

2022 AP Physics C: Mechanics International Exam Questions

Extrated from the AP Question Bank

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1 Multiple Choice Questions

The exam begins on the next page.

PHYSICS C: MECHANICS

SECTION I

Time—45 minutes

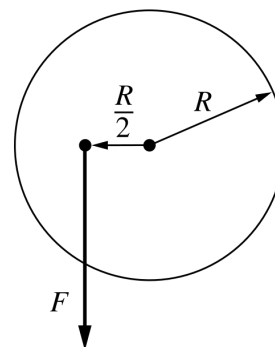
35 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding circle on the answer sheet.

Note: To simplify calculations, you may use $g = 10 \text{ m/s}^2$ in all problems.

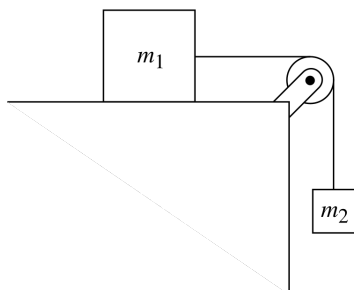
1. A rocket of mass m is launched from the surface of Earth with an initial speed equal to one-half the escape speed. The mass and the radius of Earth are 6.0×10^{24} kg and 6.4×10^6 m, respectively. What is the maximum altitude achieved by the rocket? Assume air resistance is negligible.

- (A) 9.1×10^5 m
(B) 2.1×10^6 m
(C) 6.4×10^6 m
(D) 1.9×10^7 m
(E) 4.5×10^7 m



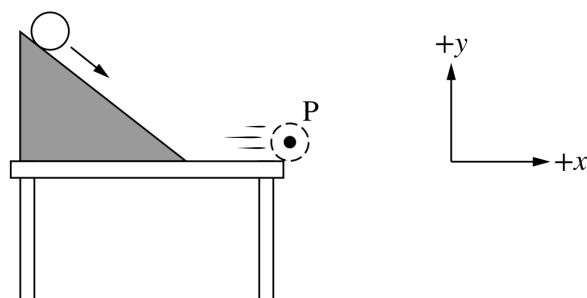
2. A solid disk of mass M and radius R is free to rotate about a fixed axis through its center. The rotational inertia of the disk around its center is $\frac{MR^2}{2}$. A force F is applied tangentially at a distance $\frac{R}{2}$ from the axis, as shown in the figure above. What is the angular acceleration of the disk?

- (A) $\frac{2F}{MR^2}$
(B) $\frac{2F}{MR}$
(C) $\frac{F}{MR}$
(D) $\frac{F}{MR^2}$
(E) $\frac{F}{2MR^2}$



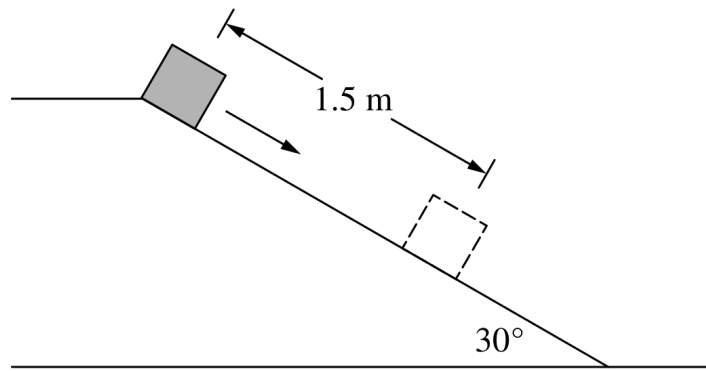
3. A large block of mass m_1 on a horizontal table is attached to a small block of mass m_2 by a light string that hangs over a pulley of mass m_0 , as shown. The table and pulley axle have negligible friction. The system is released from rest. As the blocks move, the string exerts tension F_1 on m_1 and F_2 on m_2 , as shown. The angular acceleration α of the pulley is experimentally determined. If the pulley is replaced with a new pulley that has the same radius but a much larger mass, α is found to be smaller. If frictional forces are negligible, which of the following is the best explanation for this observation?

- (A) Net torque is required to turn the pulley with mass, so F_1 and F_2 both decrease.
- (B) Net torque is required to turn the pulley with mass, so F_1 and F_2 both increase.
- (C) Net torque is required to turn the pulley with mass, so F_1 increases and F_2 decreases.
- (D) Net torque is required to turn the pulley with mass, so F_1 decreases and F_2 increases.
- (E) The angular acceleration α was incorrectly determined, because increasing the mass of the pulley should not affect α .



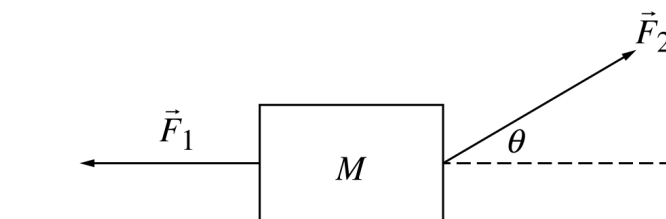
4. A ball is released from the top of a ramp and rolls off of a table, as shown in the figure. Point P is the location at which the ball has just left the edge of the table. Using the coordinate axes shown, which of the following is correct regarding the vertical acceleration a_y and the components of the velocity v of the ball at point P?

- (A) $a_y < 0$ $v_x > 0$ $v_y = 0$
- (B) $a_y = 0$ $v_x > 0$ $v_y = 0$
- (C) $a_y > 0$ $v_x > 0$ $v_y < 0$
- (D) $a_y < 0$ $v_x > 0$ $v_y > 0$
- (E) $a_y > 0$ $v_x = 0$ $v_y < 0$



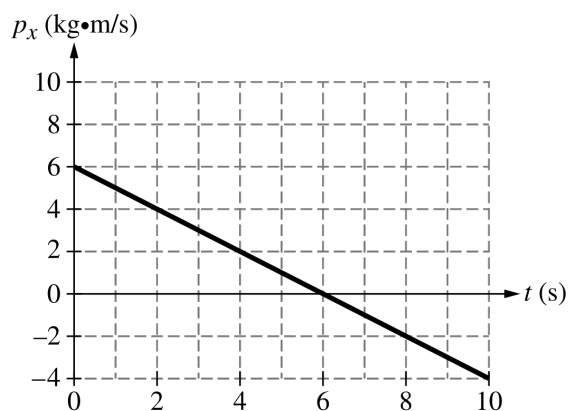
5. A box of mass 0.50 kg slides down a rough ramp with an initial speed of 2.0 m/s . The ramp is inclined at an angle of 30° above the horizontal. The box slides a distance of 1.5 m and comes to rest before reaching the bottom of the ramp. The energy dissipated as the box slides to a stop is most nearly
- (A) 0
 - (B) 1.4 J
 - (C) 2.8 J
 - (D) 4.7 J
 - (E) 8.4 J

Questions 6-7

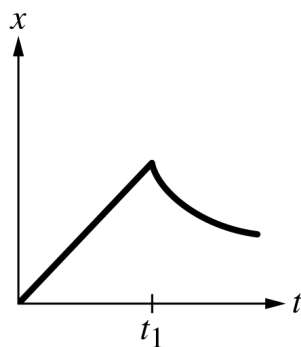


A box of mass M is pulled to the right at constant speed across a rough, horizontal surface. Two ropes are used to exert forces $\vec{F}_1$ and $\vec{F}_2$ on the box. $\vec{F}_1$ is horizontal and to the left, and $\vec{F}_2$ is at an angle θ above the horizontal, as shown. The coefficient of kinetic friction between the box and the horizontal surface is μ .

