

2021 年 Calculus BC 选择题真题

CALCULUS BC
Section I, Part A
Time—1 hour
Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solving each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the best book. Do not spend too much time on any one problem.

In this exam:

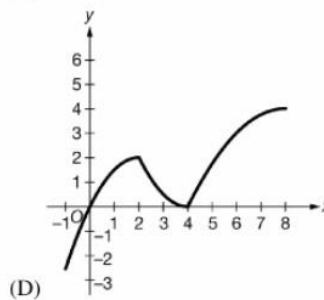
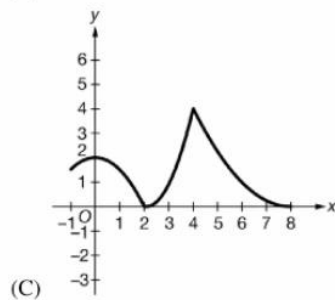
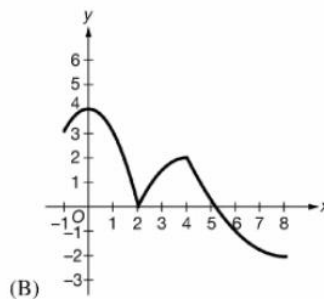
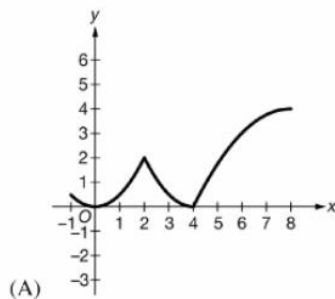
- (1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. $\int_1^3 (4x^3 - 6x + 5)dx =$.

- (A) 66
(B) 92
(C) 96
(D) 282

x	0	$0 < x < 2$	2	$2 < x < 4$	4	$4 < x < 8$
$f'(x)$	0	Negative	Does not exist	Positive	Does not exist	Negative
$f''(x)$	Negative	Negative	Does not exist	Negative	Does not exist	Positive

2. The first and second derivatives of a function f have the properties given in the table above. Which of the following could be the graph of f ?



3. If $f(x) = \sqrt[3]{5x^2 - 7}$, then $f'(x) =$

(A) $\frac{10x}{3}(5x^2 - 7)^{-\frac{2}{3}}$

(B) $\frac{1}{3}(5x^2 - 7)^{-\frac{2}{3}}$

(C) $\frac{3}{4}(5x^2 - 7)^{\frac{4}{3}}$

(D) $\frac{1}{3}(10x)^{-\frac{2}{3}}$

4. A particle moves in the xy -plane so that its position at time $t > 0$ is given by $x = 2(t) \ln t$ and $y(t) = \frac{4}{t+1}$. What is the speed of the particle when $t = 1$?

(A) 1

(B) $\sqrt{2}$

(C) 2

(D) $\sqrt{5}$

5. If $\cos b = \frac{1}{3}$ for a constant b , then $\sum_{n=0}^{\infty} (\sin^2 b)^n =$

(A) $\frac{1}{9}$

(B) $\frac{8}{9}$

(C) $\frac{9}{8}$

(D) 9

6. If $f(x) = x^2 + 4x - 2$, $g(x) = 4x + 2$, and $h(x) = \frac{f(x)}{g(x)}$, then $h'(1) =$

(A) $-\frac{2}{3}$

(B) $\frac{2}{3}$

(C) $\frac{4}{3}$

(D) $\frac{3}{2}$

7. Let f be a function with derivatives of all orders. If $P_3(x)$ is the third-degree Taylor polynomial for f about $x = 0$, which of the following statements about $P_3(x)$ must be true?

(A) $P_3'''(0) = f'''(0)$

(B) $P_3'''(0) = \frac{f'''(0)}{3!}$

(C) One of the terms of $P_3(x)$ is $\frac{x^2}{2!}$.

(D) One of the terms of $P_3(x)$ is $\frac{x^3}{3!}$.

