

4.4 Question 4

(a) g is decreasing on the interval $[2, 4]$ because g is continuous and $g'(x) < 0$ on $(2, 4]$.

(b) Average rate of change = $\frac{g(3) - g(-3)}{3 - (-3)} = \frac{\int_{-3}^3 g'(x) dx}{6} = \frac{\pi + 5}{6}$.

g is differentiable on $[-3, 3] \Rightarrow g$ is continuous on $[-3, 3]$. The Mean Value Theorem guarantees that there exists a value of c in the interval $(-3, 3)$ such that $g'(c)$ is equal to the average rate of change of g on the interval $[-3, 3]$.

OR

g' is continuous on $[-3, 3]$. The range of g' on the interval $[-3, 3]$ is $-2 \leq g'(x) \leq 4$. Because $2 < \frac{\pi + 5}{6} < 4$, there exists a value of c in the interval such that $g'(c)$ is equal to the average rate of change of g on the interval $[-3, 3]$.

(c) The graph of g will have a point of inflection where the graph of g' changes from increasing to decreasing or from decreasing to increasing. This occurs at $x = -3$, $x = -1$, $x = 0$, and $x = 3$.

(d) $H'(x) = 3e^{3x} \cdot g(x) + e^{3x} \cdot g'(x)$, then $H'(0) = 19$.

4.5 Question 5

$$(a) g(0) = \int_2^0 f(t)dt = -\int_0^2 f(t)dt = -\pi$$

$$g(3) = \int_2^3 f(t)dt = \frac{1}{2}$$

$$(b) \lim_{x \rightarrow 2} \sin(x-2) = 0$$

Since g is continuous, then $\lim_{x \rightarrow 2} g(x) = g(2) = 0$. Therefore the limit $\lim_{x \rightarrow 2} \frac{g(x)}{\sin(x-2)}$ has an indeterminate form of type $\frac{0}{0}$.

Using L'Hospital's Rule, $\lim_{x \rightarrow 2} \frac{g(x)}{\sin(x-2)} = \lim_{x \rightarrow 2} \frac{g'(x)}{\cos(x-2)} = \lim_{x \rightarrow 2} \frac{f(x)}{\cos(x-2)} = 2$.

$$(c) \frac{1}{(f(x))^2} = \frac{1}{4x-x^2} = \frac{1}{x(4-x)} = \frac{1/4}{x} + \frac{1/4}{4-x}$$

$$\int_1^2 \frac{1}{(f(x))^2} dx = \int_1^2 \left(\frac{1/4}{x} + \frac{1/4}{4-x} \right) dx = \left[\frac{1}{4} \ln|x| - \frac{1}{4} \ln|4-x| \right]_1^2 = \frac{1}{4} \ln 3$$

$$(d) \text{Perimeter} = \int_0^3 \sqrt{1+(f'(x))^2} dx + \int_0^3 \sqrt{1+(h'(x))^2} dx$$

OR

$$\text{Perimeter} = \pi + \sqrt{10} + \int_0^3 \sqrt{1+(h'(x))^2} dx$$

4.6 Question 6

$$(a) \frac{d}{dx}(y^2) = \frac{d}{dx}(2xy-3x) \Rightarrow 2y \frac{dy}{dx} = 3x \frac{dy}{dx} + 3y-3 \Rightarrow (2y-3x) \frac{dy}{dx} = 3y-3 \Rightarrow \frac{dy}{dx} = \frac{3y-3}{2y-3x}$$

$$(b) \frac{d^2y}{dx^2} = \frac{(2y-3x) \left(3 \frac{dy}{dx} \right) - (3y-3) \left(2 \frac{dy}{dx} - 3 \right)}{(2y-3x)^2}$$

$$(c) \frac{dy}{dx} = \frac{3y-3}{2y-3x} = 0 \Rightarrow y = 1$$

However, when $y = 1$, $1^2 = 3x \cdot 1 - 3x = 0$, and it does not hold.

Therefore, there is no point on the curve where $y = 1$; thus, there is no horizontal tangent line.

$$(d) 2y-3x=0 \Rightarrow y = \frac{3}{2}x.$$

$$\text{When } y = \frac{3}{2}x, \left(\frac{3}{2}x \right)^2 = 3x \cdot \frac{3}{2}x - 3x \Rightarrow \frac{9}{4}x^2 - 3x = 0 \Rightarrow 3x \left(\frac{3}{4}x - 1 \right) = 0 \Rightarrow y = 0 \text{ or } y = \frac{4}{3}$$

When $x = 0$, $y^2 = 0 - 0$, we get $y = 0$.

$$\text{When } \frac{4}{3}, \text{ then } y^2 - 4y = 4 \Rightarrow y^2 - 4y + 4 = 0 \Rightarrow y = 2.$$

The coordinates of P could be $(0,0)$ or $\left(\frac{4}{3}, 2 \right)$.