6. Which of the following is a correct expression for the magnitude of force $\vec{F}_1$?
 - (A) $F_2 \cos \theta$
 - (B) $\mu(Mg - F_2 \sin \theta)$
 - (C) $F_2 \cos \theta - \mu(Mg - F_2 \sin \theta)$
 - (D) $F_2 \cos \theta + \mu(Mg - F_2 \sin \theta)$
 - (E) $F_2 \sin \theta - \mu(Mg - F_2 \cos \theta)$
7. At a later time, the angle θ is slowly decreased without changing the magnitude of the two forces. The angle θ continues to decrease until the force $\vec{F}_2$ is horizontal, after which time the force $\vec{F}_2$ remains horizontal. Which of the following best describes the changes in the normal force and the force of friction on the box while the angle is changing?
 - (A) The normal force increases and the force of friction increases.
 - (B) The normal force increases and the force of friction decreases.
 - (C) The normal force increases and the force of friction remains constant.
 - (D) The normal force decreases and the force of friction remains constant.
 - (E) The normal force decreases and the force of friction decreases.

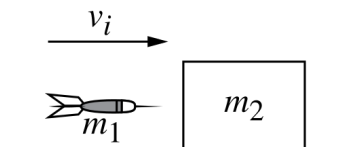


8. An object is moving in a straight line along the x -axis. The momentum of the object p_x as a function of time t is shown in the graph above. Which statement below correctly describes the net force that the object experiences during the time period shown?
- (A) The net force is decreasing over time.
 - (B) The net force is increasing over time.
 - (C) The net force is constant and positive.
 - (D) The net force is constant and negative.
 - (E) The net force is positive for $0 < t < 6$ s and negative for $6 \text{ s} < t < 10$ s.



9. An object is moving in the x direction as shown in the graph. At time $t = t_1$, the motion changes. Which of the following describes the velocity v before and acceleration a after $t = t_1$?
- | <u>Velocity before $t = t_1$</u> | <u>Acceleration after $t = t_1$</u> |
|---|--|
| (A) Zero | Positive |
| (B) Positive | Negative |
| (C) Positive | Positive |
| (D) Negative | Positive |
| (E) Positive | Zero |

Questions 10-11



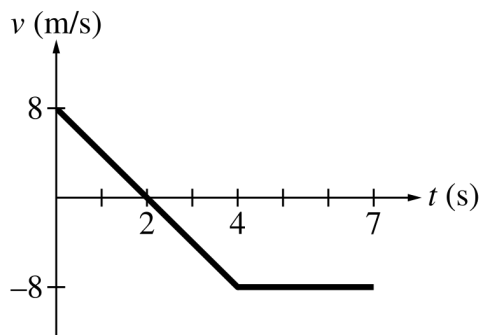
A projectile of mass m_1 is traveling with a horizontal speed v_i to the right toward a block of mass m_2 that is at rest on a horizontal surface, as shown. The projectile becomes embedded in the block, and the block and projectile move together to the right with speed v_f . There is negligible friction between the block and the surface.

10. Students measure v_f using a motion detector and compare the measured v_f to the expected theoretical value. What could contribute to the experimental value of v_f being greater than the theoretical value?

(A) Greater-than-expected friction between the block and the surface
(B) Deformation of the block as a result of the projectile's impact
(C) Friction between the projectile and the block as the projectile becomes embedded in the block
(D) An increase in temperature of the block as a result of the impact
(E) A slight downward incline of the surface the block moves on

11. The duration of the collision between the projectile and the block is Δt . Which of the following expressions is equal to the average force exerted on the block by the projectile during the collision?

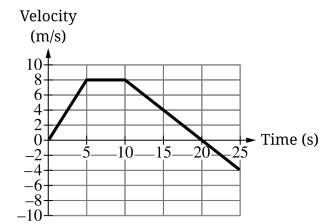
(A) $\frac{m_1 v_i}{2\Delta t}$
(B) $\frac{m_2 v_i}{2\Delta t}$
(C) $\frac{m_1 v_f}{m_2 \Delta t}$
(D) $\frac{m_1 v_f}{\Delta t}$
(E) $\frac{m_2 v_i}{\Delta t}$



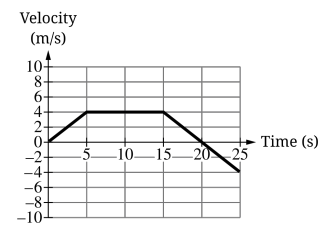
12. The graph shows the velocity of an object v as a function of time t . What is the **distance** traveled by the object from $t = 0 \text{ s}$ to $t = 7.0 \text{ s}$?

(A) -24 m
 (B) -16 m
 (C) 24 m
 (D) 40 m
 (E) 56 m

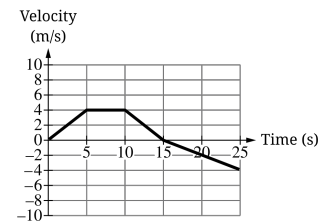
13. Five toy cars are being raced by students. The students plot the velocity as a function of time for each car. Which graph indicates the car that has the greatest displacement at the end of the race?



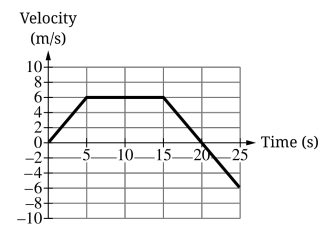
(A)



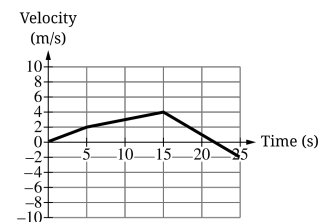
(B)



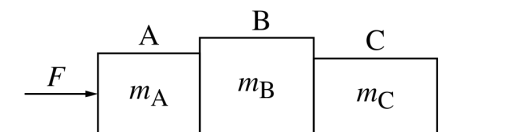
(C)



(D)



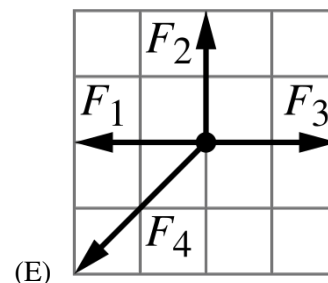
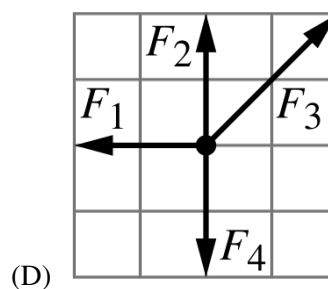
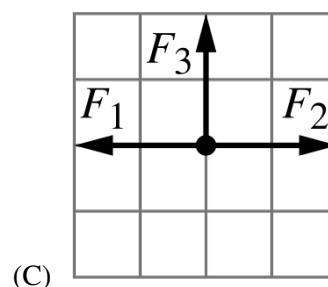
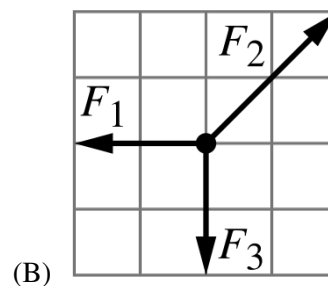
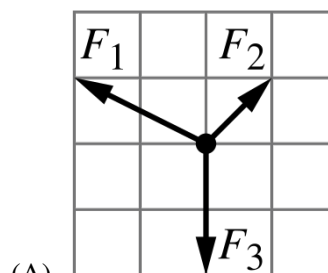
(E)



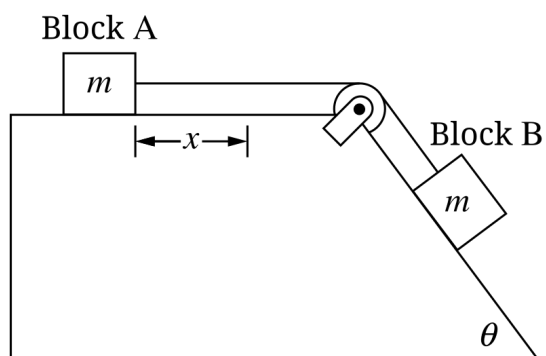
14. Boxes A, B, and C of masses m_A , m_B , and m_C , respectively, are on a horizontal surface with negligible friction. A horizontal force F is exerted on Box A, as shown. The boxes remain in contact with each other as they move along the surface. The magnitude of the force that Box B exerts on Box C is