8. $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + 27}}{\sqrt{x^2 + 27x + 3}}$ is

(A) 2

(B) 3

(C) 4

(D) infinite

9. $\lim_{x \rightarrow 0} \frac{\int_0^x \sin(3t^2) dt}{4x^3}$ is

(A) $-\frac{1}{4}$

(B) 0

(C) $\frac{1}{4}$

(D) nonexistent

$$g(x) = \begin{cases} x^2 + 1 & \text{for } x < 3 \\ 13 - x & \text{for } x \geq 3 \end{cases}$$

10. Let g be the function given above. What is the average rate of change of g on the closed interval $[1, 5]$?

- (A) $\frac{1}{2}$
- (B) $\frac{3}{2}$
- (C) 5
- (D) Undefined

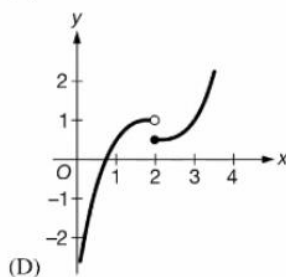
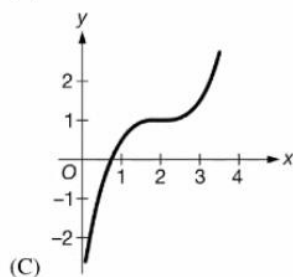
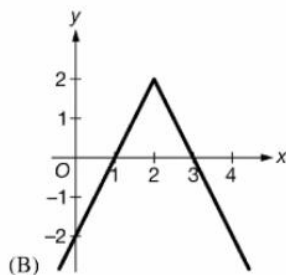
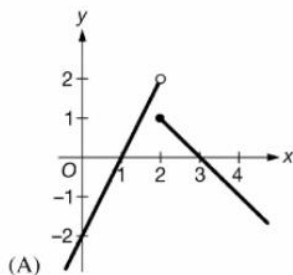
11. $\int \frac{x-1}{x+1} dx =$

- (A) $\frac{2}{(x+1)^2} + C$
- (B) $x - 2 \ln|x+1| + C$
- (C) $\frac{x^2 - 2x}{x^2 + 2x} + C$
- (D) $\left(\frac{1}{2}x^2 - x\right) \ln|x+1| + C$

12. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = \frac{1}{2}$. Using the line tangent to the graph of f at $x = 2$ as a local linear approximation for f , what is the estimate for $f(1.8)$?

(A) 2.5
(B) 2.8
(C) 2.9
(D) 3.1

13. Let g be a function such that $\lim_{x \rightarrow 2} g(x) = g(2)$ and $\lim_{x \rightarrow 2^-} g'(x) \neq \lim_{x \rightarrow 2^+} g'(x)$. Which of the following could be the graph of g ?



14. Which of the following is the solution to the differential equation $\frac{dy}{dx} = y \cos x$ with the initial condition $y\left(\frac{\pi}{2}\right) = e$?

(A) $y = \cos x + e$
(B) $y = e^{\cos x + 1}$
(C) $y = e \cdot \sin x$
(D) $y = e^{\sin x}$

15. $\int_{10}^{\infty} \frac{1000}{x^2} dx$ is

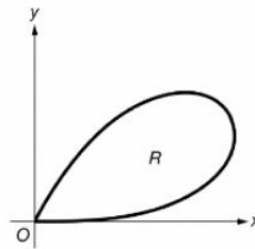
(A) $\frac{1}{3}$
(B) 2
(C) 100
(D) divergent

16. If $\frac{dy}{dx} = 16 \sin^3 x \cos x$ and $y = 16$ when $x = \frac{\pi}{2}$, what is the value of y when $x = \frac{\pi}{6}$?

(A) $\frac{49}{4}$
(B) $\frac{1}{4}$
(C) 10
(D) 13

17. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$. Which of the following statements is true?

- (A) Both series converge absolutely.
- (B) Both series converge conditionally.
- (C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges absolutely, and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges conditionally.
- (D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges conditionally, and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges absolutely.



18. Let R be the region in the first quadrant that is bounded by the polar curve $r = \sqrt{6 \sin(3\theta)}$, as shown in the figure above. What is the area of R ?

- (A) 1
- (B) 2
- (C) 4
- (D) 6

19. Consider the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \dots$. Which of the following statements is true?

- (A) The series converges, and $\frac{1}{3} < \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} < \frac{2}{3}$.
- (B) The series converges, and $\frac{2}{3} < \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} < 1$.
- (C) The series converges, and $1 < \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.
- (D) The series diverges.

