

2022 AP Calculus AB & BC International Exam Questions

Extrated from the AP Question Bank

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1 AB&BC - Common Part

1.1 Multiple Choice Questions: No Calculator

$$f(x) = \begin{cases} 4 - x^2 & \text{for } x \leq 1 \\ 5 - 2x & \text{for } x > 1 \end{cases}$$

1. Let f be the function defined as shown. Which of the following statements about f is true at $x = 1$?
- (A) f is both continuous and differentiable.
(B) f is continuous but not differentiable.
(C) f is differentiable but not continuous.
(D) f is neither continuous nor differentiable.
2. Let f be a differentiable function such that $f'(x) \geq 1$ for all x . If $a < b$, which of the following statements could be false?
- (A) $\frac{f(b) - f(a)}{b - a} \geq 1$
(B) $f(b) > f(a)$
(C) There is a value c in the open interval (a, b) such that $f(c) = 0$.
(D) There is a value c in the open interval (a, b) such that $f(c) = \frac{f(b) + f(a)}{2}$.

x	$x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$1 < x < 2$	$x = 2$	$2 < x < 3$	$x = 3$	$x > 3$
$f(x)$	$x^2 - 1$	1	$x^2 + 1$	1	$3 - x$	1	$\frac{1}{x - 2}$	1	$\frac{x^2}{9}$

3. Let f be the piecewise function defined in the table shown above. At which of the following values of x does f have a removable discontinuity?
- (A) 0
(B) 1
(C) 2
(D) 3
4. Which of the following is a solution to the differential equation $\frac{dy}{dx} = e^{2x} + 2y$?
- (A) $y = e^{2x}$
(B) $y = -e^{2x}$
(C) $y = xe^{2x}$
(D) $y = -xe^{2x}$

5. If $y = 6x^2 - x^3$, what is the maximum value of the instantaneous rate of change of y with respect to x ?
- (A) 12
(B) 16
(C) 27
(D) 32
6. What is the area of the region enclosed by the curve $y = \frac{2}{x}$, the x -axis, and the lines $x = 1$ and $x = 2$?
- (A) $\ln 4 - \ln 2$
(B) $2 \ln 2$
(C) $\frac{3}{4}$
(D) $\frac{3}{2}$
7. If $y = \sqrt{3x^2 + 1}$, then $\frac{dy}{dx} =$
- (A) $\frac{1}{2\sqrt{6x}}$
(B) $\frac{1}{2\sqrt{3x^2 + 1}}$
(C) $\frac{3x}{\sqrt{3x^2 + 1}}$
(D) $3x\sqrt{3x^2 + 1}$
8. $\int_{-1}^3 \frac{|x|}{x} dx$ is
- (A) $\frac{5}{4}$
(B) 2
(C) 4
(D) undefined
9. If $f(x) = 12x^{\frac{1}{2}} + 3x + \frac{32}{x} + 5$, then $f'(5) =$
- (A) 4
(B) 8
(C) 9
(D) 13

10. On the interval $-3 < x < 3$, the first derivative of the function f is given by $f'(x) = \frac{x^2 - 4}{(x + 3)^2}$, and the second derivative of f is given by $f''(x) = \frac{6x + 8}{(x + 3)^3}$. On which of the following intervals is the graph of f concave down?
- (A) $-3 < x < -\frac{4}{3}$
 (B) $-\frac{4}{3} < x < 3$
 (C) $-3 < x < -2$ and $2 < x < 3$
 (D) $-2 < x < 2$
11. $\int_0^{\frac{\pi}{2}} \sin\left(\frac{x}{2}\right) dx =$
- (A) $\sqrt{2} - 2$
 (B) $\frac{2 - \sqrt{2}}{4}$
 (C) $\frac{2 - \sqrt{2}}{2}$
 (D) $2 - \sqrt{2}$
12. For which of the following values of b does $\int_2^b (2x - 3) dx = 0$?
- (A) -2 only
 (B) 2 only
 (C) -2 and 2
 (D) 1 and 2
13. For time $t \geq 0$, the position of a particle moving along a straight line is given by $s(t) = \int_0^t e^{\sin x} \cos x dx$. What is the velocity of the particle at time $t = 3$?
- (A) $e^{\sin 3} - 1$
 (B) $e^{\sin 3} \cos 3$
 (C) $e^{\sin 3} \cos 3 - 1$
 (D) $e^{\sin 3} (\cos 3)^2 - e^{\sin 3} \sin 3$
14. If $f(x) = e^{7x}$, then $f^{(7)}(x) =$
- (A) $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$
 (B) $\frac{1}{7^7} e^{7x}$
 (C) e^{7x}
 (D) $7^7 e^{7x}$

15. What is the slope of the line tangent to the curve $x^2y + y^2 = 21$ at the point $(2, 3)$?

- (A) $-\frac{9}{2}$
- (B) -2
- (C) $-\frac{6}{5}$
- (D) $\frac{9}{10}$

16. $\int (x^3 + 2)^2 dx =$

- (A) $\frac{(x^3 + 2)^3}{9x^2} + C$
- (B) $\frac{1}{3}(x^3 + 2)^3 + C$
- (C) $6x^2(x^3 + 2) + C$
- (D) $\frac{1}{7}x^7 + x^4 + 4x + C$

17. Using the substitution $u = x - 1$, $\int_2^{10} 3x\sqrt{x-1}dx$ is equivalent to

- (A) $\int_1^9 3(u+1)\sqrt{u}du$
- (B) $\int_1^9 3u(u+1)du$
- (C) $\int_2^{10} (3u+1)\sqrt{u}du$
- (D) $\int_2^{10} 3u\sqrt{u+1}du$

18. If $\frac{dx}{dt} = 4t^3$ and if $x = 5$ when $t = 1$, what is the value of x when $t = 2$?

- (A) 12
- (B) 16
- (C) 20
- (D) 41

19. If $f(x) = \ln(x) \cos(4x)$, then $f'(x) =$

- (A) $-\frac{4 \sin(4x)}{x}$
(B) $\frac{\cos(4x)}{x} - \ln(x) \sin(4x)$
(C) $\frac{\cos(4x)}{x} - 4 \ln(x) \sin(4x)$
(D) $\frac{\cos(4x)}{x} + 4 \ln(x) \sin(4x)$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	5	3	2	4

20. The table shown gives values of the differentiable functions f and g and their derivatives at $x = 2$. If $h(x) = \frac{f(x)}{g(x)}$, what is the value of $h'(2)$?

- (A) $-\frac{7}{2}$
(B) $-\frac{7}{8}$
(C) $\frac{3}{4}$
(D) $\frac{13}{2}$

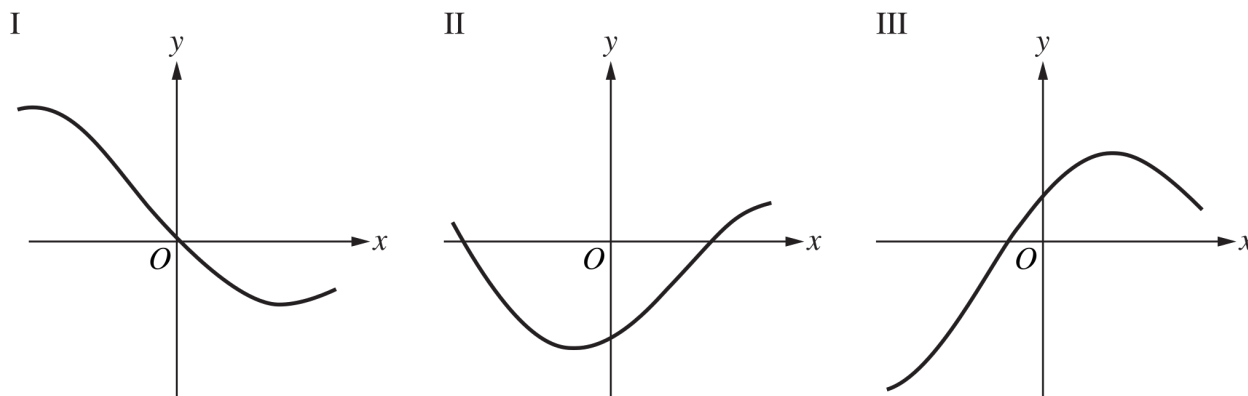
$$f(x) = \begin{cases} \frac{\sin(3x-6)}{x-2} & \text{for } x \neq 2 \\ b & \text{for } x = 2 \end{cases}$$

21. Let f be the function defined as shown, where b is a constant. For what value of b is f continuous at $x = 2$?

- (A) -3
(B) 0
(C) $\frac{1}{3}$
(D) 3

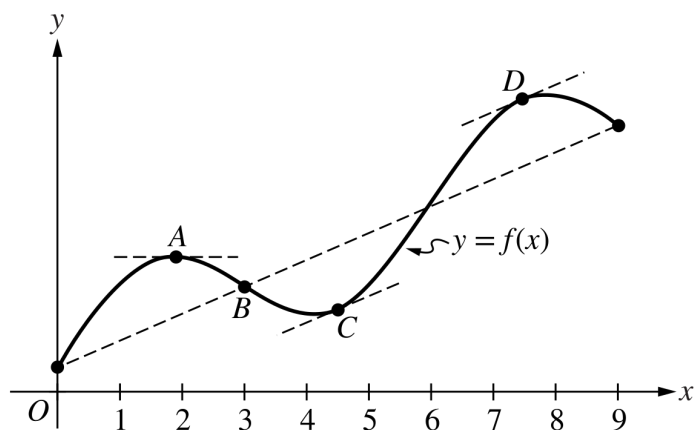
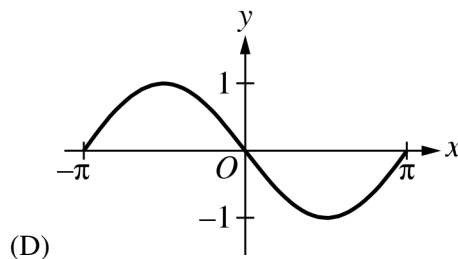
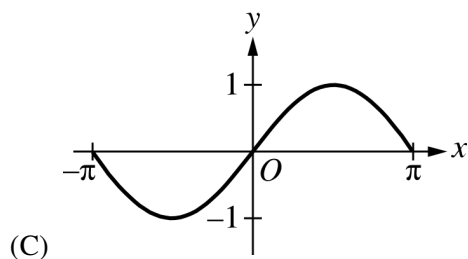
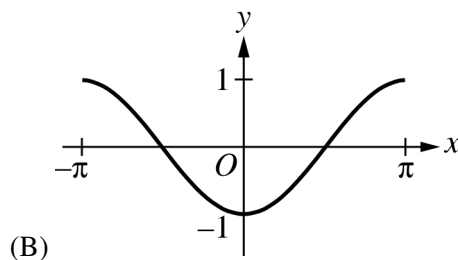
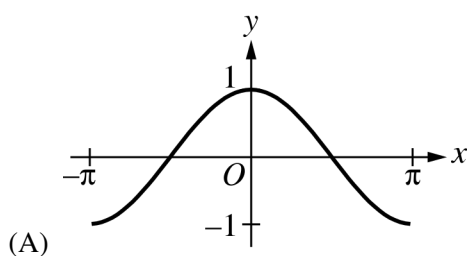
22. The function f is continuous and increasing on the interval $[0, 6]$ and $\int_0^6 f(x)dx = 8$. Consider Riemann sums with three subintervals of equal length for $\int_0^6 f(x)dx$. Which of the following statements must be true?
- I. $2f(0) + 2f(2) + 2f(4) < 8$
 - II. $2f(1) + 2f(3) + 2f(5) > 8$
 - III. $2f(2) + 2f(4) + 2f(6) > 8$
- (A) I and II only
 - (B) I and III only
 - (C) II and III only
 - (D) I, II, and III
23. For time $t \geq 0$, the position of an object moving along the x -axis is given by $x(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 1$. Which of the following statements about the object is true at time $t = 0.5$?
- (A) The object is moving to the right and slowing down.
 - (B) The object is moving to the right and speeding up.
 - (C) The object is moving to the left and slowing down.
 - (D) The object is moving to the left and speeding up.
24. The derivative of the function f is given by $f'(x) = \frac{x^3 + 2x + 6}{x^4 - 3x + 2}$. If $f(2) = 0.76$, what is the approximation for $f(2.1)$ found by using the line tangent to the graph of f at $x = 2$?
- (A) 0.61
 - (B) 0.86
 - (C) 0.91
 - (D) 2.26
25. Let f be the function given by $f(x) = \frac{e^{2x}}{1 - e^{2x}}$. Which of the following describes all asymptotes of the graph of f ?
- (A) No vertical asymptotes and two horizontal asymptotes
 - (B) One vertical asymptote and no horizontal asymptotes
 - (C) One vertical asymptote and two horizontal asymptotes
 - (D) Two vertical asymptotes and two horizontal asymptotes

26. Three graphs labeled I, II, and III are shown below. One is the graph of f , one is the graph of f' , and one is the graph of f'' . Which of the following correctly identifies each of the three graphs?



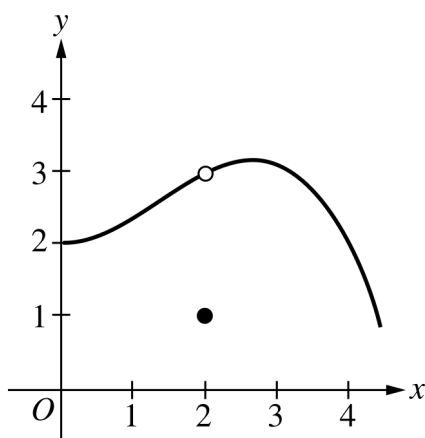
- (A) The graph of f is I, the graph of f' is II, and the graph of f'' is III.
 (B) The graph of f is I, the graph of f' is III, and the graph of f'' is II.
 (C) The graph of f is II, the graph of f' is III, and the graph of f'' is I.
 (D) The graph of f is III, the graph of f' is II, and the graph of f'' is I.

27. Which of the following could be the graph of $y = \frac{\cos(x+0.001) - \cos(x)}{0.001}$?



28. The function f shown in the figure is continuous on the closed interval $[0, 9]$ and differentiable on the open interval $(0, 9)$. Which of the following points satisfies conclusions of both the Intermediate Value Theorem and the Mean Value Theorem for f on the closed interval $[0, 9]$?