- (A) F
 (B) $\left(\frac{m_C}{m_A + m_B}\right) F$
 (C) $\left(\frac{m_A + m_B}{m_C}\right) F$
 (D) $\left(\frac{m_C}{m_A + m_B + m_C}\right) F$
 (E) $\left(\frac{m_A + m_B + m_C}{m_C}\right) F$

15. Which of the following free-body diagrams shows an object in equilibrium?

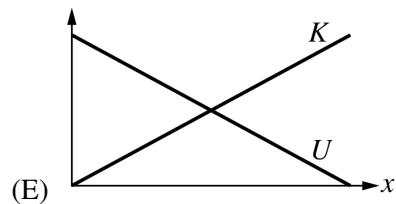
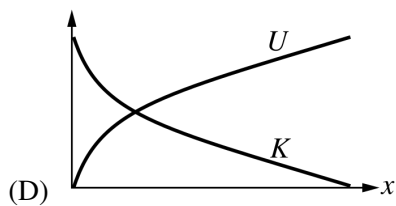
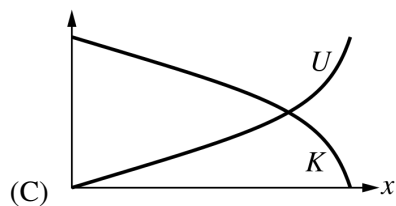
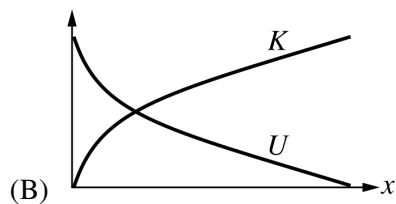
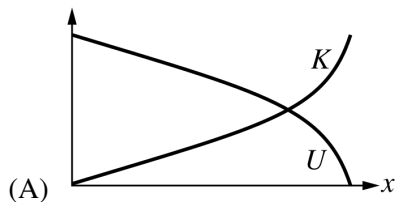


Questions 16-17



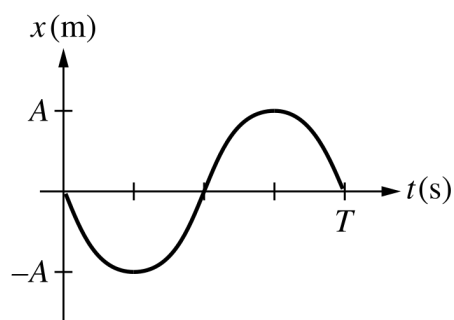
Blocks A and B, each of mass m , are connected by a string that extends over a pulley. Block A slides on a horizontal surface, and Block B is on a surface that is inclined at an angle θ above the horizontal, as shown. Both surfaces have negligible friction. The masses of the string and pulley are negligible, and the pulley rotates with negligible friction.

16. Which of the following graphs best shows the sum of the kinetic energy K of the two blocks and the potential energy U of the two-block-Earth system, as a function of the distance x Block A has moved along the horizontal surface before Block A reaches the pulley?



17. The system is released from rest. Which of the following is a correct expression for the speed of Block B when it has moved a distance x ?

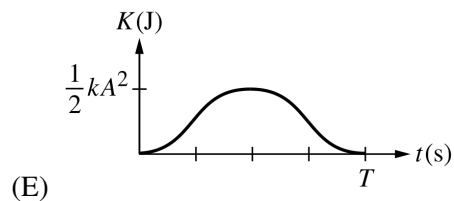
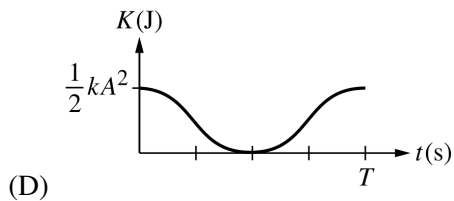
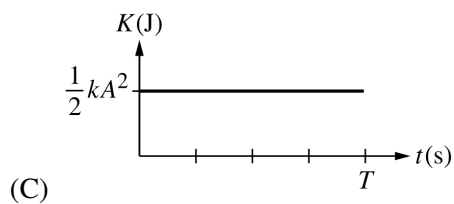
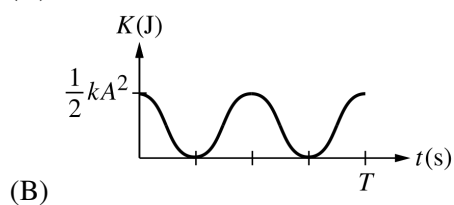
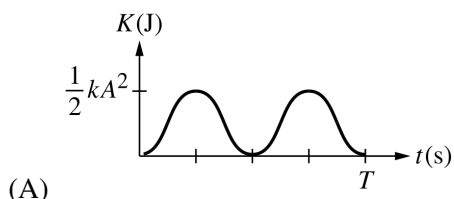
- (A) $\sqrt{gx}$
 (B) $\sqrt{gx \sin \theta}$
 (C) $\sqrt{2gx \sin \theta}$
 (D) $\sqrt{gx(1 + \sin \theta)}$
 (E) $\sqrt{2gx(1 + \sin \theta)}$



19. A particle has a potential energy given as a function of position x by $U(x) = 9x^2 - 4x$. What is the force acting on the particle at $x = a$?

- (A) $-18a + 4$
- (B) $-18a - 4$
- (C) $18a - 4$
- (D) $3a^2 - 2a^2$
- (E) $-3a^2 + 2a^2$

18. A mass on a spring of spring constant k oscillates as shown. Which of the following graphs would correctly depict the kinetic energy versus time for the oscillating mass?





Before Collision

Figure 1

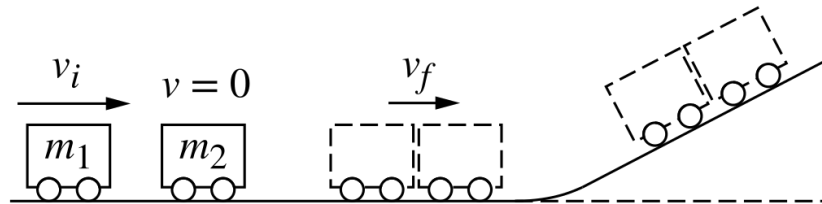


After Collision

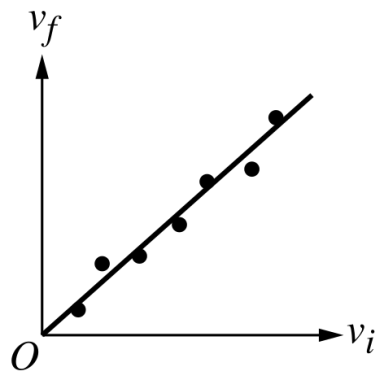
Figure 2

20. Two pucks, A and B, with unequal masses, slide with equal speeds in opposite directions on a horizontal surface with negligible friction, as shown in Figure 1. The pucks undergo a head-on collision. After the collision, both pucks move to the right. The speed of Puck A has decreased, and the speed of Puck B has increased, as shown in Figure 2. Which of the following is a plausible explanation for the observed behavior of the pucks?
- (A) The magnitude of change in momentum is the same for each puck, but Puck A has more momentum initially.
 - (B) The magnitude of change in momentum is the same for each puck, but Puck B has more momentum initially.
 - (C) The magnitude of change of momentum is the same for each puck because each puck has the same momentum initially.
 - (D) The magnitude of change in momentum of Puck A is greater than that of Puck B because Puck A has a greater initial momentum.
 - (E) The magnitude of change of momentum of Puck A is less than that of Puck B because Puck A has greater mass.

Questions 21-22



A student conducts an experiment in which a cart of mass m_1 is moving on a level horizontal surface of negligible friction with speed v_i toward a second cart of mass m_2 that is at rest. The carts collide, stick together, and move with speed v_f . The carts then travel up a ramp, momentarily come to rest, and then roll back down the ramp. The student repeats the experiment for several different values of v_i .