20. Let f be a continuous function such that $f(x) = f(-x)$ for all x . If $\int_{-2}^0 f(x) dx = 4$ and $\int_1^2 f(x) dx = 1$, what is the value of $\int_0^1 f(x) dx$?

- (A) -5
- (B) 2
- (C) 3
- (D) 5

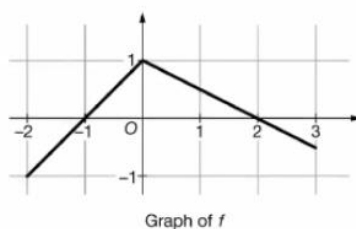
21. What are all values of x for which the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n}$ converges?

- (A) $-1 < x < 1$ only
- (B) $-1 < x \leq 1$ only
- (C) $-1 \leq x \leq 1$ only
- (D) $-\infty < x < \infty$

t	0	2	3	6
$f(t)$	12	36	48	42

22. The function f is continuous on the closed interval $[0, 6]$ and has values given in the table above. What is the approximation to $\int_0^6 f(t) dt$ obtained by using a right Riemann sum with the subintervals $[0, 2]$, $[2, 3]$, and $[3, 6]$?

- (A) 204
- (B) 225
- (C) 246
- (D) 252



23. The graph of the function f consists of two line segments, as shown in the figure above. Let g be the function defined by $g(x) = \int_{-2}^x f(t) dt$. At what value of x does g attain its global maximum on the closed interval $[-2, 3]$?

- (A) -2
- (B) -1
- (C) 0
- (D) 2

24. Let f be a differentiable function such that $\int e^{-x} f(x) dx = -e^{-x} f(x) + \int 2xe^{-x} dx$. Which of the following could be $f(x)$?

- (A) $2x$
- (B) x^2
- (C) $-e^{-x}$
- (D) e^{-x}

25. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 5x + 6}$ is

- (A) -4
- (B) 0
- (C) 4
- (D) nonexistent

26. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{10^n}{n!}$

II. $\sum_{n=1}^{\infty} e^{-n}$

III. $\sum_{n=1}^{\infty} \frac{2^n}{n(n+1)}$

- (A) I only
- (B) II only
- (C) I and II only
- (D) I, II, and III

27. Let R be the region in the first quadrant bounded by the graph of $y = e^{-x}$, the x -axis, the y -axis, and the line $x = b$. Region R is the base of a solid whose cross sections perpendicular to the x -axis are squares. If the volume of the solid is $\frac{4}{9}$, what is the value of b ?

- (A) $\ln\left(\frac{3}{\sqrt{17}}\right)$
- (B) $\ln\left(\frac{3}{\sqrt{7}}\right)$
- (C) $\ln\left(\frac{9}{5}\right)$
- (D) $\frac{1}{2}\ln 9$

28. If $x = t^4 - 1$ and $y = 4t + 1$, what $\frac{d^2y}{dx^2}$ is in terms of t ?

- (A) 0
- (B) $-\frac{3}{t^4}$
- (C) $-\frac{3}{4t^5}$
- (D) $-\frac{3}{4t^7}$

29. For which of the following integrals is $\sum_{k=1}^n \ln\left(2 + \frac{5}{n}k\right) \frac{5}{n}$ the right Riemann sum approximation with n subintervals of equal length?

(A) $\int_2^7 \ln x dx$

(B) $\int_2^7 \ln(2+x) dx$

(C) $\int_0^5 \ln(2+5x) dx$

(D) $\int_2^7 \ln(2+5x) dx$

30. If $\sum_{n=1}^{\infty} a_n = 5$ and $a_n > 0$ for all n , which of the following must be true?

(A) $\lim_{n \rightarrow \infty} a_n = 0$

(B) $\lim_{n \rightarrow \infty} a_n = 5$

(C) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$

(D) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$

CALCULUS BC
Section I, Part B
Time—45 minutes
Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solving each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the best book. Do not spend too much time on any one problem.