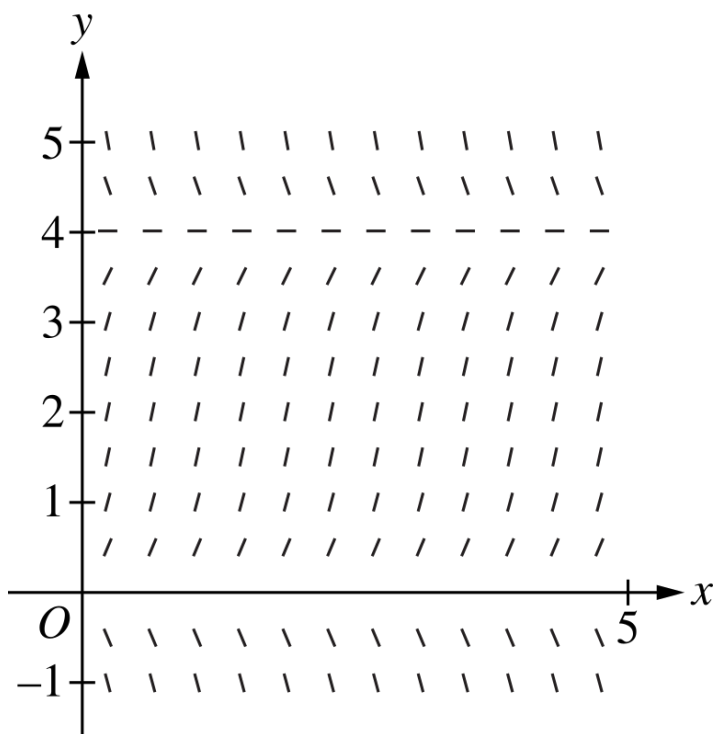
- (A) A
- (B) B
- (C) C
- (D) D



Graph of f

29. The graph of the function f is shown. What is $\lim_{x \rightarrow 2} f(x)$?

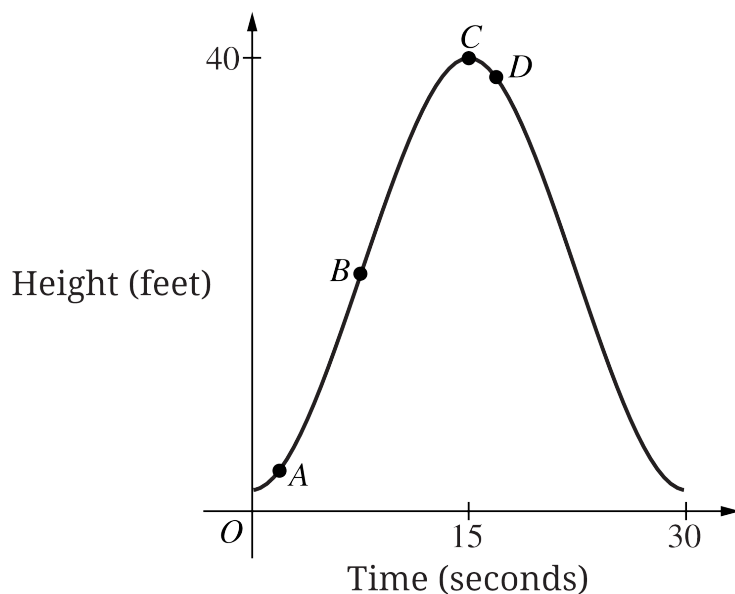
- (A) 0
- (B) 1
- (C) 3
- (D) The limit does not exist.



30. The slope field shown is for the differential equation $\frac{dy}{dx} = ky - 2y^2$, where k is a constant. What is the value of k ?

- (A) 2
- (B) 4
- (C) 6
- (D) 8

1.2 Multiple Choice Questions: Calculator Allowed



1. An amusement park ride reaches a maximum height of 40 feet above the ground. The graph shows the height above the ground, in feet, of a passenger on the ride for times $0 \leq t \leq 30$ seconds. At which of the following points is the height of the passenger above the ground changing the fastest?

(A) A
 (B) B
 (C) C
 (D) D

2. The advertising budget of a company is modeled by the differentiable function D , where $D(t)$ is measured in dollars and t is the number of years since the company was founded. For $0 \leq t \leq 10$, the company's advertising budget increased each year, but the rate of change of the advertising budget decreased each year. Which of the following could be a table of values of D ?

(A)

t	1	5	7
$D(t)$	50,000	55,000	53,000

(B)

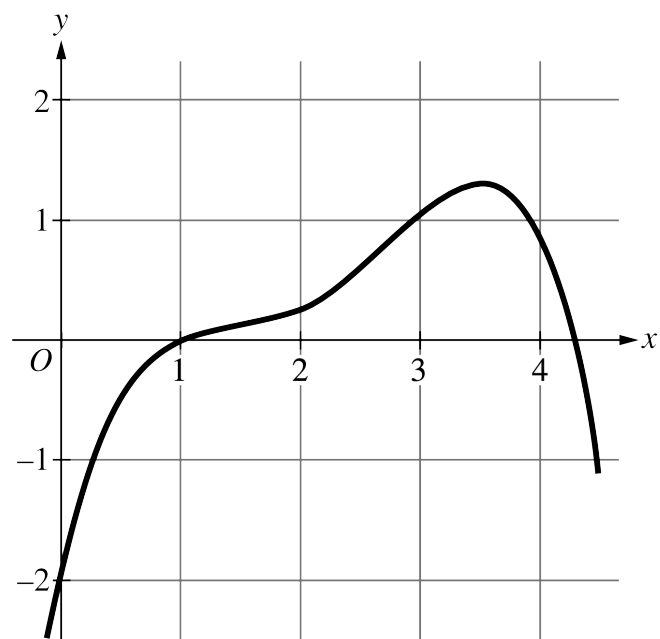
t	1	5	7
$D(t)$	50,000	54,000	60,000

(C)

t	1	5	7
$D(t)$	50,000	53,000	55,000

(D)

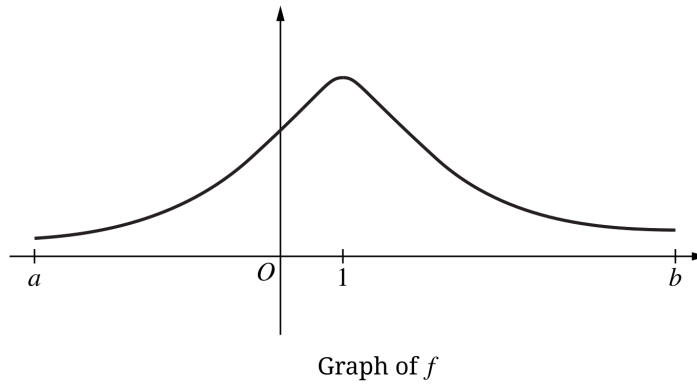
t	1	5	7
$D(t)$	50,000	58,000	60,000



Graph of f'

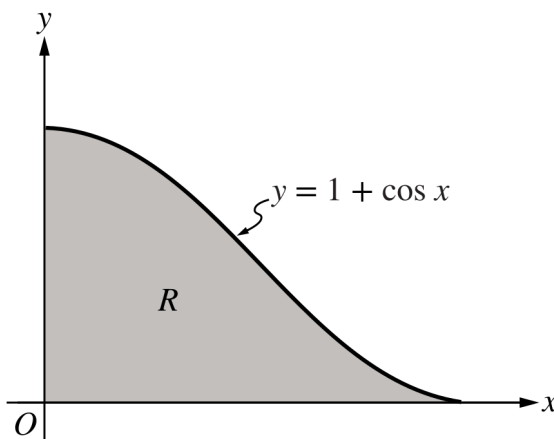
3. The graph of f' , the derivative of the function f , is shown. Which of the following differences is greatest?

- (A) $f(1) - f(0)$
- (B) $f(2) - f(1)$
- (C) $f(3) - f(2)$
- (D) $f(4) - f(3)$



4. The figure shows the graph of the function f on the open interval (a, b) . Let g be the function given by $g(x) = \int_1^x f(t) dt$. Which of the following statements must be true?
- I. g is increasing on (a, b) .
 - II. g has a relative maximum on (a, b) .
 - III. g has exactly one zero on (a, b) .
- (A) I only
 (B) II only
 (C) I and II only
 (D) I and III only
5. What is the average value of $y = e^{\sqrt{x}}$ on the interval $[1, \pi]$?
- (A) 1.479
 (B) 4.246
 (C) 4.302
 (D) 9.092
6. A particle moves along a line so that its position at any time $t \geq 0$ is given by $s(t)$. If $s''(t) = 2 - \sqrt{t^3 + 1}$ and $s'(1) = 1.5$, then the velocity of the particle at $t = 2.5$ is
- (A) positive and increasing
 (B) positive and decreasing
 (C) negative and increasing
 (D) negative and decreasing
7. Two particles move along the x -axis with velocities given by $v_1(t) = -3t^2 + 8t + 5$ and $v_2(t) = \sin t$ for time $t \geq 0$. At what time t do the two particles have equal acceleration?
- (A) 1.287
 (B) 3.194
 (C) 5.010
 (D) There is no such time t .

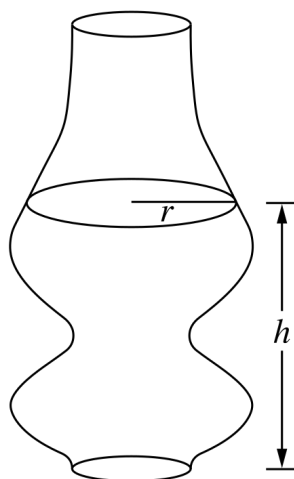
8. Let h be a function with first derivative given by $h'(x) = x \cos \sqrt{x^3}$ for $x > 0$. At what values of x does h have local maxima on the interval $0 < x < 5$?
- (A) 0.815 and 3.442
 (B) 1.351 and 3.951 only
 (C) 2.811 and 4.945 only
 (D) 1.351, 2.811, 3.951, and 4.945
9. The first derivative of the function f is given by $f'(x) = (3 - x^2) \cos x + 2x \sin x$. How many points of inflection does the graph of f have on the interval $-2 < x < 2$?
- (A) Zero
 (B) One
 (C) Two
 (D) Three



10. Let R be the shaded region in the first quadrant bounded by the x -axis, the y -axis, and the graph of $y = 1 + \cos x$, as shown in the figure. The shape of a chocolate treat can be modeled by rotating R around the x -axis. What is the volume of the chocolate treat?
- (A) 3.142
 (B) 4.712
 (C) 9.870
 (D) 14.804

11. The temperature inside a house t hours after midnight ($t = 0$) is given by the twice-differentiable function H , where $H(t)$ is measured in degrees Fahrenheit. Which of the following is an interpretation of the expression $H(0) + \int_0^8 H(t)dt$?
- (A) The temperature, in degrees Fahrenheit, inside the house at 8 A.M. ($t = 8$)
 - (B) The average temperature, in degrees Fahrenheit, inside the house between midnight and 8 A.M. ($t = 8$)
 - (C) The change in temperature, in degrees Fahrenheit, inside the house between midnight and 8 A.M. ($t = 8$)
 - (D) The average rate of change of the temperature, in degrees Fahrenheit per hour, inside the house between midnight and 8 A.M. ($t = 8$)
12. The cost, in dollars, to paint x square feet of a house is modeled by the differentiable function $C(x)$. Which of the following is the best interpretation of the statement $C'(300) = 6$?
- (A) The cost of painting 6 square feet is \$300.
 - (B) The cost of painting 300 square feet is \$6.
 - (C) The cost of painting 300 square feet is \$6 per square foot.
 - (D) When 300 square feet have been painted, the cost to paint the next square foot is approximately \$6.
13. Let g be a continuous function such that $\int_a^b 2g(x)dx = 5$ and $\int_a^c 4g(x)dx = 7$. What is the value of $\int_b^c g(x)dx$?
- (A) $-\frac{3}{4}$
 - (B) 2
 - (C) $\frac{17}{4}$
 - (D) 18
14. The radius of a circle is increasing at a constant rate of 0.5 meter per second. What is the rate of increase in the circumference of the circle at the instant when the radius of the circle is 3 meters?
- (A) 0.5 m/sec
 - (B) 1.5 m/sec
 - (C) π m/sec
 - (D) 3π m/sec
15. The function g is defined for all positive x , and $0 < g(x) < x$. Of the following, $\lim_{x \rightarrow \infty} \frac{x}{g(x)}$ could not be
- (A) 0
 - (B) 1
 - (C) 3
 - (D) ∞

1.3 Free-Response Questions: Calculator Allowed

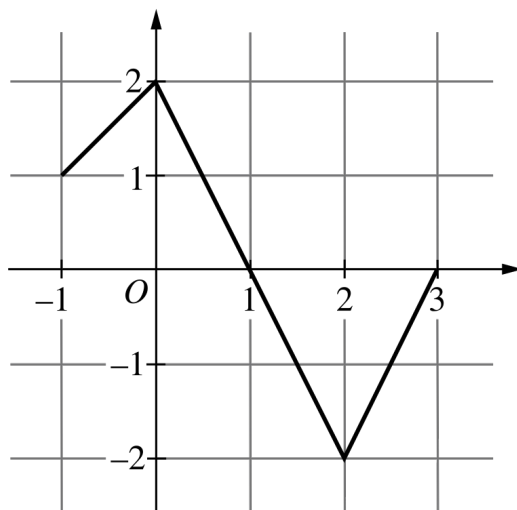


h (centimeters)	0	6.2	13	20
$r(h)$ (centimeters)	2.5	2.7	4.3	2.6

1. Juice is sold in 20-centimeter-tall bottles with horizontal cross sections parallel to the base that are circles, as shown in the figure. The radius of the circular cross section at height h above the base of the bottle is given by a differentiable function r , where h and $r(h)$ are measured in centimeters. Selected values of $r(h)$ are given in the table shown.
 - (a) Approximate $r'(3.1)$ using the average rate of change of r over the interval $0 \leq h \leq 6.2$. Show the computations that lead to your answer. Indicate units of measure.
 - (b) Interpret the meaning of $\frac{1}{20} \int_0^{20} r(h) dh$ in the context of the problem. Use a trapezoidal sum with the three subintervals $[0, 6.2]$, $[6.2, 13]$, and $[13, 20]$ to approximate the value of $\frac{1}{20} \int_0^{20} r(h) dh$.
 - (c) The radius of the circular cross section at height h above the base of the bottle can also be modeled by the function f , where h and $f(h)$ are measured in centimeters. Let $f(h) = 3\sqrt{2 - 1.25 \cos\left(\frac{\pi}{100}(h - 20)^2\right)}$ for $0 \leq h \leq 20$. Based on the model, at a height of $h = 12$ centimeters, is the radius increasing or decreasing as h increases? Give a reason for your answer.
 - (d) Using the model f defined in part (c), find the volume of the bottle.

2. The position of a particle moving along the x -axis at time t is given by the twice-differentiable function $x(t)$, where $x(0) = 2$. For $0 \leq t \leq 5$, the derivative of $x(t)$ is given by $x'(t) = \sin(t^{1.1})\sqrt{10t+4}$.
- (a) Find all times t in the open interval $0 < t < 5$ when the particle changes direction. Give a reason for your answer.
 - (b) Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.
 - (c) Find the acceleration of the particle at time $t = 4$. Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.
 - (d) Find the position of the particle at time $t = 5$. For $0 \leq t \leq 5$, is there a time t when the particle is at the origin? Justify your answer.

1.4 Free-Response Questions: No Calculator



Graph of f

1. Let f be the continuous function defined on the closed interval $[-1, 3]$ whose graph, consisting of three line segments, is shown. Let g be the function given by $g(x) = \int_0^x f(t) dt$.
- (a) On what intervals, if any, is g decreasing? Give a reason for your answer.
 - (b) Find $g''\left(-\frac{1}{2}\right)$, or explain why it does not exist.
 - (c) What is the absolute minimum value of g on the closed interval $[-1, 3]$? Justify your answer.
 - (d) Let h be the function defined by $h(x) = \ln(g(x))$ for $0 < x < 2$. Find $h'\left(\frac{3}{2}\right)$.