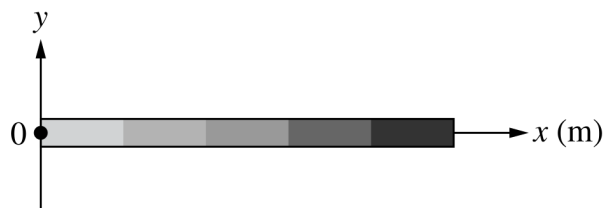
The student plots v_f as a function of v_i and draws a best-fit straight line to represent the data.

21. Which of the following expressions can be determined using the slope of the best-fit line?

- (A) $m_1 m_2$
- (B) $\frac{m_1}{m_2}$
- (C) $\frac{m_1}{m_1 + m_2}$
- (D) $\frac{m_2}{m_1 + m_2}$
- (E) $\frac{m_1 - m_2}{m_1 + m_2}$

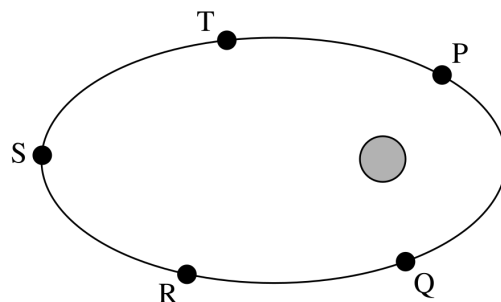
22. The student repeats the whole experiment but doubles the initial speed v_i of the cart with mass m_1 for each data point collected. When the new data points are plotted and a best-fit line is drawn, how would the slope of this best-fit line compare to the slope on the original graph?

- (A) The slope would stay the same.
- (B) The slope would be one-half as great.
- (C) The slope would be one-quarter as great.
- (D) The slope would be twice as great.
- (E) The slope would be four times as great.



23. A rod with a nonuniform mass density given by $\lambda = Ax^2$, where $A = 5.0 \text{ kg/m}^3$, is aligned on the x -axis with one end at the origin and the other end at $x = 2.0 \text{ m}$, as shown. The rotational inertia of the rod about the origin is most nearly

(A) $8 \text{ kg}\cdot\text{m}^2$
 (B) $13 \text{ kg}\cdot\text{m}^2$
 (C) $32 \text{ kg}\cdot\text{m}^2$
 (D) $53 \text{ kg}\cdot\text{m}^2$
 (E) $80 \text{ kg}\cdot\text{m}^2$

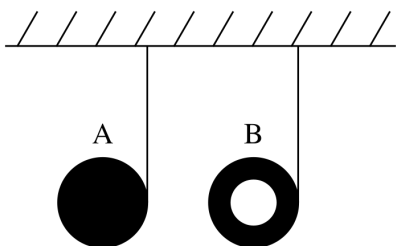


24. A satellite is orbiting a planet in an elliptical orbit. Five locations in the orbit are P, Q, R, S, and T, as shown. The distance traveled by the satellite between each point along the path is the same. Which of the following accurately represents the angular momentum about the planet and the kinetic energy of the satellite at the different locations?

	Angular Momentum	Kinetic Energy
(A)	$L_P > L_T$	$K_P = K_T$
(B)	$L_Q > L_S$	$K_Q > K_S$
(C)	$L_Q = L_R$	$K_Q > K_R$
(D)	$L_P = L_T$	$K_P < K_T$
(E)	$L_R = L_T$	$K_R > K_T$

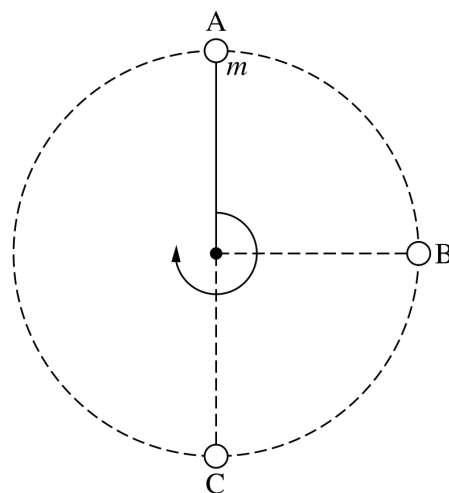
25. A sled that weighs $5.0 \times 10^3 \text{ N}$ is pulled across level snow by dogs exerting a horizontal force on it. The coefficient of kinetic friction between the sled and snow is 0.050. How much work is done by the dogs on the sled when the sled moves $1.0 \times 10^3 \text{ m}$ at constant speed?

(A) Zero
 (B) $2.5 \times 10^5 \text{ J}$
 (C) $5.0 \times 10^5 \text{ J}$
 (D) $2.5 \times 10^6 \text{ J}$
 (E) $5.0 \times 10^6 \text{ J}$



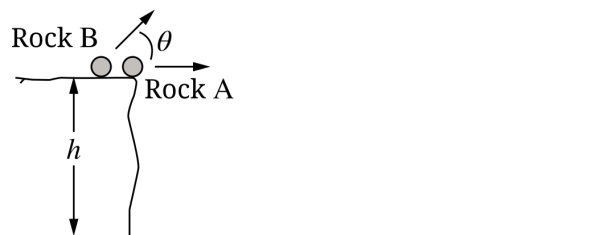
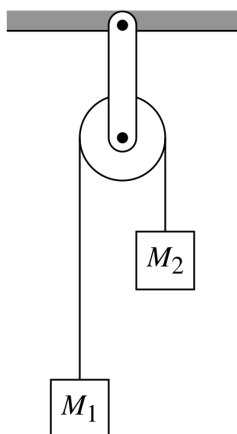
26. Two disks have equal masses and radii. Disk A is solid, and Disk B is hollow. Each disk has a thin string wrapped around it. Both strings are attached to the ceiling as shown. The disks are simultaneously released from rest from the same height. Which of the following accurately describes which disk will hit the ground first and why?

- (A) Disk A will hit the ground first. Disk A has a greater rotational inertia; therefore, the tension in the string for Disk A will be greater than the tension in the string for Disk B.
- (B) Disk B will hit the ground first. Disk B has a smaller rotational inertia; therefore, it will have a greater angular acceleration.
- (C) Disk A will hit the ground first. Disk A has a smaller rotational inertia; therefore, it will have a greater linear acceleration.
- (D) Disk B will hit the ground first. Disk B has a greater rotational inertia; therefore, it will have a greater angular acceleration.
- (E) Both disks will hit the ground at the same time. Since both disks have the same mass, they will have the same linear acceleration.



27. A sphere of mass m is connected to a lightweight rod of length L and moves at a constant speed greater than $\sqrt{Lg}$ in a vertical circle, as shown. The magnitudes of the forces the rod exerts on the sphere at positions A, B, and C are F_A , F_B , and F_C , respectively. Which of the following expressions is equal to $2mg$?

- (A) $F_A + F_B$
- (B) $F_B + F_C$
- (C) $F_C + F_A$
- (D) $F_B - F_A$
- (E) $F_C - F_A$



28. Two objects of mass M_1 and M_2 are connected by a string that passes over a pulley with negligible mass and negligible friction, as shown. A group of students wants to use this setup to measure the acceleration due to gravity g . Which of the following procedures could the students use?

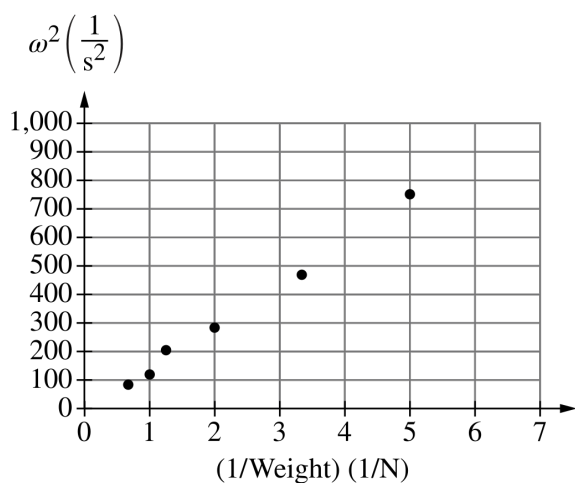
- (A) Use equal masses, collect velocity data for M_2 , and calculate its acceleration.
- (B) Use equal masses, collect position data for M_2 , and calculate its average velocity.
- (C) Use unequal masses, collect velocity data for M_2 , and calculate its acceleration.
- (D) Use unequal masses, collect position data for M_2 , and calculate its average velocity.
- (E) Use different sets of equal masses, collect velocity data for each value of M_2 , and calculate the average acceleration.