4.7 Question 7

$$(a) f(6) = \int_0^6 (t^2 - 3t - 10) dt = -42$$

$$f'(x) = x^2 - 3x - 10, \text{ then } f'(6) = 8.$$

An equation for the tangent line is $y = -42 + 8(x - 6)$.

$$(b) f'(x) = x^2 - 3x - 10 = (x - 5)(x + 2) = 0 \Rightarrow x = -2 \text{ and } x = 5.$$

f has a relative maximum at $x = -2$ because $f'(x)$ changes from positive to negative there.

f has a relative minimum at $x = 5$ because $f'(x)$ changes from negative to positive there.

$$(c) f''(x) = 2x - 3 < 0 \Rightarrow x < \frac{3}{2}$$

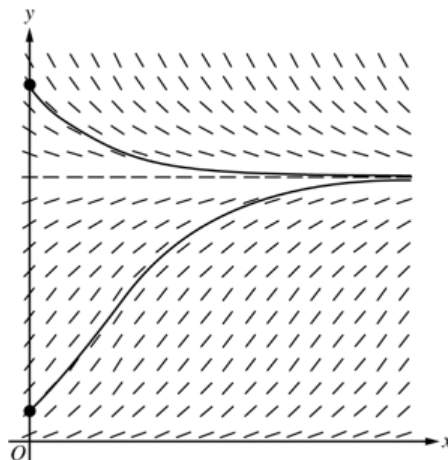
The graph of f is concave down on the interval $\left(-\infty, \frac{3}{2}\right)$ because $f''(x) < 0$ there.

$$(d) g'(x) = f'(\tan x) \cdot (\sec^2 x) = (\tan^2 x - 3 \tan x - 10)(\sec^2 x)$$

$$g'\left(\frac{\pi}{4}\right) = -24$$

4.8 Question 8

(a)



$$(b) \lim_{y \rightarrow 50} \frac{dy}{dx} = \lim_{y \rightarrow 50} -\frac{y}{4} \ln\left(\frac{y}{50}\right) = -\frac{50}{4} \ln\left(\frac{50}{50}\right) = 0$$

$$(c) f'(0) = \left. \frac{dy}{dx} \right|_{(x,y)=(0,20)} = -5 \ln\left(\frac{2}{5}\right)$$

Because $\frac{dy}{dx} > 0$ for $0 < y < 50$, f is increasing provided $f(x) < 50$.

Note that $y = 50$ is also a solution to the differential equation, so the graph of the particular solution $y = f(x)$ with $f(0) = 20$ is below the line $y = 30$ and cannot cross the line $y = 50$.

Thus, f is increasing for all x .

$$(d) \frac{d^2y}{dx^2} = -\frac{1}{4} \cdot \frac{dy}{dx} \cdot \ln\left(\frac{y}{50}\right) - \frac{y}{4} \cdot \frac{50}{y} \frac{1}{50} \frac{dy}{dx} = -\frac{1}{4} \frac{dy}{dx} \left(\ln\left(\frac{y}{50}\right) + 1 \right).$$

$$\frac{d^2y}{dx^2} = 0 \Rightarrow \left(\ln\left(\frac{y}{50}\right) + 1 \right) = 0 \Rightarrow y = 50e^{-1}.$$

Because $\frac{d^2y}{dx^2} > 0$ for $0 < y < 50e^{-1}$ and $\frac{d^2y}{dx^2} < 0$ for $50e^{-1} < y < 40$, $\frac{dy}{dx}$ attains its maximum value at $y = 50e^{-1}$.

4.9 Question 9

$$(a) \frac{f^{(20)}(0)}{20!} = \frac{(-1)^{20}}{3^{20}(20+1)} \Rightarrow f^{(20)}(0) = \frac{20!}{3^{20} \cdot 21}$$

$$(b) \left| \frac{\frac{x^{n+1}}{3^{n+1}(n+2)}}{\frac{x^n}{3^n(n+1)}} \right| = \left| x \left(\frac{n+1}{3(n+2)} \right) \right|, \text{ then } \lim_{n \rightarrow \infty} \left| x \left(\frac{n+1}{3(n+2)} \right) \right| = \left| \frac{x}{3} \right|.$$

$\left| \frac{x}{3} \right| < 1$ for $|x| < 3$, then The series converges for $(-3, 3)$.