In this exam:

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

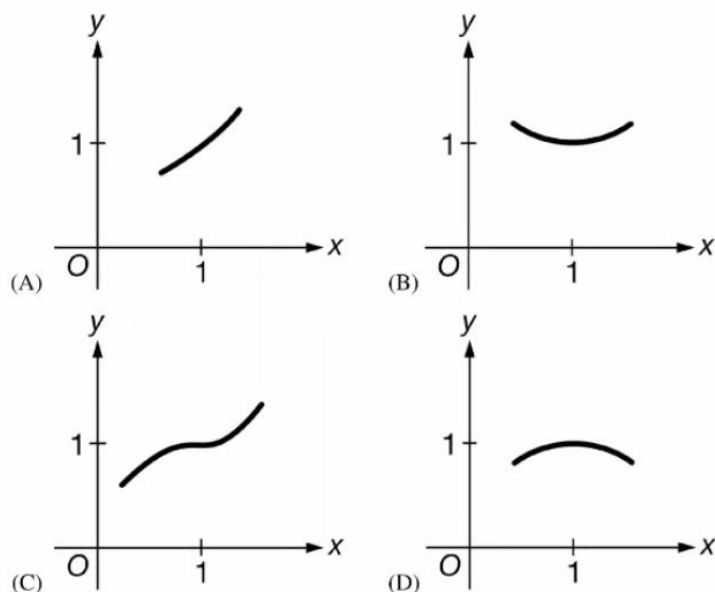
76. In a small town, water is being used at a rate of $r(t) = 12 + 9\sin(0.5t - 1.6)$ thousand gallons per hour, where t is measured in hours since midnight. To the nearest integer, how much water, in thousands of gallons, is used in the town during the 6-hour period from 3 A.M. ($t = 3$) to 9 A.M. ($t = 9$)?

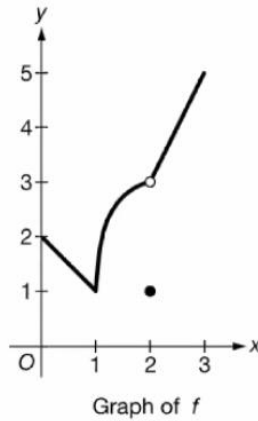
- (A) 3
- (B) 76
- (C) 107
- (D) 125

77. Let R be the region in the first quadrant bounded by the y -axis, the line $y = 2$, and the curve $y = 2\sqrt[3]{x}$. What is the volume of the solid formed by revolving R about the y -axis?

- (A) 0.500
- (B) 0.898
- (C) 1.571
- (D) 23.937

78. Which of the following could be the graph of a solution of the differential equation $\frac{dy}{dx} = (x - 1)y^2$ near the point $(1, 1)$?





79. The graph of the function f is shown above. What is $\lim_{x \rightarrow 2} f(x)$?

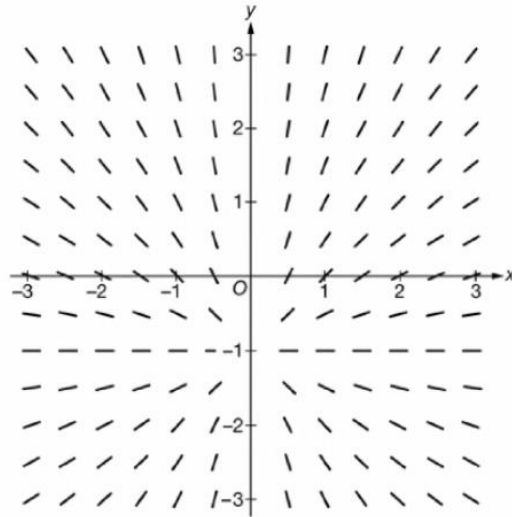
- (A) 1
- (B) 2
- (C) 3
- (D) The limit does not exist.

80. A particle moves along the curve $y = \frac{15}{x^2 + 1.3^x}$ for $x > 0$. The x -coordinate of the particle changes at a constant rate of 3 units per second. At the instant when the y -coordinate of the particle is 2, what is the rate of change of the y -coordinate of the particle, in units per second?

- (A) -0.466
- (B) -0.787
- (C) -1.397
- (D) -4.190

81. An ellipse is defined parametrically by $x(t) = 1 + 2\cos t$ and $y(t) = 3\sin t$ for $0 \leq t \leq 2\pi$. What is the perimeter of the ellipse?