29. Two rocks are thrown simultaneously from a height h above the ground, as shown. Rock A has a mass m and is thrown horizontally with a speed v_0 . Rock B has a mass $2m$ and is thrown with the same speed v_0 at an angle θ above the horizontal. Which of the following statements is true?

- (A) Rock A hits the ground with a greater speed than Rock B.
- (B) Rock B hits the ground with a greater acceleration than Rock A.
- (C) Rock A and Rock B hit the ground with the same speed.
- (D) Rock A and Rock B hit the ground simultaneously.
- (E) Rock B's acceleration changes direction during the flight.

30. A block of mass m is pulled from rest along a horizontal surface by a horizontal force whose magnitude is given by the equation $F(t) = At + B$, where A and B are constants with appropriate units. The coefficient of friction between the block and the surface is μ_k . Which expression correctly represents the velocity of the block as a function of time?

- (A) $\frac{At^2}{m} + \frac{Bt}{m} + g\mu_k t$
- (B) $\frac{At^2}{2m} + \frac{Bt}{m} + g\mu_k t$
- (C) $\frac{At^2}{2m} + \frac{Bt}{m} - g\mu_k t$
- (D) $\frac{At^2}{2m} - \frac{Bt}{m} - g\mu_k t$
- (E) $\frac{At^2}{m} - \frac{Bt}{m} + g\mu_k t$



31. Cylinders of various masses are individually hung on a spring of spring constant k and set into simple harmonic motion. The angular frequency ω is determined. Then, the square of ω is plotted as a function of the inverse of the cylinder weight, as shown in the graph. Which of the following correctly determines the spring constant k and provides correct justification?

(A) $k = 150 \text{ N/m}$, because the slope of the best-fit line is equal to k .

(B) $k = 15 \text{ N/m}$, because the slope of the best-fit line is equal to $g \cdot k$.

(C) $k = 6.7 \text{ N/m}$, because the slope of the best fit line is equal to $\frac{1}{k}$.

(D) $k = 6.7 \times 10^{-2} \text{ N/m}$, because the slope of the best fit line is equal to $\frac{g}{k}$.

(E) k cannot be determined from the graph, because ω does not depend on k .

32. A rock weighs 2.0 N on the surface of Earth. How far away from the center of a star with mass $2.0 \times 10^{30} \text{ kg}$ must the same rock be in order to exert a gravitational force on the star with a magnitude of 2.0 N?

(A) $3.7 \times 10^9 \text{ m}$

(B) $8.1 \times 10^9 \text{ m}$

(C) $1.2 \times 10^{10} \text{ m}$

(D) $1.3 \times 10^{19} \text{ m}$

(E) $6.7 \times 10^{19} \text{ m}$

33. A satellite is orbiting Earth at an altitude of one-half Earth's radius. If the weight of the satellite while in orbit is 9,000 N, what is the weight of the satellite on Earth's surface?

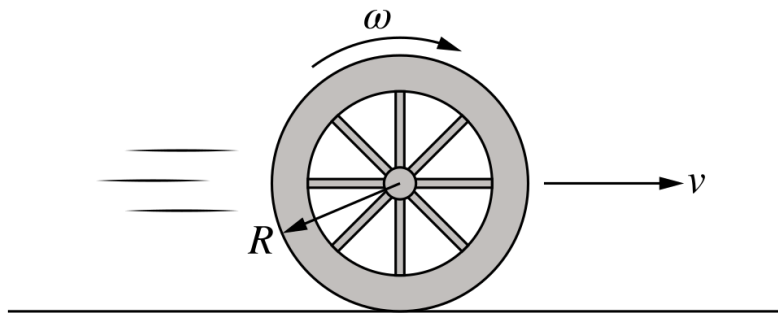
(A) 4,000 N

(B) 6,000 N

(C) 9,000 N

(D) 13,500 N

(E) 20,250 N



34. At time $t = 0$, the wheel shown in the diagram has clockwise rotation with an angular speed of 20 rad/s . The center of mass of the wheel is initially moving to the right at 2 m/s . The wheel rolls on a flat rough surface and has a radius of 0.2 m . Which of the following is correct about the type of motion, the angular speed, and the linear speed of the wheel at time $t = 0$?
- (A) The type of motion is rolling with slipping, the angular speed remains the same, and the linear speed is decreasing.
 - (B) The type of motion is rolling with slipping, the angular speed is decreasing, and the linear speed is increasing.
 - (C) The type of motion is rolling with slipping, the angular speed is decreasing, and the linear speed is decreasing.
 - (D) The type of motion is rolling without slipping, the angular speed is increasing, and the linear speed is increasing.
 - (E) The type of motion is rolling without slipping, the angular speed is decreasing, and the linear speed is decreasing.

Item #35 was not scored.

**STOP
END OF SECTION I.**

2 Free-Response Questions

The exam begins on the next page.

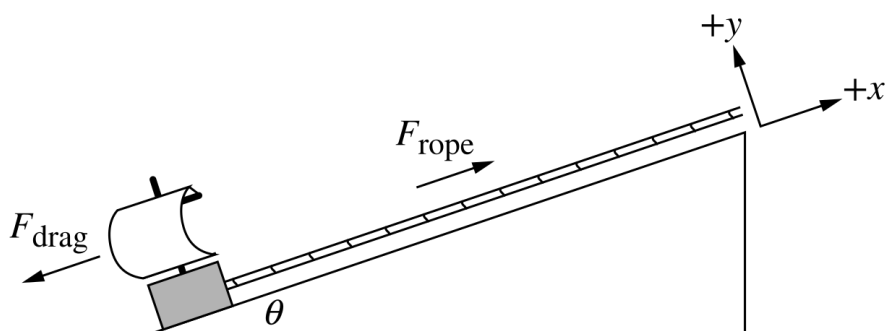
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

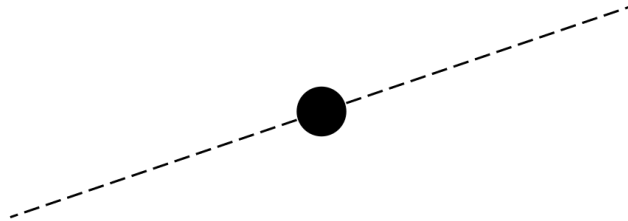
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

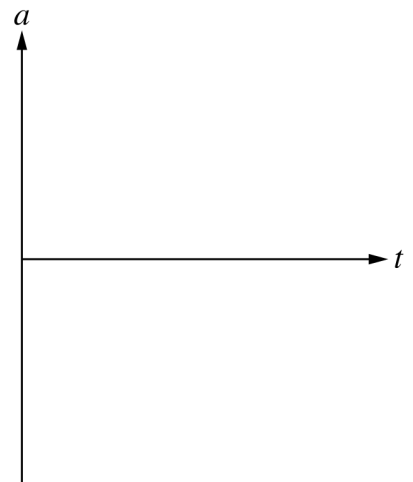
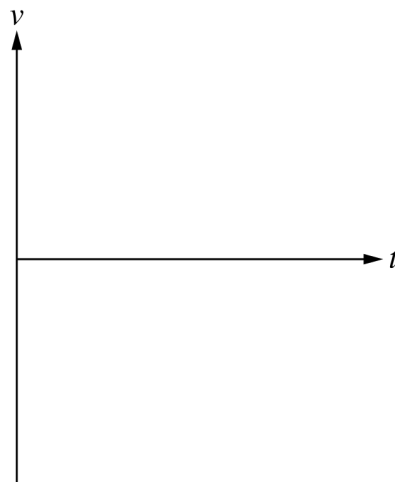
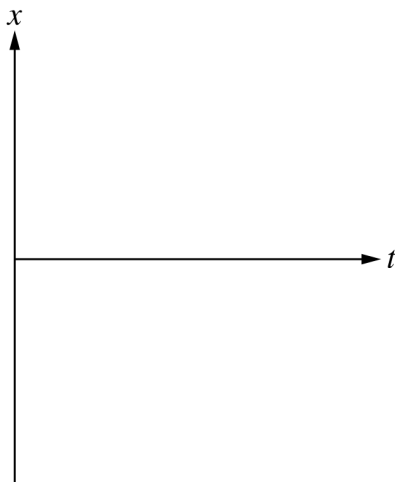