When $x = -3$, the series is $\sum_{n=0}^{\infty} \frac{1}{n+1}$. This is the harmonic series, which diverges.

When $x = 3$ the series is $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$. This is the alternating harmonic series, which converges.

Thus, the interval of convergence of the Maclaurin series for f is $(-3, 3]$. (c) $f(1) \approx 1 - \frac{1}{6} + \frac{1}{27} = \frac{47}{54}$

(d) The series for $f(1)$ is an alternating series whose terms decrease in magnitude (absolute value) to 0. The alternating series error bound is the absolute value of the fourth term of the series. $\left| f(1) - \frac{47}{54} \right| < \left| \frac{(-1)^3(1)^3}{3^3(4)} \right| = \frac{108}{1} < \frac{1}{100}$.

4.10 Points of Allocation

4.10.1 Question 1

- (a) 1: Definite Integral; 1: Answer
- (b) 1: Integrand; 1: Answer
- (c) 1: Value; 1: Interpretation with units
- (d) 1: $\frac{dr}{dt} = \frac{dr}{dh} \cdot \frac{dh}{dr}$; 1: Use $\frac{dh}{dt}|_{h=10} = 0.6$ or $\frac{dh}{dt}|_{h=10} = r'(10)$; 1: Answer

4.10.2 Question 2

- (a) 1: Right Riemann sum set up; 1: Approximation; 1: Underestimate with reason
- (b) 1: Value; 1: Interpretation
- (c) 1: Definite Integral; 1: Answer
- (d) 1: $\int_0^{40} B(t)dt$; 1: Answer

4.10.3 Question 3

- (a) 1: Answer
- (b) 1: $x(2)$; 1: Definite Integral; 1: $y(2)$
- (c) 1: Answer with reason
- (d) 1: Sets $\frac{dy}{dx} = -1$; 1: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$; 1: $\frac{dx}{dt} = -2$; 1: Answer

4.10.4 Question 4

- (a) 1: Answer with reason
- (b) 1: Difference quotient or definite integral; 1: Average rate of change; 1: Differentiable implies continuous; 1: Conclusion using Mean Value Theorem **OR** Conclusion using Intermediate Value Theorem
- (c) 1: Answer; 1: Justification
- (d) 1: $H'(x)$; 1: Answer

4.10.5 Question 5

- (a) 1: $g(0)$; 1: $g(3)$
- (b) 1: Uses L'Hospital's Rule; 1: Answer with justification
- (c) 1: Uses partial fractions; 1: Antiderivatives; 1: Answer
- (d) 1: $\int_0^3 \sqrt{1 + (h'(x))^2} dx$; 1: Answer

4.10.6 Question 6

- (a) 1: Implicit differentiation; 1: Verification
- (b) 1: Quotient Rule; 1: $\frac{d^2y}{dx^2}$
- (c) 1: Considers $\frac{dy}{dx} = 0$; 1: Answer with explanation
- (d) 1: Considers $2y - 3x = 0$; 1: Considers both x -coordinates OR both y -coordinates OR one coordinate pair; 1: second coordinate pair

4.10.7 Question 7

- (a) 1: $f(6)$; 1: $f'(x)$; 1: Equation for tangent line
- (b) 1: Critical points; 1: Answers with justification
- (c) 1: Find $f''(x)$ and compare to zero; 1: Answer with reason
- (d) 1: Chain rule; 1: Answer

4.10.8 Question 8

- (a) 1: Top solution curve; 1: Bottom solution curve
- (b) 1: Limit
- (c) 1: $f'(0)$; 1: Explanation
- (d) 1: Product rule; 1: Chain Rule; 1: Attempts to solve $\frac{d^2y}{dx^2} = 0$ for y ; 1: Answer with justification

4.10.9 Question 9

- (a) 1: Answer
- (b) 1: Sets up appropriate ratio; 1: Computes limit of ratio; 1: Identifies interior of interval of convergence; 1: Considers both endpoints; 1: Analysis and interval of convergence
- (c) 1: Approximation
- (d) 1: Uses fourth term; 1: Justification