2. At an aquarium, a tank is kept full of water. At time $t = 0$, where t is measured in minutes, the water in the tank contains no salt. For time $t > 0$, water containing salt flows into the tank at a constant rate and mixes with the water in the tank. The mixed water leaves the tank through an overflow drain at the same constant rate. The number of pounds of salt in the water in the tank at time $t \geq 0$ is modeled by the function S , which satisfies the differential equation $\frac{dS}{dt} = -\frac{1}{6}(S - 36)$ with initial condition $S(0) = 0$.
- Is the amount of salt in the water in the tank changing more rapidly when the water contains 6 pounds of salt or when the water contains 30 pounds of salt? Explain your reasoning.
 - Write an expression for $\frac{d^2S}{dt^2}$ in terms of S . Find the value of $\frac{d^2S}{dt^2}$ at the instant when there are 20 pounds of salt in the water in the tank. Using correct units, interpret this value in the context of the problem.
 - Use separation of variables to find $y = S(t)$, the particular solution to the differential equation $\frac{dS}{dt} = -\frac{1}{6}(S - 36)$ with initial condition $S(0) = 0$.

3. Consider the curve given by $x^2 - xy + 2y^2 = 7$.

- (a) Show that $\frac{dy}{dx} = \frac{y-2x}{4y-x}$.
- (b) Determine the y -coordinate of each point on the curve at which the line tangent to the curve at that point is vertical. Justify your answer.
- (c) Find $\frac{d^2y}{dx^2}$ in terms of x , y , and $\frac{dy}{dx}$. The line tangent to the curve at the point $(1, 2)$ is horizontal. Determine whether the curve is concave up or concave down at the point $(1, 2)$.

4. The function f is given by $f(x) = \frac{x^3}{e^x}$.

- (a) Write an equation for the line tangent to the graph of f at $x = 5$.
- (b) Write an equation that involves a limit and demonstrates that the x -axis is an asymptote for the graph of f .
- (c) Find the x -coordinate of each critical point of f . Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.
- (d) Find the average value of f' , the derivative of f , on the interval $[-1, 5]$.

2 BC ONLY

2.1 Multiple Choice Questions: No Calculator

1. If $y' = \frac{2x-1}{\sqrt{2x^2-2x+3}}$, then $y =$

(A) $\sqrt{2x^2-2x+3} + C$

(B) $2\sqrt{2x^2-2x+3} + C$

(C) $\frac{1}{2}\sqrt{2x^2-2x+3} + C$

(D) $\frac{x^2-x}{\sqrt{\frac{2x^3}{3}-x^3+3x}} + C$

2. If $y = \frac{x^3}{x+2}$, then $\frac{dy}{dx} =$

(A) $3x^2$

(B) $\frac{-2x^3-6x^2}{(x+2)^2}$

(C) $\frac{2x^3+6x^2}{(x+2)^2}$

(D) $\frac{4x^3+6x^2}{(x+2)^2}$

3. Let f be a function with $f(3) = -7$ and $f'(3) = 5$. What is the approximation of $f(2.8)$ obtained by using the line tangent to the graph of f at $x = 3$?

(A) -8

(B) -6

(C) 4

(D) 6

x	$f(x)$	$f'(x)$	$f''(x)$
2	-2	1	3
7	4	5	2

4. The table shown gives selected values of a twice-differentiable function f and its first two derivatives. What is the value of $\int_2^7 f'(t) dt$?

(A) 6

(B) 5

(C) 4

(D) 2

5. Which of the following series converge?

I. $\sum_{k=1}^{\infty} \frac{1}{k^2 + 1}$

II. $\sum_{k=1}^{\infty} \frac{k}{k^2 + 1}$

III. $\sum_{k=1}^{\infty} \frac{k^2}{k^2 + 1}$

- (A) None
- (B) I only
- (C) I and II only
- (D) I, II, and III

$$1 + (x-3) + \frac{(x-3)^2}{2 \cdot 2!} + \frac{(x-3)^3}{3 \cdot 3!} + \cdots + \frac{(x-3)^n}{n \cdot n!} + \cdots$$

6. Which of the following statements about the power series above is true?

- (A) The series diverges for all x .
- (B) The series converges for $x = 3$ only.
- (C) The series converges for $2 \leq x < 4$ only.
- (D) The series converges for all x .

7. Consider the series $\sum_{n=0}^{\infty} (-1)^n a_n$, where $a_n > 0$ for all n and $\lim_{n \rightarrow \infty} a_n = 0$. Which of the following statements is true?

- (A) The series converges conditionally.
- (B) The series converges absolutely.
- (C) The series converges, but there is not enough information to determine if the series converges absolutely.
- (D) There is not enough information to determine if the series converges or diverges.

The function f is continuous at $x = 3$.

$$f(3) = 1 + \lim_{x \rightarrow 3^-} f(x) + \lim_{x \rightarrow 3^+} f(x)$$

8. Based on the statements above, which of the following must be true?

- (A) $f(3) = -1$
- (B) $f(3) = 1$
- (C) $f(3) \neq \lim_{x \rightarrow 3} f(x)$
- (D) f is differentiable at $x = 3$.

$$f(x) = \begin{cases} x^2 - 4x & \text{for } x < 2 \\ x^3 - 12 & \text{for } x \geq 2 \end{cases}$$

9. Let f be the function defined as shown. Which of the following statements about f is false?

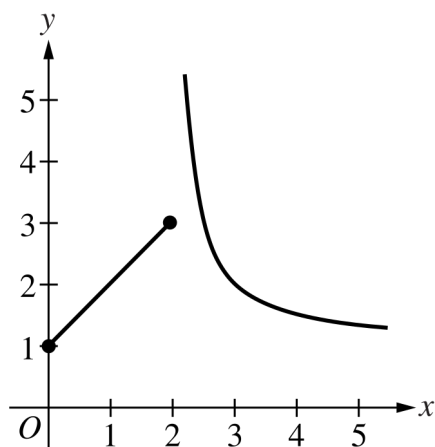
- (A) $\lim_{x \rightarrow 2^-} f(x)$ exists.
- (B) $\lim_{x \rightarrow 2} f(x)$ exists.
- (C) f is continuous at $x = 2$.
- (D) f is differentiable at $x = 2$.

10. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)!}$ converges to S . Based on the alternating series error bound, what is the least number of terms in the series that must be summed to guarantee a partial sum that is within 0.001 of S ?

- (A) Two
- (B) Three
- (C) Four
- (D) Five

11. The length of the graph of $y = f(x)$ from $x = 0$ to $x = 1$ is given by $\int_0^1 \sqrt{1 + e^{2x}} dx$. Of the following, which could be $f(x)$?

- (A) e^{-x}
- (B) e^x
- (C) e^{-2x}
- (D) e^{2x}



Graph of f

12. The figure shows the graph of a piecewise function f with domain $[0, \infty)$. The function f has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 1$. Which of the following statements are true?

- I. $\lim_{x \rightarrow 2^-} f(x)$ exists.
- II. $\lim_{x \rightarrow 2} f(x)$ exists.
- III. $\lim_{x \rightarrow \infty} f(x) = \infty$

- (A) I only
- (B) I and II only
- (C) I and III only
- (D) II and III only

$$f(x) = \begin{cases} \frac{\sin(x^2)}{x^2} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

13. What are the first three nonzero terms of the Maclaurin series for the piecewise function defined as shown above?

- (A) $\frac{1}{x} - \frac{x^3}{2!} + \frac{x^7}{4!}$
- (B) $x - \frac{x^5}{3!} + \frac{x^9}{5!}$
- (C) $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!}$
- (D) $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!}$

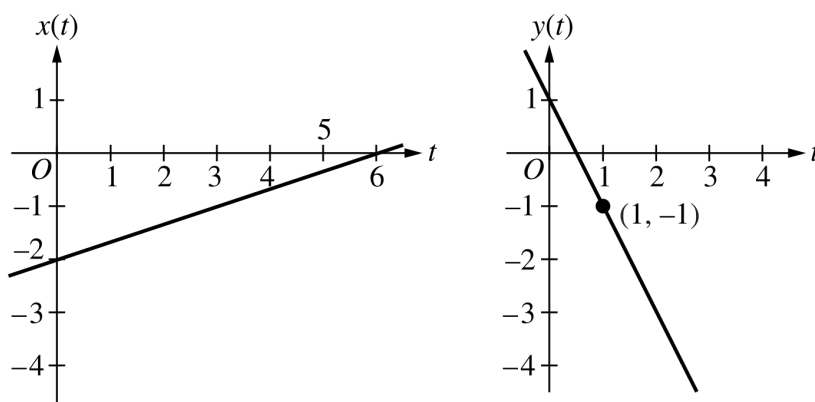
14. The Maclaurin series for $\frac{1}{1+x^2}$ is $1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots$. Which of the following is a power series expansion for $x \tan^{-1} x$?

(A) $-2x + 12x^3 - 30x^5 + 64x^7 - \cdots$

(B) $\frac{x^2}{2} - \frac{x^4}{4} + \frac{x^6}{6} - \frac{x^8}{8} - \cdots$

(C) $x^2 - \frac{x^4}{3} + \frac{x^6}{5} - \frac{x^8}{7} - \cdots$

(D) $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} - \cdots$



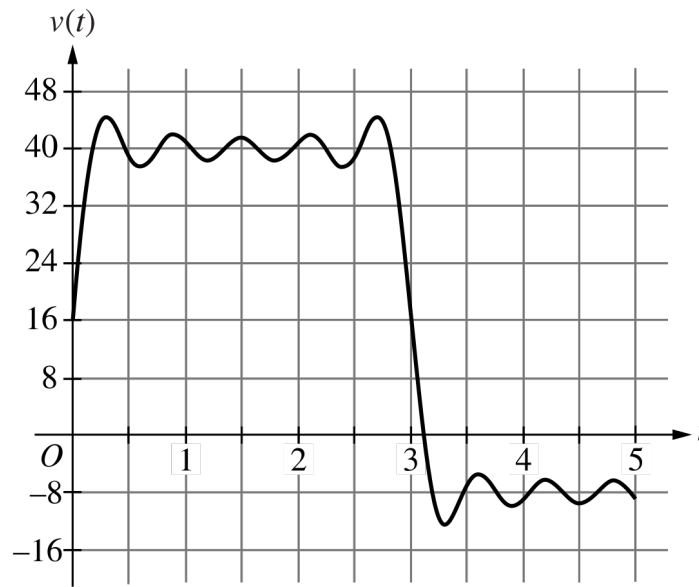
15. The position of a particle moving in the xy -plane is given by the parametric functions $x(t)$ and $y(t)$, whose graphs are shown. What is the slope of the line tangent to the path of the particle at the point at which $t = 2$?

(A) $\frac{9}{4}$

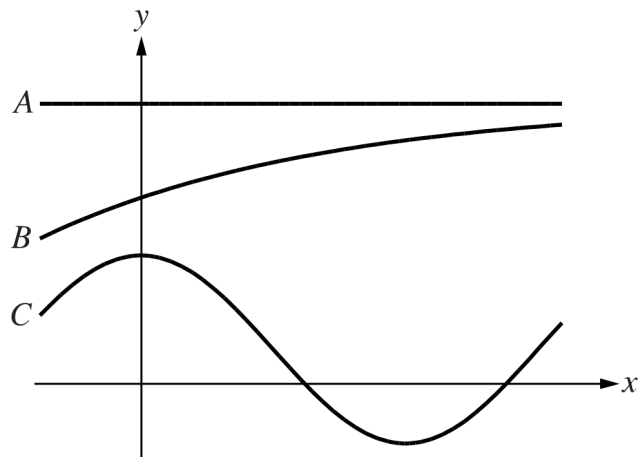
(B) $-\frac{1}{6}$

(C) -2

(D) -6



16. For time $t \geq 0$, a particle moves along a straight line with velocity $v(t)$ meters per second. The graph of v is shown. Of the following, which is the best estimate for the total distance, in meters, traveled by the particle from time $t = 0$ to $t = 5$ seconds?
- (A) 24
 (B) 80
 (C) 104
 (D) 136



17. Of the graphs shown, which could be the graph of a solution to a differential equation whose slope field has slopes that depend only on the y -coordinate at each point?
- (A) Graph A only
 (B) Graph B only
 (C) Graphs A and B only
 (D) Graphs B and C only

x	$f'(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	2	-1	3	5
2	3	-2	7	-1

18. The table above gives values of a function f and its derivatives at selected values of x . Which of the following is the third-degree Taylor polynomial for f about $x = 2$?

- (A) $2 - x + \frac{3}{2}x^2 + \frac{5}{6}x^3$
 (B) $2 - x + \frac{3}{2}x^2 + \frac{5}{3}x^3$
 (C) $3 - 2(x - 2) + \frac{7}{2}(x - 2)^2 - \frac{1}{6}(x - 2)^3$
 (D) $3 - 2(x - 2) + \frac{7}{2}(x - 2)^2 - \frac{1}{3}(x - 2)^3$

19. Let f be the function defined by $f(x) = -2x + \int_2^x \sqrt{1+t^2} dt$. Which of the following statements is true on the interval $0 < x < 1$?

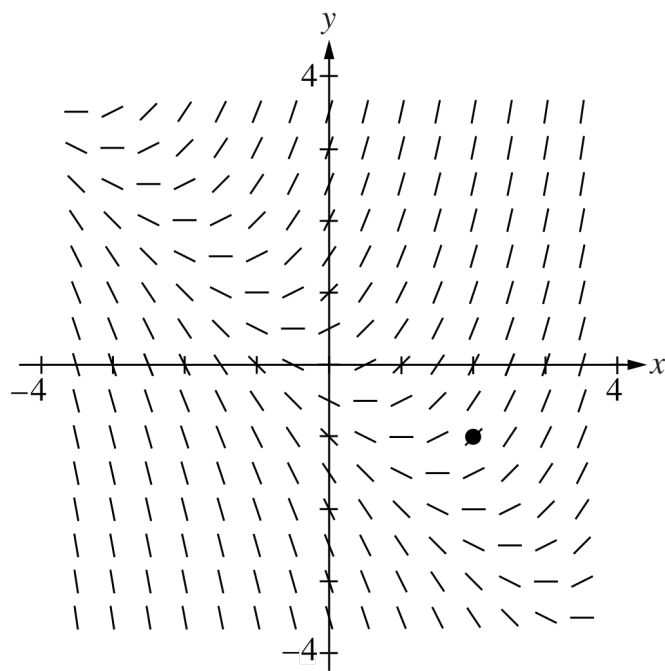
- (A) f is decreasing, and the graph of f is concave up.
 (B) f is decreasing, and the graph of f is concave down.
 (C) f is increasing, and the graph of f is concave up.
 (D) f is increasing, and the graph of f is concave down.