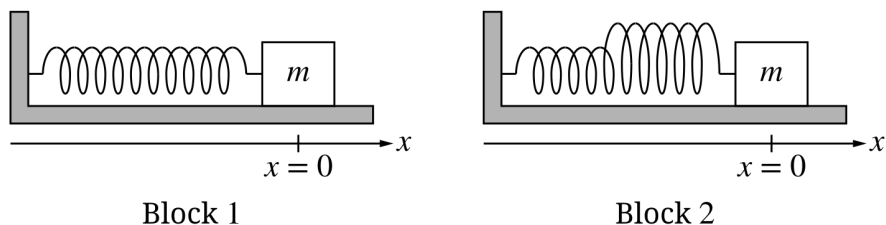
1. A block-sail system of mass m is pulled up a ramp by a rope. There is negligible friction between the system and the ramp, which is inclined at an angle θ from the horizontal, as shown. The rope exerts a constant force of magnitude $F_{\text{rope}} = 2mg$ on the system. Air exerts a drag force of magnitude $F_{\text{drag}} = bv$ on the system, where v is the speed of the system and b is a positive constant with units of kg/s. The system is at position $x = 0$ at rest at time $t = 0$.

- (a) On the dot below, which represents the block-sail system a short time after time $t = 0$, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



- (b) Write, but do NOT solve, a differential equation that could be used to solve for the speed $v(t)$ of the block-sail system as a function of time t . Express your answer in terms of m , θ , b , v , and physical constants as appropriate.
- (c) The velocity of the block-sail system can be modeled by the equation $v(t) = v_{\text{terminal}} \left(1 - e^{-\frac{bt}{m}} \right)$.
- Derive an expression for the position of the block-sail system as a function of time t in terms of θ , m , and b . Do not include v_{terminal} in your final expression.
 - Derive an expression for the change in kinetic energy of the block-sail system as a function of time t .
- (d) On the axes below, sketch the position x , velocity v , and acceleration a of the block-sail system as functions of time t .





2. Two identical blocks of mass $m = 0.65 \text{ kg}$ are initially at rest on horizontal surfaces at position $x = 0$. There is negligible friction between the blocks and the surfaces. One end of a spring is attached to each block, and the other end of the spring is attached to a fixed vertical support. The force exerted by the spring on each block is given as a function of the position x of the block, as indicated by the following equations.

Block 1: $F = -k_1x$ where $k_1 = 10.0 \text{ N/m}$

Block 2: $\begin{cases} F_1 = -k_1x & \text{for } |x| \leq 0.50\text{m, where } k_1 = 10.0\text{N/m} \\ F_1 = -k_2x^3 & \text{for } |x| > 0.50\text{m, where } k_2 = 40.0\text{N/m}^3 \end{cases}$

Both blocks are pulled 0.80 m to the right and released from rest.

- (a) Which block or blocks, if either, exhibits simple harmonic motion?

_____ Block 1 only _____ Block 2 only _____ Both _____ Neither

Briefly justify your answer.

- (b) i. Calculate the amount of work required to pull Block 1 from the equilibrium position to a position $x = 0.80 \text{ m}$.

- ii. Calculate the amount of work required to pull Block 2 from the equilibrium position to a position $x = 0.80 \text{ m}$.

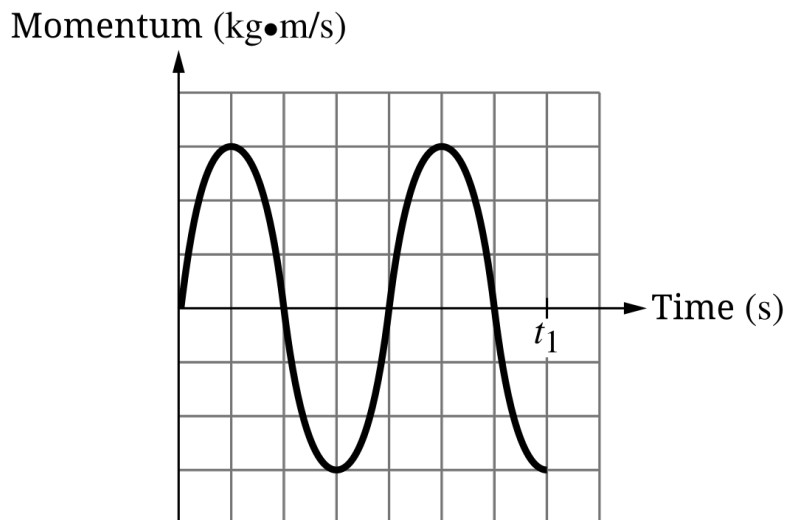
- (c) i. At time $t = 0$, Block 1 is released from rest at $x = 0.80 \text{ m}$. Calculate the time t_{eq} at which Block 1 passes through the equilibrium position the first time.

- ii. If both blocks are released simultaneously from rest at $x = 0.80 \text{ m}$, which block passes through its equilibrium first?

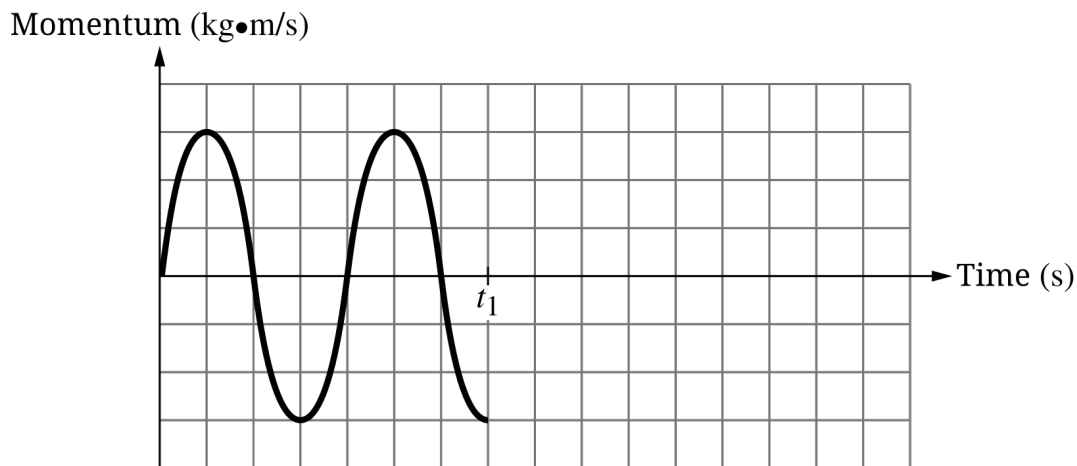
_____ Block 1 _____ Block 2 _____ Same time

Briefly justify your answer.

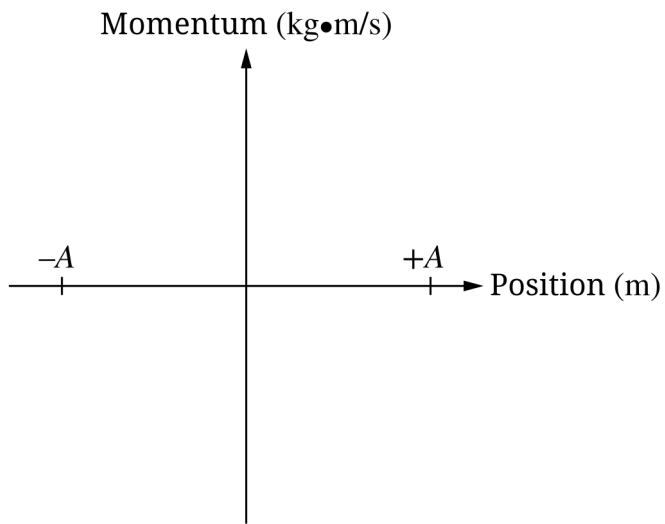
Block 1 is pressed against the spring by 0.40 m and released from rest. The momentum of the block as a function of time is shown below.