- (A) 6.283
- (B) 15.865
- (C) 16.574
- (D) 40.841



82. Shown above is a slope field for which of the following differential equations?

- (A) $\frac{dy}{dx} = \frac{y+1}{x}$
- (B) $\frac{dy}{dx} = -\frac{y+1}{x}$
- (C) $\frac{dy}{dx} = \frac{y-1}{x}$
- (D) $\frac{dy}{dx} = -\frac{y-1}{x}$

$$g(x) = \begin{cases} 3x^2 - 5x + 1 & \text{if } x \leq 0 \\ \frac{1}{x(x+1)} & \text{if } x > 0 \end{cases}$$

83. Let g be the function given above. Which of the following statements is true?

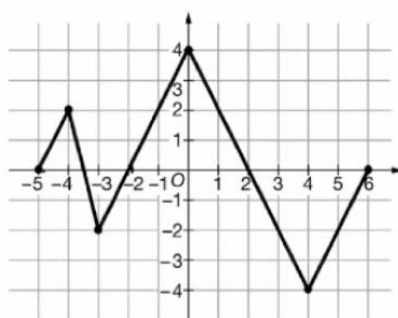
- (A) g is continuous at $x = 0$.
- (B) g has a jump discontinuity at $x = 0$.
- (C) g has a removable discontinuity at $x = 0$.
- (D) g has a discontinuity due to a vertical asymptote at $x = 0$.

84. The first derivative of the function f is given by $f'(x) = xe^{\sin x}$. How many points of inflection does the graph of f have on the interval $-3 < x < 6$?

- (A) One
- (B) Two
- (C) Three
- (D) Four

85. For time $t \geq 0$, the velocity of a particle moving in the xy -plane is given by the vector $\langle t^5, (1 + \sqrt{t})^2 \rangle$. At time $t = 1$, the position of the particle is $(4, 3)$. What is the distance between the position of the particle at time $t = 1$ and the position of the particle at time $t = 2$?

- (A) 11.603
- (B) 12.129
- (C) 28.142
- (D) 76.011



Graph of f

86. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-5}^x f(t) dt$, how many zeros does g have on the closed interval $[-5, 6]$?

- (A) One
- (B) Three
- (C) Four
- (D) Five

87. The derivative of the function f is given by $f'(x) = 5(x-1)^2 \sin(x^2)$. On the interval $(-1, 3)$, at which of the following values of x does f have a local minimum?
- (A) 1.558
(B) 1.772
(C) 2.248
(D) 2.507
88. The derivative of the function f is given by $f'(x) = 1 - 5 \cos(3x^2 - 2)$. For how many values of x in the interval $0 \leq x \leq 2$ is the line tangent to the graph of $y = f(x)$ at the point $(x, f(x))$ parallel to the line $y = 1 + 4x$?
- (A) One
(B) Two
(C) Three
(D) Four
89. Information is spreading through a population. The number of people who have the information at time t hours is modeled by an increasing function $y(t)$ that satisfies a logistic differential equation. If the information is spreading most rapidly when 7500 people have the information, which of the following differential equations could represent this model?
- (A) $\frac{dy}{dt} = k(7500 - y)$
(B) $\frac{dy}{dt} = k(15000 - y)$
(C) $\frac{dy}{dt} = ky(7500 - y)$
(D) $\frac{dy}{dt} = ky(15000 - y)$
90. Paint spills onto a floor in a circular pattern. The radius of the spill increases at a constant rate of 2.5 inches per minute. How fast is the area of the spill increasing when the radius of the spill is 18 inches?
- (A) $5\pi \text{ in}^2/\text{min}$
(B) $36\pi \text{ in}^2/\text{min}$
(C) $45\pi \text{ in}^2/\text{min}$
(D) $90\pi \text{ in}^2/\text{min}$

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART B ONLY.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

1 - 5 ABADD
6 - 10 BAACB
11-15 BCBDC
16-20 ADBBC
21-25 CCDBA
26-30 CDDAA
76-80 CBBCD
81-85 BADBA
86-90 BDCDD