20. If $f(x) = \sqrt{x^4 + 8}$, then $f'(-1) =$

- (A) $-\frac{4}{3}$
 (B) $-\frac{2}{3}$
 (C) $-\frac{1}{6}$
 (D) $\frac{1}{6}$

21. $\lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{5} + h\right) - \cos\left(\frac{\pi}{5}\right)}{h}$

- (A) $-\sin\left(\frac{\pi}{5}\right)$
 (B) $\sin\left(\frac{\pi}{5}\right)$
 (C) $\cos\left(\frac{\pi}{5}\right)$
 (D) nonexistent



22. A slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown. The point $(2, -1)$ is indicated in the figure. Let $y = f(x)$ be the solution to the differential equation with initial condition $f(2) = -1$. If $f(3)$ is approximated by using Euler's method with two steps of equal size starting at $x = 2$, which of the following statements is true?
- (A) $f(3) \approx \frac{1}{2}$, and this value is an underapproximation of $f(3)$.
- (B) $f(3) \approx \frac{1}{2}$, and this value is an overapproximation of $f(3)$.
- (C) $f(3) \approx 0$, and this value is an underapproximation of $f(3)$.
- (D) $f(3) \approx 0$, and this value is an overapproximation of $f(3)$.
23. What are all values of x for which $\sum_{n=0}^{\infty} \left(\frac{x}{2+x} \right)^n$ converges?
- (A) $x > -2$ only
- (B) $x > -1$ only
- (C) $x \geq 0$ only
- (D) $-1 < x < 1$ only
24. $\int_3^{\infty} \frac{1}{x^3} dx$ is
- (A) $\frac{1}{18}$
- (B) $\frac{1}{9}$
- (C) $\frac{2}{9}$
- (D) divergent

25. $\int x \sin(2x) dx =$
- (A) $-\frac{1}{2}x \cos(2x) + \frac{1}{4} \sin(2x) + C$
 (B) $-\frac{1}{2}x \cos(2x) - \frac{1}{4} \sin(2x) + C$
 (C) $-x \cos(2x) + \sin(2x) + C$
 (D) $-2x \cos(2x) + 4 \sin(2x) + C$
26. Let f be the function defined by $f(x) = \frac{(x-2)(x^2-k^2)}{(x^2-4)(x-k)}$, where k is a constant. For what value of k , if any, does $\lim_{x \rightarrow 2} f(x) = 3$?
- (A) 2
 (B) 3
 (C) 10
 (D) There is no such value of k .
27. $\int \frac{5}{(2x+1)(x-2)} dx =$
- (A) $-2 \ln|2x+1| + \ln|x-2| + C$
 (B) $-\ln|2x+1| + \ln|x-2| + C$
 (C) $\frac{1}{2} \ln|2x+1| - 2 \ln|x-2| + C$
 (D) $5 \ln|2x+1| + 5 \ln|x-2| + C$
28. The path of a particle in the xy -plane is described by the parametric equations $x(t) = \sin t$ and $y(t) = e^t - t^2$. Which of the following gives the total distance traveled by the particle from $t = 1$ to $t = 2$?
- (A) $\int_1^2 \sqrt{\sin^2 t + (e^t - t^2)^2} dt$
 (B) $\int_1^2 \sqrt{\cos^2 t + (e^t - 2t)^2} dt$
 (C) $\int_1^2 \sqrt{\cos t + (e^t - 2t)} dt$
 (D) $\int_1^2 (\cos^2 t + (e^t - 2t)^2) dt$
29. The position of an object moving in the xy -plane is given by the parametric equations $x(t) = 2t^3 - 3t^2 - 4$ and $y(t) = 2t^3 - 9t^2 + 12t$. At what times t is the object at rest?
- (A) 1 only
 (B) 0 and 1 only
 (C) 1 and 2 only
 (D) 0, 1, and 2

30. Let f be a continuous function. If $\int_4^5 f(x)dx = 20$, $\int_5^k f(x)dx = -5$, and $\int_4^k (f(x) + 2)dx = 45$, what is the value of k ?
- (A) 13
(B) 15
(C) 17
(D) 19
31. A spherical snowball is melting in such a way that it maintains its shape. The volume of the snowball is decreasing at a constant rate of 5 cubic inches per minute. At the instant when the radius of the snowball is decreasing at a rate of $\frac{3}{5}$ inch per minute, what is the radius of the snowball, in inches? (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)
- (A) $\sqrt{\frac{5}{4\pi}}$
(B) $\frac{25}{12\pi}$
(C) $\frac{5}{\sqrt{12\pi}}$
(D) $\frac{125}{36\pi}$

2.2 Multiple Choice Questions: Calculator Allowed

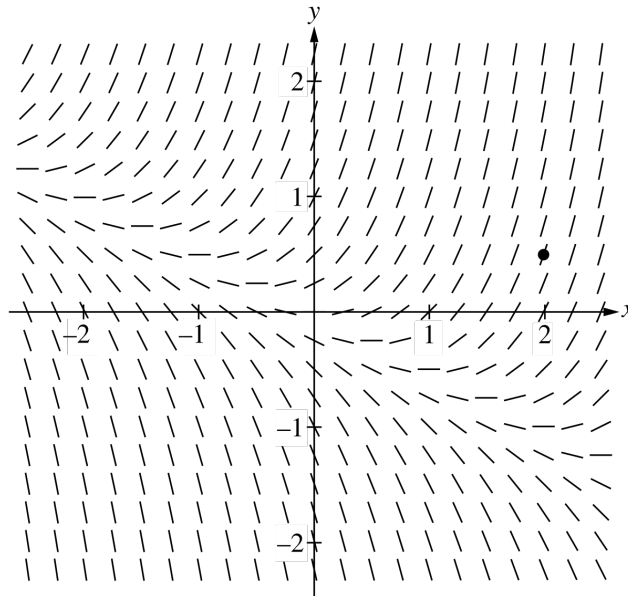
- For $t \geq 0$ hours, the velocity of a ship is modeled by $v(t) = \frac{1530}{t^2 - 24t + 172}$ miles per hour. What is the average velocity of the ship, in miles per hour, from $t = 1$ to $t = 15$ hours?
(A) 2.220
(B) 25.810
(C) 31.578
(D) 33.834
- The velocity of a particle moving in the xy -plane can be described by the parametric equations $x'(t) = 3.4\cos(3t)$ and $y'(t) = 6t$ for time $t \geq 0$. If the particle is at the point $(5, -2)$ at time $t = 0$, what is the position of the particle at time $t = 6$?
(A) $(4.149, 34)$
(B) $(4.149, 106)$
(C) $(7.245, 34)$
(D) $(7.245, 106)$
- What is the sum of the areas of the three regions enclosed by the graph of the polar curve $r = \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{3\pi}$?
(A) 0.394
(B) 0.657
(C) 0.788
(D) 1.315
- Let f be the function with derivative given by $f'(x) = \sin(e^{\frac{x}{4}})$ on the open interval $(5, 8)$. Of the following, which is the largest subinterval of $(5, 8)$ on which the graph of f is concave up?
(A) $(5, 7.448)$
(B) $(5, 8)$
(C) $(6.201, 8)$
(D) $(7.352, 8)$

2.3 Free-Response Questions: Calculator Allowed

1. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.
- (a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.
 - (b) Find the x -coordinate of the particle's position at time $t = 4$.
 - (c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.
 - (d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

2.4 Free-Response Questions: No Calculator

1. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with initial condition $f(2) = 0.5$.
- (a) A portion of the slope field for the differential equation $\frac{dy}{dx} = x + 2y$ is given. Sketch the solution curve through the point $(2, 0.5)$.



- (b) Write an equation for the line tangent to the graph of f at $x = 2$. Use the equation to approximate $f(2.4)$.
- (c) Write the second-degree Taylor polynomial for f about $x = 2$. Use the Taylor polynomial to approximate $f(2.4)$.
- (d) It is known that $0 < f'''(x) < 36$ for $2 \leq x \leq 2.4$. Use the Lagrange error bound to show that the approximation from part (c) differs from the actual value of $f(2.4)$ by less than 0.5.

2. The Taylor series for a function f about $x = 7$ is given by $\sum_{n=1}^{\infty} \frac{(x-7)^n}{n \cdot 5^n}$ and converges to $f(x)$ for all values of x in the interval of convergence.
- (a) Find the interval of convergence of the Taylor series for f about $x = 7$.
 - (b) Write the first four nonzero terms of the Taylor series for f' , the derivative of f , about $x = 7$.
 - (c) Let $g(x) = 2e^{f(x)}$. Find $g'(7)$. Show the work that leads to your answer.

3 Answer for Multiple Choice Questions

3.1 AB: No Calculator

Question	Answer	Question	Answer	Question	Answer
1	A	11	D	21	D
2	C	12	D	22	B
3	B	13	B	23	A
4	C	14	D	24	C
5	A	15	C	25	C
6	B	16	D	26	A
7	C	17	A	27	D
8	B	18	C	28	C
9	A	19	C	29	C
10	A	20	A	30	D

3.2 AB: Calculator Allowed

Question	Answer	Question	Answer
1	B	11	A
2	D	12	D
3	D	13	A
4	D	14	C
5	B	15	C
6	B		
7	A		
8	B		
9	D		
10	D		

3.3 BC: No Calculator

Question	Answer	Question	Answer	Question	Answer	Question	Answer
1	A	11	B	21	A	31	C
2	C	12	A	22	A		
3	A	13	B	23	B		
4	A	14	C	24	A		
5	B	15	D	25	A		
6	D	16	D	26	C		
7	D	17	C	27	B		
8	A	18	C	28	B		
9	D	19	A	29	A		
10	B	20	B	30	D		

3.4 BC: Calculator Allowed

Answer: DBBC

4 Scoring Guidelines for Free-Response Questions

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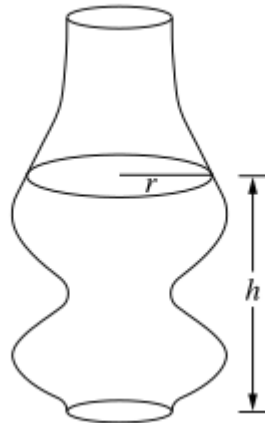
1.  A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



h (centimeters)	0	6.2	13	20
$r(h)$ (centimeters)	2.5	2.7	4.3	2.6

Juice is sold in 20-centimeter-tall bottles with horizontal cross sections parallel to the base that are circles, as shown in the figure. The radius of the circular cross section at height h above the base of the bottle is given by a differentiable function r , where h and $r(h)$ are measured in centimeters. Selected values of $r(h)$ are given in the table shown.

- (a) Approximate $r'(3.1)$ using the average rate of change of r over the interval $0 \leq h \leq 6.2$. Show the computations that lead to your answer. Indicate units of measure.

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- (b) Interpret the meaning of $\frac{1}{20} \int_0^{20} r(h) dh$ in the context of the problem. Use a trapezoidal sum with the three subintervals $[0, 6.2]$, $[6.2, 13]$, and $[13, 20]$ to approximate the value of $\frac{1}{20} \int_0^{20} r(h) dh$.
- (c) The radius of the circular cross section at height h above the base of the bottle can also be modeled by the function f , where h and $f(h)$ are measured in centimeters. Let $f(h) = 3\sqrt{2 - 1.25 \cos\left(\frac{\pi}{100}(h - 20)^2\right)}$ for $0 \leq h \leq 20$. Based on the model, at a height of $h = 12$ centimeters, is the radius increasing or decreasing as h increases? Give a reason for your answer.
- (d) Using the model f defined in part (c), find the volume of the bottle.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

To earn the first point a response must provide a difference and a quotient. A response of only $\frac{2.7-2.5}{6.2}$ or $\frac{0.2}{6.2-0}$ earns the first point.

To earn the second point a response must have $\frac{\text{cm}}{\text{cm}}$ (or the equivalent) attached to a numerical value. These units do not reduce to 1 because the numerator, cm, refers to radius of the bottle and the denominator, cm, refers to height of the bottle.



0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ Answer
- ☐ Units

Solution:

$$r'(3.1) \approx \frac{r(6.2) - r(0)}{6.2 - 0} = \frac{2.7 - 2.5}{6.2}$$

0.032 centimeter per centimeter

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

To earn the first point, a response must provide “average,” “radius,” and “the interval from 0 to 20” (or equivalent such as “throughout the bottle” or “from the bottom to the top”). Units are not required to earn the first point.

To earn the second point, at least five of the six factors in the trapezoidal sum must be correct.

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If there is any error in the trapezoidal sum, the response does not earn the third point.

A response of $\frac{1}{20} \left(6.2 \cdot \frac{2.5+2.7}{2} + 6.8 \cdot \frac{2.7+4.3}{2} + 7 \cdot \frac{4.3+2.6}{2} \right)$ earns the second and third points, unless there is a subsequent error in simplification, in which case the response would earn only the second point.

A response that is missing the factor of $\frac{1}{2}$ in the trapezoidal sum does not earn the second and third points.



0	1	2	3
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The student response accurately includes **all three** of the criteria below.

- ☐ Interpretation
- ☐ Form of trapezoidal sum
- ☐ Approximation

Solution:

$\frac{1}{20} \int_0^{20} r(h) \, dh$ is the average value of the radius of the juice bottle, in centimeters, over the interval $0 \leq h \leq 20$ centimeters.

$$\begin{aligned}
 & \frac{1}{20} \int_0^{20} r(h) \, dh \\
 & \approx \frac{1}{20} \left(6.2 \cdot \frac{r(0)+r(6.2)}{2} + 6.8 \cdot \frac{r(6.2)+r(13)}{2} + 7 \cdot \frac{r(13)+r(20)}{2} \right) \\
 & = \frac{1}{20} \left(6.2 \cdot \frac{2.5+2.7}{2} + 6.8 \cdot \frac{2.7+4.3}{2} + 7 \cdot \frac{4.3+2.6}{2} \right) \\
 & = \frac{1}{20} \cdot 64.07 = 3.2035
 \end{aligned}$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

To earn the second point, a response does not need to provide a value for $f'(12)$; the sign of $f'(12)$ is sufficient.

Any presented value of $f'(12)$ must be correct for the digits presented, up to three decimal places.

If a response presents an incorrect sign or value for $f'(12)$, the response does not earn the second point.

Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $f'(12) = -0.0006660598$. If a response presents this value, it earns the first point but is not eligible to earn the second point.

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0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ Considers $f'(12)$
- ☐ Answer with reason

Solution:

$$f'(12) = -0.535902$$

At a height of 12 centimeters, the radius is decreasing as h increases because $f'(12) < 0$.

Part D

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The first point is earned for a definite integral with an integrand of $(f(h))^2$, regardless of any subsequent errors in presentation of $f(h)$. A response that *begins* with an error in copying $f(h)$ into an integrand of the form $(f(h))^2$ does not earn the first point, but is eligible to earn the second point for a correct answer (1007.112 or 1007.111).

An answer of 320.574π rounds and truncates to 1007.028, and therefore does not earn the second point. Similarly, an answer of 320.573π truncates to 1007.109 and rounds to 1007.110, and therefore does not earn the second point. However, answers of 320.5736π or 320.5737π do round and truncate to the correct answer and earn the second point.

Degree mode: $\pi \int_0^{20} (f(h))^2 dh = 427.508$ (or 427.507) or 136.0799156π . A response that did not earn the second point in part (c) because of degree mode is eligible to earn the answer point in part (d) for this value. Furthermore, in degree mode 136.079π rounds to 427.505 and truncates to 427.504, and therefore does not earn the second point. However, in degree mode 136.080π is accurate to three decimals and is eligible for the second point.



0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ Integrand
- ☐ Answer

Solution:

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$$\text{Volume} = \pi \int_0^{20} (f(h))^2 dh$$

= 1007.112 (or 1007.111) cubic centimeters

2. A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

The position of a particle moving along the x -axis at time t is given by the twice-differentiable function $x(t)$, where $x(0) = 2$. For $0 \leq t \leq 5$, the derivative of $x(t)$ is given by $x'(t) = \sin(t^{1.1})\sqrt{10t+4}$.

- (a) Find all times t in the open interval $0 < t < 5$ when the particle changes direction. Give a reason for your answer.
- (b) Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.
- (c) Find the acceleration of the particle at time $t = 4$. Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.
- (d) Find the position of the particle at time $t = 5$. For $0 \leq t \leq 5$, is there a time t when the particle is at the origin? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The first point cannot be earned implicitly by reporting $t = 2.831095$; the setup must be shown.

A response that finds an incorrect value of t does not earn the second point.

A response may find and report extraneous solutions to the equation $x'(t) = 0$. If the response does not specifically eliminate the extraneous solutions, it will not earn the second point.