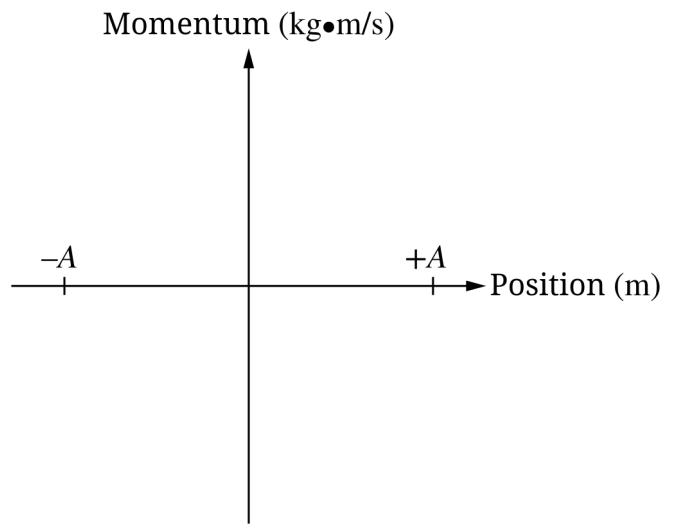
- (d) At time t_1 , a second block of mass $3m$ falls straight down and sticks to Block 1. The two blocks then oscillate together. On the grid below, sketch a graph of the momentum of the two-block system as a function of time after time $t = t_1$.



- (e) The second block is removed from the top of Block 1 and Block 1 oscillates with an amplitude A .
Use the axes below on the left to sketch a graph of the momentum of Block 1 as a function of position as Block 1 moves from left to right.
Use the axes below on the right to sketch a graph of the momentum of Block 1 as a function of position as Block 1 moves from right to left.



Moving Left to Right



Moving Right to Left

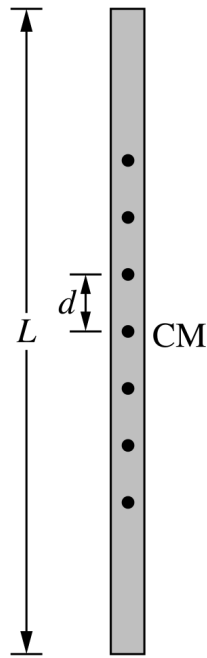


Figure 1

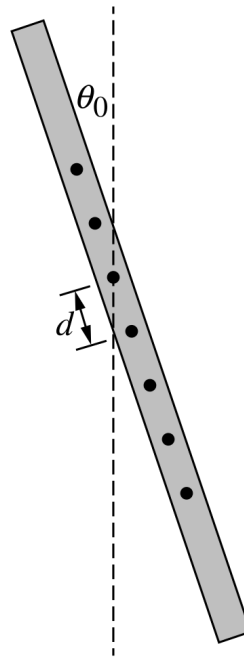


Figure 2

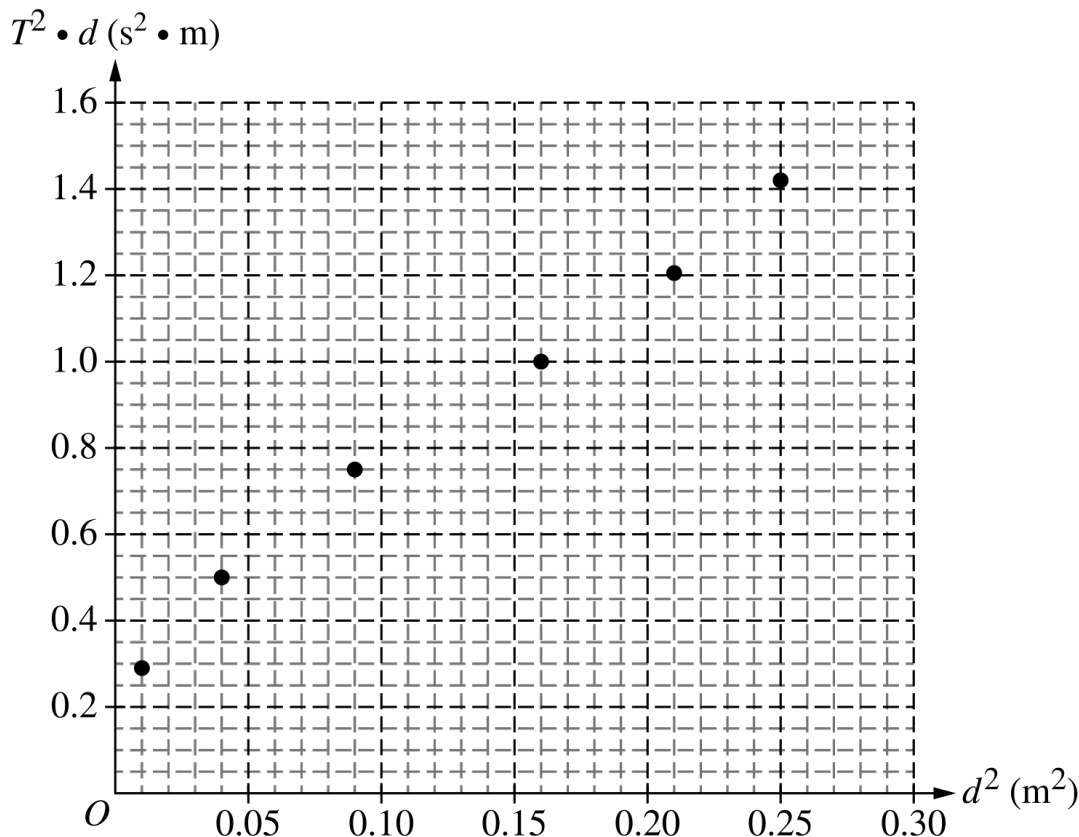
3. A group of students perform experiments using a long, uniform, thin rod that can be pivoted at various points along its length, as shown in Figure 1. The length of the rod is L and the mass of the rod is M . The point CM is the center of mass of the rod. The moment of inertia of the rod about point CM is $I_{CM} = \frac{1}{12}ML^2$.

The rod is attached to a pivot that is located a distance d from point CM . The rod is then displaced by a small angle θ_0 from the vertical and released from rest, as shown in Figure 2.

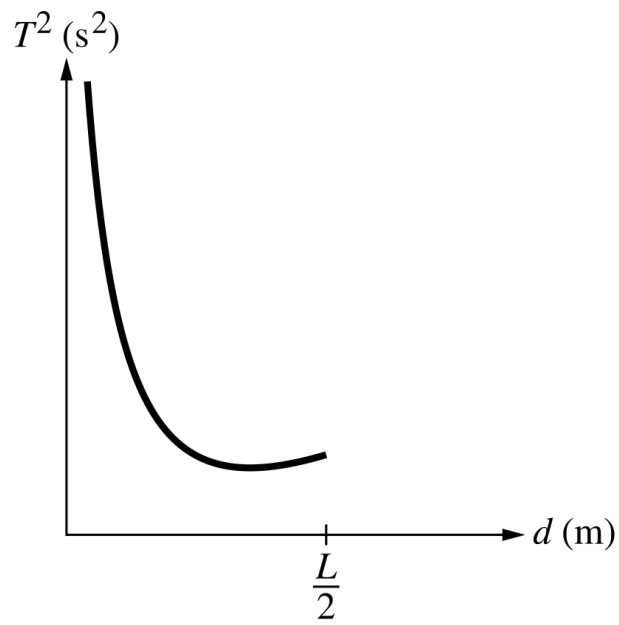
- (a) Derive an expression for the initial angular acceleration α of the rod. Express your answer in terms of M , L , d , θ_0 , and physical constants, as appropriate.