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The second point can be earned by applying the Second Derivative Test. This includes reporting that $x'(t) = 0$ at $t = 2.831$, and that $x''(2.831) < 0$, and therefore the particle changes direction when $t = 2.831$. Such a response need not report the value of $x''(2.831)$, but if a value is presented it must be correct for the presented digits, up to three decimal places ($x''(2.831) = -6.938$).

Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $t = 112.26587$, which is not in $0 < t < 5$. Therefore, such a response does not earn the second point.



0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ Sets $x'(t) = 0$
- ☐ Answer with reason

Solution:

$$x'(t) = 0 \Rightarrow t = 2.831095$$

The particle changes direction when $t = 2.831$ because the velocity, $x'(t)$, changes from positive to negative at this value.

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

A response that presents incorrect limits or no limits does not earn the first point, but, in the presence of the correct answer, does earn the second point.

A response that omits the absolute value on the integrand or uses $x(t)$ for the integrand does not earn the first point, but, in the presence of the correct answer, does earn the second point.

A response that presents $\left| \int_1^4 x'(t) dt \right|$ does not earn the first point, but, in the presence of the correct answer, does earn the second point.

Degree mode: In degree mode, $\int_1^4 |x'(t)| dt = 0.813$ (or 0.812). (See degree mode statement in part (a).)



0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ Integral(s)
- ☐ Answer

Solution:

$$\int_1^4 |x'(t)| dt = 10.5556$$

The total distance traveled by the particle from time $t = 1$ to time $t = 4$ is 10.556 (or 10.555).

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The minimum work needed to earn the first point is $x''(4) = -1.732$ (or -1.731).

$a(4) = -1.732$ (or -1.731) is not sufficient to earn the first point. The connection $a(4) = x''(4)$, or $a(t) = x''(t)$, must be explicitly made.

A response does not need to find the value of $v(4)$; the sign is sufficient. However, a response that reports an incorrect sign or value of $v(4)$ does not earn the second point. A response must declare a value for $x''(4)$ to be eligible for the second point.

If a value for $v(4)$ is presented, that value must be accurate for the digits shown, up to three decimal places.

A response that declares an incorrect value of $x''(4)$ is eligible for the second point. The conclusion must be consistent with the presented value of $x''(4)$ and the correct sign/value of $v(4)$.

Degree mode: In degree mode, $x''(4) = a(4) = 0.206$ (or 8.415) and $x'(4) = v(4) = 0.531 > 0$. (See degree mode statement in part (a).)



0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ $x''(4)$
- ☐ Conclusion with reason

Solution:

$$x''(4) = a(4) = -1.732 \text{ (or } -1.731)$$

$$x'(4) = v(4) = -6.587438$$

Because the values of the velocity and acceleration have the same sign at $t = 4$, the speed is increasing at time $t = 4$.

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Part D

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

A response that presents an integral of $\int_0^5 |x'(t)| dt$ does not earn the first point.

· This response will earn the second point only for a value of $x(5) = 2 + \int_0^5 |x'(t)| dt = 19.612$.

· If this response earns the second point, it will earn the third point only for reasoning that there is not sufficient information to determine whether there is a time t , for $0 \leq t \leq 5$, when the particle is at the origin. The reasoning must reference the Intermediate Value Theorem.

To earn the third point, the response must include:

- differentiability implies continuity (the continuity of $x(t)$ by itself is insufficient),
- $x(5) < 0 < x(0)$ or -0.566 (or -0.565) $< 0 < 2$ (this may be expressed in words or symbols), and
- “yes.”

A response of $x(5) = \int_0^5 x'(t) dt = -2.566$ (or -2.565) earns the first point, does not earn the second point, and is eligible to earn the third point.

A response with incorrect or no limits of integration does not earn the first point. Such a response that presents the correct value of $x(5)$ does earn the second point, and is eligible to earn the third point. A response with incorrect or no limits that presents an incorrect value of $x(5)$ will not earn any points.

A response that solves the equation $2 + \int_0^t x'(s) ds = 0$ for t and reports a value of $t = 4.841$ earns the third point, but must correctly find $x(5)$ in order to earn the first 2 points.

Degree mode: In degree mode, $x(5) = 2 + \int_0^5 x'(t) dt = 3.479$. (See degree mode statement in part (a).)



0	1	2	3
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The student response accurately includes **one** of the criteria below.

- ☐ $\int_0^5 x'(t) dt$
- ☐ $x(5)$
- ☐ Answer with justification

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$$x(5) = 2 + \int_0^5 x'(t) dt = -0.565953$$

The position of the particle at time $t = 5$ is -0.566 (or -0.565).

The particle is at the origin when $x(t) = 0$.

$x(t)$ is twice-differentiable. $\Rightarrow x(t)$ is continuous.

Because $x(5) < 0 < x(0) = 2$ and $x(t)$ is continuous, by the Intermediate Value Theorem there must be a time t in $0 \leq t \leq 5$ when the particle is at the origin.

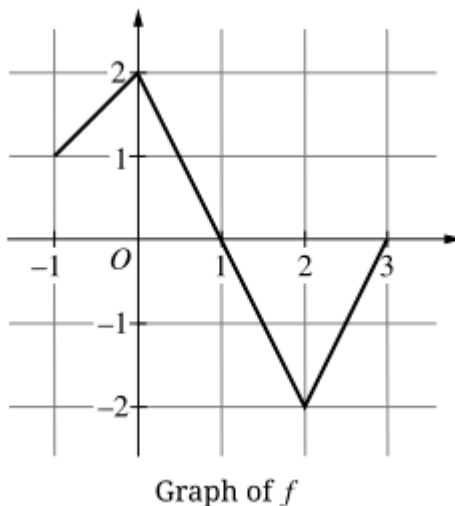
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3. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Let f be the continuous function defined on the closed interval $[-1, 3]$ whose graph, consisting of three line segments, is shown. Let g be the function given by $g(x) = \int_0^x f(t) dt$.

- (a) On what intervals, if any, is g decreasing? Give a reason for your answer.
- (b) Find $g''\left(-\frac{1}{2}\right)$, or explain why it does not exist.
- (c) What is the absolute minimum value of g on the closed interval $[-1, 3]$? Justify your answer.
- (d) Let h be the function defined by $h(x) = \ln(g(x))$ for $0 < x < 2$. Find $h'\left(\frac{3}{2}\right)$.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

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The first point is earned for $g'(x) = f(x)$ in any part of the problem.

· A response of $g''(x) = f'(x)$ or $g''(-\frac{1}{2}) = f'(-\frac{1}{2})$ in part (b) is sufficient to earn the first point.

The interval $[1, 3]$ can be reported $(1, 3)$, $(1, 3]$, or $(1, 3]$.

The second point can be earned without the first by presenting a correct interval and a statement that $f(x) < 0$ on this interval.



0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ $g'(x) = f(x)$ in any part of response
- ☐ Answer with reason

Solution:

$g'(x) = f(x) < 0$ on the open interval $(1, 3)$. Therefore, g is decreasing on the closed interval $[1, 3]$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

An answer of 1 with no supporting work earns the point.

Any work shown must be correct in order to earn the point.



0	1
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The student response accurately includes the criteria below.

- ☐ Answer

Solution:

$$g''(x) = f'(x)$$

$$g''(-\frac{1}{2}) = f'(-\frac{1}{2}) = 1$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

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The first point is earned for any presented consideration of $x = -1$ and $x = 3$.

A response that finds only that $g(-1) = -1.5$ and declares this as the minimum earns only the second point.

A response that evaluates $g(x)$ at any values of x in addition to $x = -1$, $x = 1$, and $x = 3$ does not earn the third point.

To earn the third point, a response must:

- have earned the first point,
- have no errors in the evaluation of $g(-1)$ and $g(3)$,
- conclude that $g(3) > g(-1)$, and
- either evaluate $g(1)$ as part of the Candidates Test or include a global argument that specifically addresses the behavior of $g(x)$ at $x = 1$ (e.g., “ $g'(x) = f(x)$ changes signs from positive to negative at $x = 1$ ” or “ $g'(1) = f(1) = 0$ and there is a maximum at $x = 1$ ”).



0	1	2	3
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The student response accurately includes **all three** of the criteria below.

- ☐ Considers $x = -1$ and $x = 3$
- ☐ Answer
- ☐ Justification

Solution:

Because $g'(x) = f(x) > 0$ for $-1 < x < 1$ and $g'(x) = f(x) < 0$ for $1 < x < 3$, the minimum value of g must occur at either $x = -1$ or $x = 3$.

$$g(-1) = \int_0^{-1} f(t)dt = - \int_{-1}^0 f(t)dt = -1.5$$

$$g(3) = \int_0^3 f(t)dt = -1$$

Therefore, the minimum value of g is $g(-1) = -1.5$.

Alternate solution using Candidates Test:

$$g'(x) = f(x) = 0 \Rightarrow x = 1$$

$$g(-1) = \int_0^{-1} f(t)dt = - \int_{-1}^0 f(t)dt = -1.5$$

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$$g(1) = \int_0^1 f(t) \, dt = 1$$

$$g(3) = \int_0^3 f(t) \, dt = -1$$

Therefore, the minimum value of g is $g(-1) = -1.5$.

Part D

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The first point is earned for either $h'(x) = \frac{1}{g(x)} \cdot g'(x)$, $h'\left(\frac{3}{2}\right) = \frac{1}{g\left(\frac{3}{2}\right)} \cdot g'\left(\frac{3}{2}\right)$, or $\frac{1}{\frac{3}{4}} \cdot (-1)$.

To earn the second point a response must use the correct value of either $g\left(\frac{3}{2}\right)$ or $g'\left(\frac{3}{2}\right)$ (or both) in an expression for $h'\left(\frac{3}{2}\right)$.

The minimal answer that earns all 3 points is $\frac{1}{\frac{3}{4}} \cdot (-1)$.

An answer of $-\frac{4}{3}$ with incorrect or without supporting work does not earn any points.

The third point is earned only for $-\frac{4}{3}$ or equivalent.



0	1	2	3
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The student response accurately includes **all three** of the criteria below.

- ☐ $h'(x)$
- ☐ Correct value of $g\left(\frac{3}{2}\right)$ or $g'\left(\frac{3}{2}\right)$
- ☐ Answer

Solution:

$$h'(x) = \frac{1}{g(x)} \cdot g'(x)$$

$$h'\left(\frac{3}{2}\right) = \frac{1}{g\left(\frac{3}{2}\right)} \cdot g'\left(\frac{3}{2}\right) = \frac{1}{\frac{3}{4}} \cdot (-1) = -\frac{4}{3}$$

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4. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

At an aquarium, a tank is kept full of water. At time $t = 0$, where t is measured in minutes, the water in the tank contains no salt. For time $t > 0$, water containing salt flows into the tank at a constant rate and mixes with the water in the tank. The mixed water leaves the tank through an overflow drain at the same constant rate. The number of pounds of salt in the water in the tank at time $t \geq 0$ is modeled by the function S , which satisfies the differential equation $\frac{dS}{dt} = -\frac{1}{6}(S - 36)$ with initial condition $S(0) = 0$.

(a) Is the amount of salt in the water in the tank changing more rapidly when the water contains 6 pounds of salt or when the water contains 30 pounds of salt? Explain your reasoning.

(b) Write an expression for $\frac{d^2S}{dt^2}$ in terms of S . Find the value of $\frac{d^2S}{dt^2}$ at the instant when there are 20 pounds of salt in the water in the tank. Using correct units, interpret this value in the context of the problem.

(c) Use separation of variables to find $y = S(t)$, the particular solution to the differential equation $\frac{dS}{dt} = -\frac{1}{6}(S - 36)$ with initial condition $S(0) = 0$.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The first point is earned for an attempt to evaluate $\frac{dS}{dt}$ at either $S = 6$ or $S = 30$.

Correct, labeled values of $\frac{dS}{dt}$ at both $S = 6$ and $S = 30$ are required to earn the second point.



0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ Uses $\frac{dS}{dt}$
- ☐ Answer with reason

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Solution:

$$\left. \frac{dS}{dt} \right|_{S=6} = -\frac{1}{6}(6 - 36) = 5$$

$$\left. \frac{dS}{dt} \right|_{S=30} = -\frac{1}{6}(30 - 36) = 1$$

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

A response must present $\frac{d^2S}{dt^2}$ in terms of S in order to earn the first point.

A response must present a negative value of $\left. \frac{d^2S}{dt^2} \right|_{S=20}$ in order to be eligible for the third point.

A response with a sign error in the second derivative (i.e., $\frac{d^2S}{dt^2} = -\frac{1}{36}(S - 36)$) can earn the second point for calculating $\left. \frac{d^2S}{dt^2} \right|_{S=20} = -\frac{1}{36}(20 - 36) = \frac{4}{9}$. This response is not eligible for the third point and thus earns a maximum of 1 point in part (b).

The interpretation requires “at the moment when there are 20 pounds of salt,” “rate of the amount of salt,” “decreasing,” and “at a rate of |the presented value| pounds per minute per minute” (or the equivalent) to earn the third point.



0	1	2	3
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The student response accurately includes **all three** of the criteria below.

- ☐ $\frac{d^2S}{dt^2}$
- ☐ $\left. \frac{d^2S}{dt^2} \right|_{S=20}$
- ☐ Interpretation with units

Solution:

$$\frac{d^2S}{dt^2} = -\frac{1}{6} \frac{dS}{dt} = -\frac{1}{6} \left(-\frac{1}{6} \right) (S - 36)$$

$$\left. \frac{d^2S}{dt^2} \right|_{S=20} = -\frac{1}{6} \left(-\frac{1}{6} \right) (20 - 36) = -\frac{4}{9}$$

At the instant when there are 20 pounds of salt in the tank, the rate of change of the number of pounds of salt is decreasing at a rate of $\frac{4}{9}$ pound per minute per minute.

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

A response with no separation of variables earns 0 out of 4 points.

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A response of $-\frac{1}{6} \int \frac{1}{S-36} dS = \int dt$ is a bad separation and does not earn the first point. However, this response is eligible for the second and third points. It cannot earn the fourth point.

A response that enters with both correct antiderivatives earns the first 2 points.

If the absolute value is missing on the antiderivative of $\frac{1}{S-36}$ the response earns the second point and is eligible for the third point. The response is not eligible for the fourth point.

A response with no constant of integration can earn at most the first 2 points.

A response is eligible for the third point if it has earned the first point and has at least one of the two antiderivatives correct. A response earns the third point by correctly including the constant of integration in an equation involving the presented antiderivatives and then replacing t with 0 and S with 0. This point is earned even in the presence of nonreal values (e.g., $\ln(-36)$) or severe algebraic errors.

A response is eligible for the fourth point only if it has earned the first 3 points and has correctly handled absolute value in all steps.

There are alternate (equivalent) algebraic forms of the correct equation, including, but not limited to:
 $S(t) = 36 - e^{\ln 36 - t/6}$ and $S(t) = 36 - \frac{36}{e^{t/6}}$.



0	1	2	3	4
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The student response accurately includes **all four** of the criteria below.

- ☐ Separation of variables
- ☐ Antiderivatives
- ☐ Constant of integration and uses initial condition
- ☐ Solves for S

Solution:

$$\frac{dS}{dt} = -\frac{1}{6}(S - 36)$$

$$\int \frac{1}{S-36} dS = \int -\frac{1}{6} dt$$

$$\ln|S - 36| = -\frac{1}{6}t + C$$

$$\text{Because } 0 \leq S < 36, |S - 36| = 36 - S.$$

$$\ln(36 - 0) = -\frac{1}{6}(0) + C \Rightarrow \ln(36) = C$$

$$36 - S = 36e^{-t/6}$$

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$$S(t) = 36 - 36e^{-t/6}, \quad t \geq 0$$

5. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Consider the curve given by $x^2 - xy + 2y^2 = 7$.

(a) Show that $\frac{dy}{dx} = \frac{y-2x}{4y-x}$.

(b) Determine the y -coordinate of each point on the curve at which the line tangent to the curve at that point is vertical. Justify your answer.

(c) Find $\frac{d^2y}{dx^2}$ in terms of x , y , and $\frac{dy}{dx}$. The line tangent to the curve at the point $(1, 2)$ is horizontal. Determine whether the curve is concave up or concave down at the point $(1, 2)$.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

Responses may use alternative notations for $\frac{dy}{dx}$ such as y' .

The first point is earned if there is at most one sign error for the derivative of the term $-xy$ and the remainder of the equation is correct.

The first point is earned with a completely correct derivative of $x^2 - xy + 2y^2$.

The second point cannot be earned if there are any errors.

It is sufficient to present $(4y - x)\frac{dy}{dx} = y - 2x$ to earn the second point, provided that there are no subsequent errors. It is not necessary to simplify to $\frac{dy}{dx} = \frac{y-2x}{4y-x}$.



0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ Implicit differentiation
- ☐ Verification

Solution:

$$2x - \left(y + x \frac{dy}{dx}\right) + 4y \frac{dy}{dx} = 0$$

$$\Rightarrow 4y \frac{dy}{dx} - x \frac{dy}{dx} = y - 2x \Rightarrow (4y - x) \frac{dy}{dx} = y - 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-2x}{4y-x}$$

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The second point is earned by substituting $x = 4y$ or $y = \frac{1}{4}x$ into the equation of the curve.

$y - 2x \neq 0$ is not required to earn the first point, but it is required to earn the third point.

A response that imports an incorrect expression for $\frac{dy}{dx}$ from part (a) can earn the first and second points only if the expression for $\frac{dy}{dx}$ is a quotient of two linear polynomials in x and y with both variables appearing in the numerator and denominator. Such a response cannot earn the third point.



0	1	2	3
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The student response accurately includes **all three** of the criteria below.

- ☐ Sets $4y - x = 0$
- ☐ Substitution of $x = 4y$
- ☐ Answer with justification

Solution:

The line tangent to the curve is vertical when $4y - x = 0$ and $y - 2x \neq 0$.

$$4y - x = 0 \Rightarrow x = 4y$$

$$(4y)^2 - (4y)y + 2y^2 = 7$$

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$$\Rightarrow 16y^2 - 4y^2 + 2y^2 = 7$$

$$\Rightarrow 14y^2 = 7 \Rightarrow y^2 = \frac{7}{14}$$

$$\Rightarrow y = -\sqrt{\frac{1}{2}} \text{ or } y = \sqrt{\frac{1}{2}}$$

Because $x = 4y$ and $y \neq 0$, $y - 2x \neq 0$.

Therefore, the lines tangent to the curve at $y = \pm\sqrt{\frac{1}{2}}$ are vertical.

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The first point is earned for a correct application of the quotient rule (or product rule), with or without the chain rule.

The second point is earned if the derivatives of all of the y terms are correct (i.e., $\frac{dy}{dx}$ and $4\frac{dy}{dx}$) AND the presented $\frac{d^2y}{dx^2}$ satisfies exactly one of the following:

- $\frac{d^2y}{dx^2}$ is completely correct.
- The quotient rule was used and
 - o there are sign errors in front of either or both of the two terms in the numerator of $\frac{d^2y}{dx^2}$, and there are no other errors.
 - o the numerator of $\frac{d^2y}{dx^2}$ is correct but the denominator is not squared, and there are no other errors.
 - o the numerator of $\frac{d^2y}{dx^2}$ has missing parentheses around the factors in one or both terms, and there are no other errors.
- The product rule was used and there is a sign error between the two terms, and there are no other errors.
- The derivative of one of either $-2x$ or $-x$ is incorrect, and there are no other errors.

To earn the third point, a response must have earned at least 1 of the first 2 points, presented a value of $\frac{d^2y}{dx^2}$ at $(1, 2)$, and have a conclusion consistent with this presented value.

To earn the fourth point, a response must have earned the first 3 points and must present a $\frac{d^2y}{dx^2}$ equivalent to $-\frac{2}{7}$ with no computational errors.

A response that imports an incorrect expression for $\frac{dy}{dx}$ from part (a) can earn the first and second points only if the expression for $\frac{dy}{dx}$ is a quotient of two linear polynomials in x and y with both variables appearing in the numerator and denominator. Such a response cannot earn the third or fourth points.



0	1	2	3	4
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The student response accurately includes **all four** of the criteria below.

2022 Form I FRQ

- ☐ Quotient rule (or product rule)
- ☐ Chain rule
- ☐ Answer
- ☐ Reason

Solution:

$$\frac{d^2y}{dx^2} = \frac{(4y-x)\left(\frac{dy}{dx}-2\right)-(y-2x)\left(4\frac{dy}{dx}-1\right)}{(4y-x)^2}$$

Because the line tangent to the curve at $(1, 2)$ is horizontal, $\left.\frac{dy}{dx}\right|_{(x,y)=(1,2)} = 0$.

Therefore,

$$\left.\frac{d^2y}{dx^2}\right|_{(x,y)=(1,2)} = \frac{(8-1)(0-2)-(2-2)(0-1)}{(8-1)^2} = -\frac{14}{49} = -\frac{2}{7}.$$

Because $\left.\frac{d^2y}{dx^2}\right|_{(x,y)=(1,2)} < 0$, the curve is concave down at the point $(1, 2)$.

6. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

The function f is given by $f(x) = \frac{x^3}{e^x}$.

- (a) Write an equation for the line tangent to the graph of f at $x = 5$.
- (b) Write an equation that involves a limit and demonstrates that the x -axis is an asymptote for the graph of f .
- (c) Find the x -coordinate of each critical point of f . Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.
- (d) Find the average value of f' , the derivative of f , on the interval $[-1, 5]$.

Part A

2022 Form I FRQ

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

$f'(x) = \frac{3x^2 - x^3}{e^x}$ earns the first 2 points.

An unlabeled correct $f'(x)$ earns the first 2 points.

To earn the first point:

· If the quotient rule was used, the presented derivative must include a sum or difference, a factor of e^x , and a polynomial in each term of the numerator.

· If the product rule was used, the presented derivative must include a factor of e^{-x} in each term.

A response that includes $f'(x) = \frac{e^x \cdot 3x^2 - x^3 e^x}{e^{x^2}}$ earns the first point, does not earn the second point, and is eligible for the third point. In this case the response must correctly evaluate this expression at $x = 5$, and use the resulting value as the slope of the tangent line.

The second point cannot be earned without the first. The second point is earned for a correct expression for $f'(x)$ or a correct value of $f'(5)$.

A response that labels the values $f(5)$ and $f'(5)$ correctly with no other supporting work does not earn the first 2 points but is eligible for the third point.

A response that presents a correct $f'(x)$ but an incorrect $f'(5)$ earns the first 2 points, but does not earn the third point.

A response that presents a correct equation of the tangent line with no supporting work earns no points in this part.

A response with any arithmetic error in $f'(x)$ does not earn the third point.



0	1	2	3
---	---	---	---

The student response accurately includes **all three** of the criteria below.

- ☐ Form of quotient rule (or product rule)
- ☐ $f'(x)$ or $f'(5)$
- ☐ Answer

Solution:

$$f'(x) = \frac{e^x \cdot 3x^2 - x^3 e^x}{(e^x)^2} = \frac{3x^2 - x^3}{e^x}$$

$$f'(5) = \frac{75 - 125}{e^5} = \frac{-50}{e^5}$$

$$f(5) = \frac{125}{e^5}$$

An equation for the tangent line is $y = \frac{125}{e^5} - \frac{50}{e^5}(x - 5)$.

2022 Form I FRQ

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

$\lim_{x \rightarrow \infty} f(x) = 0$ or $\lim_{x \rightarrow \infty} \frac{x^3}{e^x} = 0$ earns the point.

A response that presents an expression, rather than an equation, does not earn the point.

A response that presents any additional incorrect limits does not earn the point.



0	1
---	---

The student response accurately includes the criteria below.

· Equation

Solution:

$$\lim_{x \rightarrow \infty} f(x) = 0$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

$(3x^2 - x^3) = 0$ earns the first point.

A response that presents only one of $x = 0$ or $x = 3$ along with correct answer and justification earns 1 of the last 2 points.

A response that lists additional values of x as critical points does not earn either of the last 2 points.

The second derivative may be used to justify the relative maximum at $x = 3$. Since $f''(x) = 0$ at $x = 0$, it is not sufficient to use the second derivative to conclude that $x = 0$ is neither a maximum or minimum value; more justification is needed.

A sign chart with no explanation is not sufficient justification.

Because the interval is not closed, the Extreme Value Theorem may not be used.

A response that imports e^{x^2} in the denominator of the derivative from part (a) is eligible for all 3 points.

A response that imports either $f'(x) = \frac{e^x \cdot 3x^2 + x^3 e^x}{(e^x)^2}$ or $f'(x) = \frac{-e^x \cdot 3x^2 + x^3 e^x}{(e^x)^2}$ from part (a) is eligible for all 3 points.

· An import of $f'(x) = \frac{e^x \cdot 3x^2 + x^3 e^x}{(e^x)^2}$ should find critical points $x = -3$ and $x = 0$ AND conclude that f has a relative minimum at $x = -3$ and neither a minimum nor a maximum at $x = 0$.

2022 Form I FRQ

An import of $f'(x) = \frac{-e^x \cdot 3x^2 + x^3 e^x}{(e^x)^2}$ should find critical points $x = 0$ and $x = 3$ AND conclude that f has neither a minimum nor a maximum at $x = 0$ and a relative minimum at $x = 3$.



0	1	2	3
---	---	---	---

The student response accurately includes **all three** of the criteria below.

- ☐ Considers $f'(x) = 0$
- ☐ $x = 0$ and $x = 3$ only
- ☐ Answer with justification

Solution:

$$f'(x) = \frac{e^x \cdot 3x^2 - x^3 e^x}{(e^x)^2} = \frac{x^2 e^x (3-x)}{e^{2x}} = 0 \Rightarrow x = 0, x = 3$$

$f'(x)$ does not change sign at $x = 0$. Therefore, f has neither a minimum nor a maximum at $x = 0$.

$f'(x)$ changes sign from positive to negative at $x = 3$. Therefore, f has a relative maximum at $x = 3$.

Part D

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

A factor of $\frac{1}{6}$ is not needed to earn the first point.

The definite integral can be presented in a single or multiple steps to earn the first point. For example, the following response earns both points: $\int f'(x) dx = f(x)$, so the average value is $\frac{1}{6} \left(\frac{125}{e^5} + \frac{1}{e^{-1}} \right)$.

$f(5) - f(-1)$ earns the first point and $\frac{1}{6} \left(\frac{125}{e^5} - \left(\frac{-1}{e^{-1}} \right) \right)$ earns both points, provided there are no subsequent errors in simplification.

A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points:

$$\int_{-1}^5 f'(x) dx = \left(\frac{125}{e^5} + e \right) = \frac{1}{6} \left(\frac{125}{e^5} + e \right).$$

$\int_{-1}^5 f'(x) dx$ with an incorrect $f'(x)$ imported from part (a) or part (c) earns the first point.

A response with incorrect limits earns no points.

A response of $\frac{1}{6} \cdot \frac{125}{e^5} - \frac{-1}{e^{-1}}$ or $\frac{1}{6} \cdot f(5) - f(1)$ earns the first point but not the second point.

A response that uses an incorrect value of $f(5)$ or $f(-1)$ does not earn the second point.

2022 Form I FRQ



0	1	2
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The student response accurately includes **both** of the criteria below.

☐ Integral

☐ Answer

Solution:

$$\begin{aligned} & \frac{1}{5-(-1)} \int_{-1}^5 f'(x) \, dx \\ &= \frac{1}{6} (f(5) - f(-1)) = \frac{1}{6} \left(\frac{125}{e^5} - \left(\frac{-1}{e^{-1}} \right) \right) = \frac{1}{6} \left(\frac{125}{e^5} + e \right) \end{aligned}$$

2022 Form I BC

1. If $y' = \frac{2x-1}{\sqrt{2x^2-2x+3}}$, then $y =$

(A) $\sqrt{2x^2 - 2x + 3} + C$ ✓

(B) $2\sqrt{2x^2 - 2x + 3} + C$

(C) $\frac{1}{2}\sqrt{2x^2 - 2x + 3} + C$

(D) $\frac{x^2-x}{\sqrt{\frac{2x^3}{3}-x^2+3x}} + C$

2. If $y = \frac{x^3}{x+2}$, then $\frac{dy}{dx} =$

(A) $3x^2$

(B) $\frac{-2x^3-6x^2}{(x+2)^2}$

(C) $\frac{2x^3+6x^2}{(x+2)^2}$ ✓

(D) $\frac{4x^3+6x^2}{(x+2)^2}$

3. Let f be a function with $f(3) = -7$ and $f'(3) = 5$. What is the approximation of $f(2.8)$ obtained by using the line tangent to the graph of f at $x = 3$?

(A) -8 ✓

(B) -6

(C) 4

(D) 6

4.

x	$f(x)$	$f'(x)$	$f''(x)$
2	-2	1	3
7	4	5	2

The table shown gives selected values of a twice-differentiable function f and its first two derivatives. What is the value of $\int_2^7 f'(t) dt$?

(A) 6 ✓

(B) 5

(C) 4

(D) 2

2022 Form I BC

5. Which of the following series converge?

I. $\sum_{k=1}^{\infty} \frac{1}{k^2+1}$

II. $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$

III. $\sum_{k=1}^{\infty} \frac{k^2}{k^2+1}$

(A) None

(B) I only

(C) I and II only

(D) I, II, and III

6. $1 + (x - 3) + \frac{(x-3)^2}{2 \cdot 2!} + \frac{(x-3)^3}{3 \cdot 3!} + \cdots + \frac{(x-3)^n}{n \cdot n!} + \cdots$

Which of the following statements about the power series above is true?

(A) The series diverges for all x .

(B) The series converges for $x = 3$ only.

(C) The series converges for $2 \leq x < 4$ only.

(D) The series converges for all x .

7. Consider the series $\sum_{n=0}^{\infty} (-1)^n a_n$, where $a_n > 0$ for all n and $\lim_{n \rightarrow \infty} a_n = 0$. Which of the following statements is true?

(A) The series converges conditionally.

(B) The series converges absolutely.

(C) The series converges, but there is not enough information to determine if the series converges absolutely.

(D) There is not enough information to determine if the series converges or diverges.

8. The function f is continuous at $x = 3$.

$$f(3) = 1 + \lim_{x \rightarrow 3^-} f(x) + \lim_{x \rightarrow 3^+} f(x)$$

Based on the statements above, which of the following must be true?

2022 Form I BC

(A) $f(3) = -1$



(B) $f(3) = 1$

(C) $f(3) \neq \lim_{x \rightarrow 3} f(x)$

(D) f is differentiable at $x = 3$.

9. $f(x) = \begin{cases} x^2 - 4x & \text{for } x < 2 \\ x^3 - 12 & \text{for } x \geq 2 \end{cases}$

Let f be the function defined as shown. Which of the following statements about f is false?

(A) $\lim_{x \rightarrow 2^-} f(x)$ exists.

(B) $\lim_{x \rightarrow 2} f(x)$ exists.

(C) f is continuous at $x = 2$.

(D) f is differentiable at $x = 2$.



10. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)!}$ converges to S . Based on the alternating series error bound, what is the least number of terms in the series that must be summed to guarantee a partial sum that is within 0.001 of S ?

(A) Two

(B) Three



(C) Four

(D) Five

11. The length of the graph of $y = f(x)$ from $x = 0$ to $x = 1$ is given by $\int_0^1 \sqrt{1 + e^{2x}} dx$. Of the following, which could be $f(x)$?

(A) e^{-x}

(B) e^x

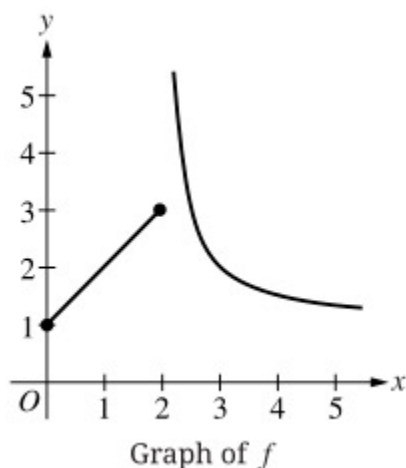


(C) e^{-2x}

(D) e^{2x}

2022 Form I BC

12.



The figure shows the graph of a piecewise function f with domain $[0, \infty)$. The function f has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 1$. Which of the following statements are true?

- I. $\lim_{x \rightarrow 2^-} f(x)$ exists.
- II. $\lim_{x \rightarrow 2} f(x)$ exists.
- III. $\lim_{x \rightarrow \infty} f(x) = \infty$

(A) I only



(B) I and II only

(C) I and III only

(D) II and III only

13.
$$f(x) = \begin{cases} \frac{\sin(x^2)}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

What are the first three nonzero terms of the Maclaurin series for the piecewise function defined as shown?

(A) $\frac{1}{x} - \frac{x^3}{2!} + \frac{x^7}{4!}$

(B) $x - \frac{x^5}{3!} + \frac{x^9}{5!}$



(C) $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!}$

(D) $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!}$

14. The Maclaurin series for $\frac{1}{1+x^2}$ is $1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots$. Which of the following is a power series expansion for $x \tan^{-1} x$?

2022 Form I BC

(A) $-2x + 12x^3 - 30x^5 + 64x^7 - \dots$

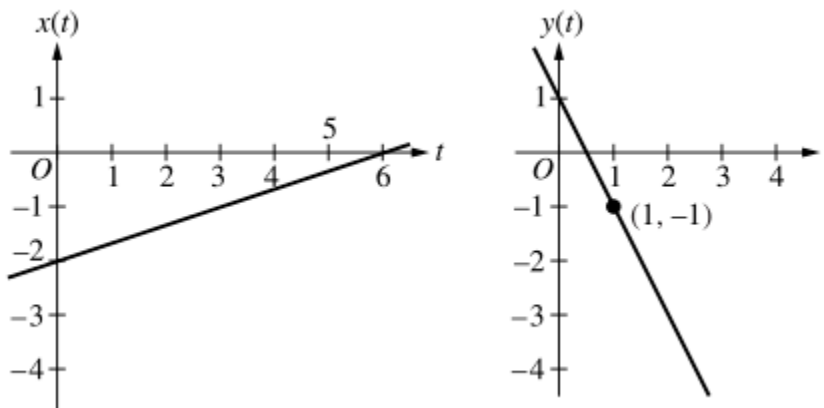
(B) $\frac{x^2}{2} - \frac{x^4}{4} + \frac{x^6}{6} - \frac{x^8}{8} + \dots$

(C) $x^2 - \frac{x^4}{3} + \frac{x^6}{5} - \frac{x^8}{7} + \dots$

(D) $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \dots$



15.



The position of a particle moving in the xy -plane is given by the parametric functions $x(t)$ and $y(t)$, whose graphs are shown. What is the slope of the line tangent to the path of the particle at the point at which $t = 2$?

(A) $\frac{9}{4}$

(B) $-\frac{1}{6}$

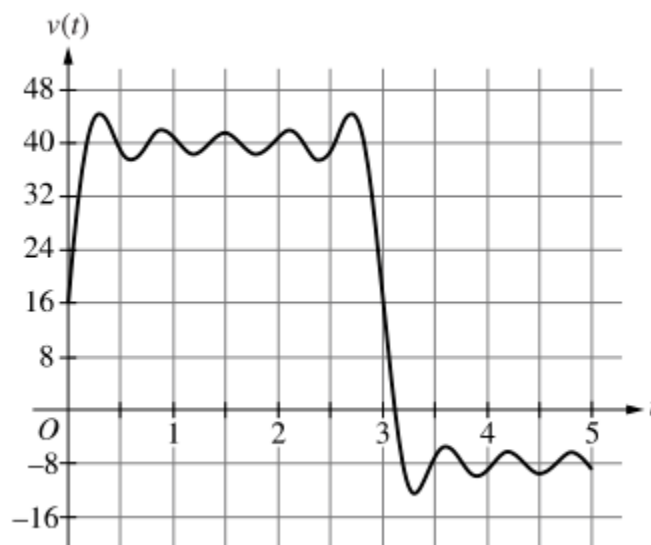
(C) -2

(D) -6



2022 Form I BC

16.

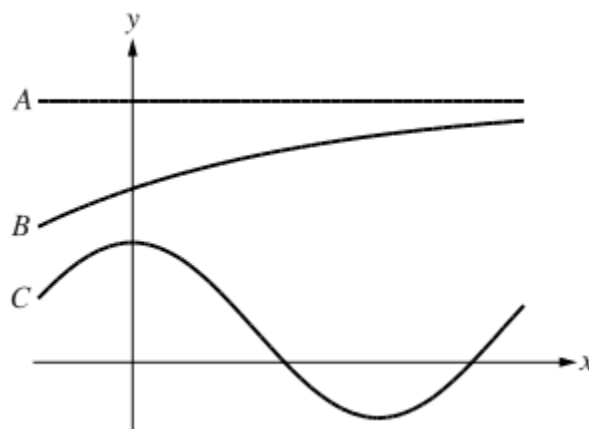


For time $t \geq 0$, a particle moves along a straight line with velocity $v(t)$ meters per second. The graph of v is shown. Of the following, which is the best estimate for the total distance, in meters, traveled by the particle from time $t = 0$ to $t = 5$ seconds?

- (A) 24
- (B) 80
- (C) 104
- (D) 136



17.



Of the graphs shown, which could be the graph of a solution to a differential equation whose slope field has slopes that depend only on the y -coordinate at each point?

2022 Form I BC

(A) Graph A only(B) Graph B only(C) Graphs A and B only(D) Graphs B and C only

18.

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	2	-1	3	5
2	3	-2	7	-1

The table above gives values of a function f and its derivatives at selected values of x . Which of the following is the third-degree Taylor polynomial for f about $x = 2$?

(A) $2 - x + \frac{3}{2}x^2 + \frac{5}{6}x^3$ (B) $2 - x + \frac{3}{2}x^2 + \frac{5}{3}x^3$ (C) $3 - 2(x - 2) + \frac{7}{2}(x - 2)^2 - \frac{1}{6}(x - 2)^3$ (D) $3 - 2(x - 2) + \frac{7}{2}(x - 2)^2 - \frac{1}{3}(x - 2)^3$

19.

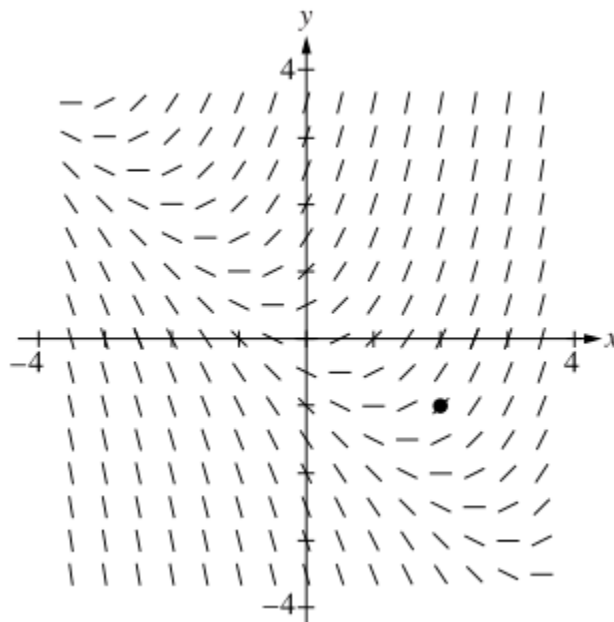
Let f be the function defined by $f(x) = -2x + \int_2^x \sqrt{1+t^2} dt$. Which of the following statements is true on the interval $0 < x < 1$?

(A) f is decreasing, and the graph of f is concave up.(B) f is decreasing, and the graph of f is concave down.(C) f is increasing, and the graph of f is concave up.(D) f is increasing, and the graph of f is concave down.20. If $f(x) = \sqrt{x^4 + 8}$, then $f'(-1) =$ (A) $-\frac{4}{3}$ (B) $-\frac{2}{3}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{6}$ 21. $\lim_{h \rightarrow 0} \frac{\cos(\frac{\pi}{5} + h) - \cos(\frac{\pi}{5})}{h}$ is(A) $-\sin\left(\frac{\pi}{5}\right)$ (B) $\sin\left(\frac{\pi}{5}\right)$ (C) $\cos\left(\frac{\pi}{5}\right)$

(D) nonexistent

2022 Form I BC

22.



A slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown. The point $(2, -1)$ is indicated in the figure. Let $y = f(x)$ be the solution to the differential equation with initial condition $f(2) = -1$. If $f(3)$ is approximated by using Euler's method with two steps of equal size starting at $x = 2$, which of the following statements is true?

(A) $f(3) \approx \frac{1}{2}$, and this value is an underapproximation of $f(3)$. ✓

(B) $f(3) \approx \frac{1}{2}$, and this value is an overapproximation of $f(3)$.

(C) $f(3) \approx 0$, and this value is an underapproximation of $f(3)$.

(D) $f(3) \approx 0$, and this value is an overapproximation of $f(3)$.

23. What are all values of x for which $\sum_{n=0}^{\infty} \left(\frac{x}{2+x}\right)^n$ converges?

(A) $x > -2$ only

(B) $x > -1$ only ✓

(C) $x \geq 0$ only

(D) $-1 < x < 1$ only

24. $\int_3^{\infty} \frac{1}{x^3} dx$ is

(A) $\frac{1}{18}$ ✓

(B) $\frac{1}{9}$

(C) $\frac{2}{9}$

(D) divergent

2022 Form I BC

25. $\int x \sin(2x) dx =$

(A) $-\frac{1}{2}x \cos(2x) + \frac{1}{4}\sin(2x) + C$ ✓

(B) $-\frac{1}{2}x \cos(2x) - \frac{1}{4}\sin(2x) + C$

(C) $-x \cos(2x) + \sin(2x) + C$

(D) $-2x \cos(2x) + 4 \sin(2x) + C$

26. Let f be the function defined by $f(x) = \frac{(x-2)(x^2-k^2)}{(x^2-4)(x-k)}$, where k is a constant. For what value of k , if any, does $\lim_{x \rightarrow 2} f(x) = 3$?

(A) 2

(B) 3

(C) 10 ✓

(D) There is no such value of k .

27. $\int \frac{5}{(2x+1)(x-2)} dx =$

(A) $-2 \ln |2x+1| + \ln |x-2| + C$

(B) $-\ln |2x+1| + \ln |x-2| + C$ ✓

(C) $\frac{1}{2} \ln |2x+1| - 2 \ln |x-2| + C$

(D) $5 \ln |2x+1| + 5 \ln |x-2| + C$

28. The path of a particle in the xy -plane is described by the parametric equations $x(t) = \sin t$ and $y(t) = e^t - t^2$. Which of the following gives the total distance traveled by the particle from $t = 1$ to $t = 2$?

(A) $\int_1^2 \sqrt{\sin^2 t + (e^t - t^2)^2} dt$

(B) $\int_1^2 \sqrt{\cos^2 t + (e^t - 2t)^2} dt$ ✓

(C) $\int_1^2 \sqrt{\cos t + (e^t - 2t)} dt$

(D) $\int_1^2 \left(\cos^2 t + (e^t - 2t)^2 \right) dt$

29. The position of an object moving in the xy -plane is given by the parametric equations $x(t) = 2t^3 - 3t^2 - 4$ and $y(t) = 2t^3 - 9t^2 + 12t$. At what times t is the object at rest?

2022 Form I BC

(A) 1 only



(B) 0 and 1 only

(C) 1 and 2 only

(D) 0, 1, and 2

30. Let f be a continuous function. If $\int_4^5 f(x) dx = 20$, $\int_5^k f(x) dx = -5$, and $\int_4^k (f(x) + 2) dx = 45$, what is the value of k ?

(A) 13

(B) 15

(C) 17

(D) 19



31. A spherical snowball is melting in such a way that it maintains its shape. The volume of the snowball is decreasing at a constant rate of 5 cubic inches per minute. At the instant when the radius of the snowball is decreasing at a rate of $\frac{3}{5}$ inch per minute, what is the radius of the snowball, in inches? (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) $\sqrt{\frac{5}{4\pi}}$ (B) $\frac{25}{12\pi}$ (C) $\frac{5}{\sqrt{12\pi}}$ (D) $\frac{125}{36\pi}$ 

32.  For $t \geq 0$ hours, the velocity of a ship is modeled by $v(t) = \frac{1530}{t^2 - 24t + 172}$ miles per hour. What is the average velocity of the ship, in miles per hour, from $t = 1$ to $t = 15$ hours?

(A) 2.220

(B) 25.810

(C) 31.578

(D) 33.834



33.  The velocity of a particle moving in the xy -plane can be described by the parametric equations $x'(t) = 3.4 \cos(3t)$ and $y'(t) = 6t$ for time $t \geq 0$. If the particle is at the point $(5, -2)$ at time $t = 0$, what is the position of the particle at time $t = 6$?

(A) $(4.149, 34)$ (B) $(4.149, 106)$ (C) $(7.245, 34)$ (D) $(7.245, 106)$ 

2022 Form I BC

34.  What is the sum of the areas of the three regions enclosed by the graph of the polar curve $r = \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{3\pi}$?
- (A) 0.394
- (B) 0.657 
- (C) 0.788
- (D) 1.315
35.  Let f be the function with derivative given by $f'(x) = \sin\left(e^{\frac{x}{4}}\right)$ on the open interval $(5, 8)$. Of the following, which is the largest subinterval of $(5, 8)$ on which the graph of f is concave up?
- (A) $(5, 7.448)$
- (B) $(5, 8)$
- (C) $(6.201, 8)$ 
- (D) $(7.352, 8)$
36.  For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.
- (a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.
- (b) Find the x -coordinate of the particle's position at time $t = 4$.
- (c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.
- (d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

Part A

1 point is earned for stating moving to the right with reason

1 point is earned for considering $\frac{dy/dt}{dx/dt}$

1 point is earned for slope at $t = 2$

$$\left. \frac{dx}{dt} \right|_{t=2} = \frac{2}{e^2}$$

Because $\left. \frac{dx}{dt} \right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$$\left. \frac{dy}{dx} \right|_{t=2} = \frac{dy/dt|_{t=2}}{dx/dt|_{t=2}} = 3.055 \text{ (or } 3.054\text{)}$$

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0	1	2	3
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The student earns all of the following points:

1 point is earned for stating moving to the right with reason

1 point is earned for considering $\frac{dy/dt}{dx/dt}$

1 point is earned for slope at $t = 2$

$$\left. \frac{dx}{dt} \right|_{t=2} = \frac{2}{e^2}$$

Because $\left. \frac{dx}{dt} \right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$$\left. \frac{dy}{dx} \right|_{t=2} = \frac{dy/dt|_{t=2}}{dx/dt|_{t=2}} = 3.055 \text{ (or 3.054)}$$

Part B

1 point is earned for the integral

1 point is earned for the answer

$$x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t} dt$$

$$= 1.253 \text{ (or 1.252)}$$



0	1	2
---	---	---

The student earns all of the following points:

1 point is earned for the integral

1 point is earned for the answer

$$x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t} dt$$

$$= 1.253 \text{ (or 1.252)}$$

Part C

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1 point is earned for the speed

1 point is earned for the acceleration

Speed =

$$\sqrt{((x'(4))^2 + (y'(4))^2}$$

$$= 0.575 \text{ (or } 0.574)$$

$$\begin{aligned}\text{Acceleration} &= \langle x''(4), y''(4) \rangle \\ &= \langle -0.041, 0.989 \rangle\end{aligned}$$



0	1	2
---	---	---

The student earns all of the following points:

1 point is earned for the speed

1 point is earned for the acceleration

Speed =

$$\sqrt{((x'(4))^2 + (y'(4))^2}$$

$$= 0.575 \text{ (or } 0.574)$$

$$\begin{aligned}\text{Acceleration} &= \langle x''(4), y''(4) \rangle \\ &= \langle -0.041, 0.989 \rangle\end{aligned}$$

Part D

1 point is earned for the integral

1 point is earned for the answer

Distance =

$$\int_2^4 \sqrt{((x'(t))^2 + (y'(t))^2} dt$$

$$= 0.651 \text{ (or } 0.650)$$

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0	1	2
---	---	---

The student earns all of the following points:

1 point is earned for the integral

1 point is earned for the answer

Distance =

$$\int_2^4 \sqrt{((x'(t))^2 + (y'(t))^2} dt$$

= 0.651 (or 0.650)

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37. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

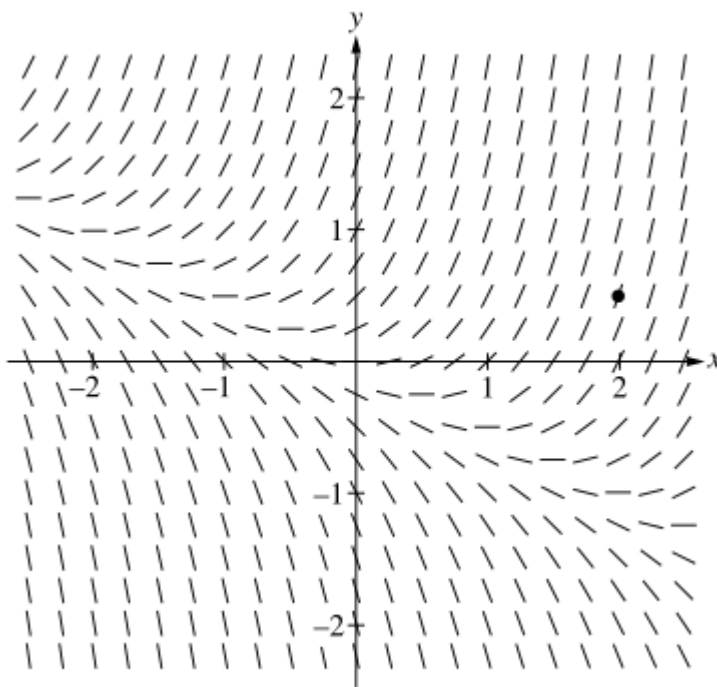
Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with initial condition $f(2) = 0.5$.

(a) A portion of the slope field for the differential equation $\frac{dy}{dx} = x + 2y$ is given. Sketch the solution curve through the point $(2, 0.5)$.



(b) Write an equation for the line tangent to the graph of f at $x = 2$. Use the equation to approximate $f(2.4)$.

(c) Write the second-degree Taylor polynomial for f about $x = 2$. Use the Taylor polynomial to approximate $f(2.4)$.

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(d) It is known that $0 < f'''(x) < 36$ for $2 \leq x \leq 2.4$. Use the Lagrange error bound to show that the approximation from part (c) differs from the actual value of $f(2.4)$ by less than 0.5.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The solution curve must pass through the point $(2, 0.5)$, extend reasonably close (within one row or column of the edge of the slope field) to the left edge and top-right corner of the square, and have no obvious conflicts with the given slope lines.

Do not consider any portion of the solution curve that extends beyond the square.



0

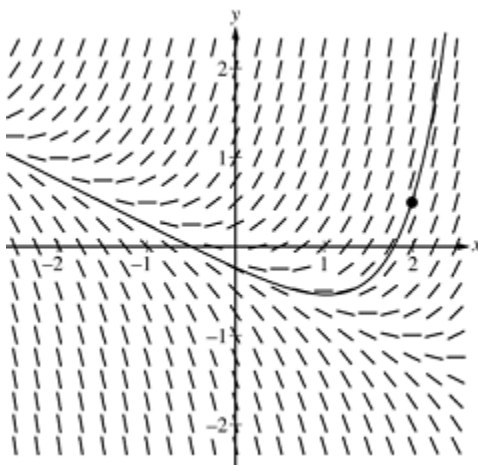
1

The student response accurately includes the criteria below.

☐

Solution curve

Solution:



Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The tangent line equation can be presented in any equivalent form.

A response with incorrect tangent line equation with a slope of 3 will not earn the first point but earns the second point for a consistent approximation.

A response of only $0.5 + 3(2.4 - 2)$ earns the second point but not the first.

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0	1	2
---	---	---

The student response accurately includes **both** of the criteria below.

- ☐ Tangent line equation
- ☐ Approximation

Solution:

$$\left. \frac{dy}{dx} \right|_{(x, y)=(2, 0.5)} = 2 + 2(0.5) = 3$$

An equation for the tangent line is $y = 0.5 + 3(x - 2)$.

$$f(2.4) \approx 0.5 + 3(2.4 - 2) = 1.7$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

To earn the first point a response must present work to support $f''(2) = 7$.

The minimal supporting work is $\frac{d^2y}{dx^2} = 1 + 2\frac{dy}{dx}$.

The expression $\frac{f''(2)}{2!}(x - 2)^2$ will earn the second point even if a value of $f''(2)$ is never presented. If an incorrect value of $f''(2)$ is presented, it must be nonzero in order to earn the second point.

The third point is for adding the tangent line expression and an expression (such as $\frac{f''(2)}{2!}(x - 2)^2$ or $\frac{7}{2}(x - 2)^2$) which has earned the second point.

- An incorrect tangent line expression may be imported from part (b).
- Numeric values of $f(2)$, $f'(2)$, and $f''(2)$ must be included in the polynomial expression in order to earn the third point.
- A correct second-degree polynomial followed by “+ . . .” can earn the third point provided the response uses only the second-degree polynomial in calculating the approximation.
- A response that includes additional terms in a presented polynomial will not earn the third point, but is eligible for the fourth point provided the approximation is found using only the second-degree polynomial.

An answer of just $0.5 + 3(0.4) + \frac{7}{2}(0.4)^2$ earns only the fourth point.

A response that presents an incorrect approximation will earn the fourth point if it has earned the third point and correctly evaluates the presented (incorrect) second-degree Taylor polynomial about $x = 2$ at $x = 2.4$.

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0	1	2	3	4
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The student response accurately includes **all four** of the criteria below.

- ☐ $f''(2) = 7$
- ☐ $\frac{f''(2)}{2!}(x-2)^2$ -OR- $\frac{7}{2}(x-2)^2$
- ☐ Second-degree Taylor polynomial about $x = 2$
- ☐ Approximation

Solution:

$$f(2) = 0.5$$

$$f'(2) = 3$$

$$\frac{d^2y}{dx^2} = 1 + 2\frac{dy}{dx} = 1 + 2(x + 2y) = 1 + 2x + 4y$$

$$f''(2) = 1 + 2(2) + 4(0.5) = 7$$

$$P_2(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2$$

$$= 0.5 + 3(x-2) + \frac{7}{2}(x-2)^2$$

$$f(2.4) \approx P_2(2.4) = 0.5 + 3(0.4) + \frac{7}{2}(0.4)^2 = 2.26$$

Part D

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

· The minimal response to earn the first point is $(\max f'''(x)) \cdot \frac{0.4^3}{3!}$ or $36 \cdot \frac{0.4^3}{3!}$.

· In order to earn the second point, a response must indicate that $|f(2.4) - P_2(2.4)| \leq 36 \cdot \frac{0.4^3}{3!} < 0.5$,
Error bound = $36 \cdot \frac{0.4^3}{3!} < 0.5$, or **Error** $\leq 36 \cdot \frac{0.4^3}{3!} < 0.5$.

A response that only shows $36 \cdot \frac{0.4^3}{3!} < 0.5$ does not earn the second point.

A response that equates the error with the bound (e.g., **Error** = $36 \cdot \frac{0.4^3}{3!}$) does not earn the second point.



0	1	2
---	---	---

The student response accurately includes **both** of the criteria below.

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- ☐ Form of error bound
- ☐ Answer with supporting work

Solution:

By the Lagrange error bound,

$$|f(2.4) - P_2(2.4)| \leq \left(\max_{2 \leq x \leq 2.4} |f'''(x)| \right) \cdot \frac{|2.4-2|^3}{3!}$$

$$< 36 \cdot \frac{0.4^3}{6} = \frac{6 \cdot 64}{1000} < \frac{500}{1000} = 0.5$$

38. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

The Taylor series for a function f about $x = 7$ is given by $\sum_{n=1}^{\infty} \frac{(x-7)^n}{n \cdot 5^n}$ and converges to $f(x)$ for all values of x in the interval of convergence.

- (a) Find the interval of convergence of the Taylor series for f about $x = 7$.
- (b) Write the first four nonzero terms of the Taylor series for f' , the derivative of f , about $x = 7$.
- (c) Let $g(x) = 2e^{f(x)}$. Find $g'(7)$. Show the work that leads to your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The first point is earned by writing the ratio $\frac{\frac{(x-7)^{n+1}}{(n+1) \cdot 5^{n+1}}}{\frac{(x-7)^n}{n \cdot 5^n}}$ or equivalent. The ratio can be written with or without absolute value symbols. Once earned this point cannot be lost. Any errors in simplification or evaluation of the limit will not earn the second point.

The ratio may be expressed in either of the forms $\frac{a_{n+1}}{a_n}$ or $\frac{a_n}{a_{n-1}}$. An expression equivalent to $\frac{a_n}{a_{n+1}}$ earns the first point, and is eligible for the remaining points.

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Read for the second and third points with incorrect ratios.

A response that presents no ratio does not earn the second point.

Proper limit notation must be used in order to earn the second point.

The third point is earned for $2 < x < 12$, or for $\frac{1}{5} |x - 7| < 1$, with or without inclusion of the endpoints.

A response that presents an incorrect limit is eligible for the third point. In this case the point would be earned for finding an interior interval of convergence that is consistent with a presented limit of the form $\left| \frac{x-7}{b} \right|$, where $b \neq 0$.

A response that presents an infinite interval of convergence is not eligible for the fourth or fifth points.

A response that begins with $\frac{1}{5} |x - 7| < 1$ or $2 < x < 12$ does not earn the first 2 points but is eligible for the fourth and fifth points.

The fourth point is earned for considering both endpoints of the correct interval or both endpoints of an incorrect interval that has earned the third point.

To earn the fifth point, the response must have earned the fourth point, must correctly analyze the series at each of the endpoints, and must state the test(s) used for each analysis. A response does not need to address the conditions necessary to apply the test at each endpoint.

The fifth point cannot be earned with an incorrect interval. If the analysis at either endpoint is incorrect, the fifth point is not earned.

Root test: A response that uses the root test instead of the ratio test is scored as follows for the first 2 points (with third, fourth, and fifth points scored as described above):

· First point (Sets up root): $\sqrt[n]{|a_n|} = \sqrt[n]{\left| \frac{(x-7)^n}{n5^n} \right|} = \frac{|x-7|}{5\sqrt[n]{n}}$

· Second point (Computes limit of root): $\lim_{n \rightarrow \infty} \frac{|x-7|}{5\sqrt[n]{n}} = \left| \frac{x-7}{5} \right|$

Note: $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$ can be stated without additional work shown.



0	1	2	3	4	5
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The student response accurately includes **all five** of the criteria below.

- ☐ Sets up ratio
- ☐ Computes limit of ratio
- ☐ Identifies interior of interval of convergence
- ☐ Considers both endpoints
- ☐ Analysis and interval of convergence

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Solution:

$$\left| \frac{\frac{(x-7)^{n+1}}{(n+1) \cdot 5^{n+1}}}{\frac{(x-7)^n}{n \cdot 5^n}} \right| = \left| \frac{(x-7)^{n+1}}{(n+1) \cdot 5^{n+1}} \cdot \frac{n \cdot 5^n}{(x-7)^n} \right| = \frac{1}{5} \cdot \frac{n}{n+1} \cdot |x-7|$$

$$\lim_{n \rightarrow \infty} \left(\frac{1}{5} \cdot \frac{n}{n+1} \right) |x-7| = \frac{1}{5} |x-7|$$

$$\frac{1}{5} |x-7| < 1 \Rightarrow |x-7| < 5 \Rightarrow 2 < x < 12$$

The series converges for $2 < x < 12$.

When $x = 2$, the series is $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$. This series converges by the alternating series test.

When $x = 12$, the series is $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$, the harmonic series, which diverges.

Therefore, the interval of convergence is $2 \leq x < 12$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

A series written about $x = 0$ earns neither point.

Terms could be presented in a list, as a third degree polynomial, or as part of a Taylor series.

Any additional terms presented are not to be considered.



0	1	2
---	---	---

The student response accurately includes **both** of the criteria below.

- ☐ Two terms
- ☐ Remaining terms

Solution:

$$f(x) = \frac{x-7}{5} + \frac{(x-7)^2}{2 \cdot 25} + \frac{(x-7)^3}{3 \cdot 125} + \frac{(x-7)^4}{4 \cdot 625} + \dots$$

$$f'(x) = \frac{1}{5} + \frac{x-7}{25} + \frac{(x-7)^2}{125} + \frac{(x-7)^3}{625} + \dots$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The first point is earned for $g'(x) = 2e^{f(x)} \cdot f'(x)$, $g'(7) = 2e^{f(7)} \cdot f'(7)$, or $g'(7) = 2e^0 \cdot \frac{1}{5}$.

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A response of just $2e^0 \cdot \frac{1}{5}$ earns both points, but a response of just $2 \cdot \frac{1}{5}$ or $\frac{2}{5}$ earns neither point.

The second point can be earned by consistent use of an incorrect, nonzero value for $f'(7)$ imported from part (b).



0	1	2
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The student response accurately includes **both** of the criteria below.

- ☐ $g'(x)$
- ☐ Answer

Solution:

$$g'(x) = 2e^{f(x)} \cdot f'(x)$$

$$g'(7) = 2e^{f(7)} \cdot f'(7) = 2e^0 \cdot \frac{1}{5} = \frac{2}{5}$$