The period of oscillation of the rod is described by the relationship $T^2 d = (2\pi)^2 \left(\frac{d^2 + L^2/12}{g} \right)$.

The students now conduct an experiment to determine the value of the gravitational constant g . The students use a rod of mass $M = 0.200$ kg and length $L = 1.0$ m and measure the period T of the oscillations of the rod for several different values of d . The students then create the plot of $T^2 d$ as a function of d^2 shown below.



- (b) i. Draw a best-fit line for the data.
 ii. Using the slope of the best-fit line, calculate the acceleration due to gravity g .
 iii. Using the y-intercept of the best-fit line, calculate the acceleration due to gravity g .
- (c) The students measure the length of the rod, and they determine that the length of the rod is not exactly 1.0 m.
- i. Did this difference in length affect the value of g calculated using the slope of the line, the y-intercept, both, or neither?
 _____ Slope _____ The y-intercept _____ Both _____ Neither
 Briefly justify your answer.
- ii. Is the actual length of the rod greater than or less than 1.0 m?
 _____ Less than _____ Greater than _____ Not enough information to determine
 Briefly justify your answer.



- (d) Students correctly sketch the graph above, of the square of the period, T^2 , of the rod as a function of d . In terms of torque, explain the physical significance of the vertical asymptote as d approaches zero.

STOP
END OF EXAM

3 Answers for Multiple Choice Questions

Question	Answer	Question	Answer	Question	Answer	Question	Answer	Question	Answer
1	B	8	D	15	B	22	A	29	C
2	C	9	C	16	E	23	C	30	C
3	D	10	E	17	B	24	C	31	B
4	A	11	E	18	B	25	B	32	A
5	D	12	D	19	A	26	C	33	E
6	C	13	A	20	A	27	E	34	B
7	A	14	D	21	C	28	C	35	-

4 Scoring Guidelines

2022 Form I

1. A rocket of mass m is launched from the surface of Earth with an initial speed equal to one-half the escape speed. The mass and the radius of Earth are 6.0×10^{24} kg and 6.4×10^6 m, respectively. What is the maximum altitude achieved by the rocket? Assume air resistance is negligible.

(A) 9.1×10^5 m

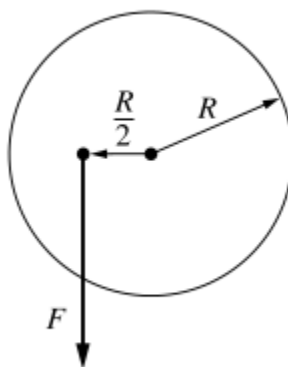
(B) 2.1×10^6 m

(C) 6.4×10^6 m

(D) 1.9×10^7 m

(E) 4.5×10^7 m

2.



A solid disk of mass M and radius R is free to rotate about a fixed axis through its center. The rotational inertia of the disk around its center is $MR^2/2$. A force F is applied tangentially at a distance $R/2$ from the axis, as shown in the figure above. What is the angular acceleration of the disk?

(A) $\frac{2F}{MR^2}$

(B) $\frac{2F}{MR}$

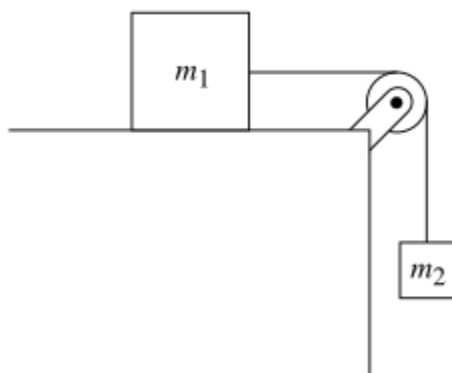
(C) $\frac{F}{MR}$

(D) $\frac{F}{MR^2}$

(E) $\frac{F}{2MR^2}$

2022 Form I

3.

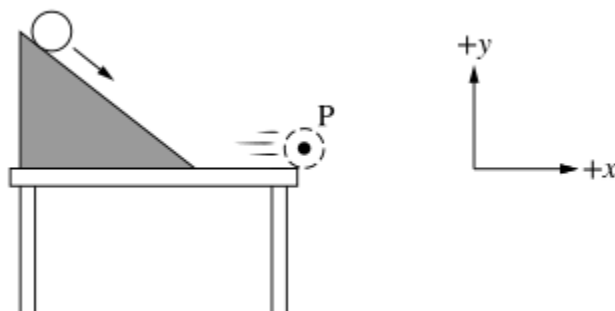


A large block of mass m_1 on a horizontal table is attached to a small block of mass m_2 by a light string that hangs over a pulley of mass m_0 , as shown. The table and pulley axle have negligible friction. The system is released from rest. As the blocks move, the string exerts tension F_1 on m_1 and F_2 on m_2 , as shown. The angular acceleration α of the pulley is experimentally determined. If the pulley is replaced with a new pulley that has the same radius but a much larger mass, α is found to be smaller. If frictional forces are negligible, which of the following is the best explanation for this observation?

- (A) Net torque is required to turn the pulley with mass, so F_1 and F_2 both decrease.
- (B) Net torque is required to turn the pulley with mass, so F_1 and F_2 both increase.
- (C) Net torque is required to turn the pulley with mass, so F_1 increases and F_2 decreases.
- (D) Net torque is required to turn the pulley with mass, so F_1 decreases and F_2 increases. ✓
- (E) The angular acceleration α was incorrectly determined, because increasing the mass of the pulley should not affect α .

2022 Form I

4.



A ball is released from the top of a ramp and rolls off of a table, as shown in the figure. Point P is the location at which the ball has just left the edge of the table. Using the coordinate axes shown, which of the following is correct regarding the vertical acceleration a_y and the components of the velocity v of the ball at point P?

(A) $a_y < 0$ $v_x > 0$ $v_y = 0$ ✓

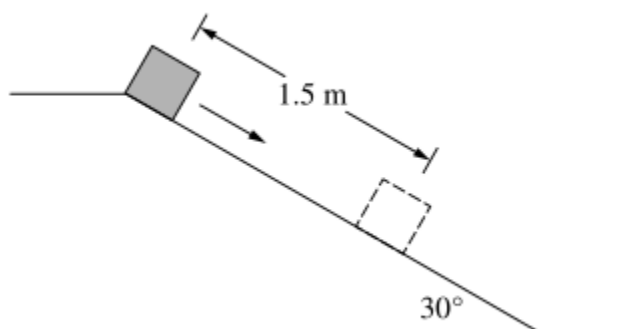
(B) $a_y = 0$ $v_x > 0$ $v_y = 0$

(C) $a_y > 0$ $v_x > 0$ $v_y < 0$

(D) $a_y < 0$ $v_x > 0$ $v_y > 0$

(E) $a_y > 0$ $v_x = 0$ $v_y < 0$

5.



A box of mass 0.50 kg slides down a rough ramp with an initial speed of 2.0 m/s. The ramp is inclined at an angle of 30° above the horizontal. The box slides a distance of 1.5 m and comes to rest before reaching the bottom of the ramp. The energy dissipated as the box slides to a stop is most nearly

2022 Form I

- (A) 0
(B) 1.4 J
(C) 2.8 J
(D) 4.7 J
(E) 8.4 J



A box of mass M is pulled to the right at constant speed across a rough, horizontal surface. Two ropes are used to exert forces $\vec{F}_1$ and $\vec{F}_2$ on the box. $\vec{F}_1$ is horizontal and to the left, and $\vec{F}_2$ is at an angle θ above the horizontal, as shown. The coefficient of kinetic friction between the box and the horizontal surface is μ .

6. Which of the following is a correct expression for the magnitude of force $\vec{F}_1$?

- (A) $F_2 \cos \theta$
(B) $\mu(Mg - F_2 \sin \theta)$
(C) $F_2 \cos \theta - \mu(Mg - F_2 \sin \theta)$
(D) $F_2 \cos \theta + \mu(Mg - F_2 \sin \theta)$
(E) $F_2 \sin \theta - \mu(Mg - F_2 \cos \theta)$



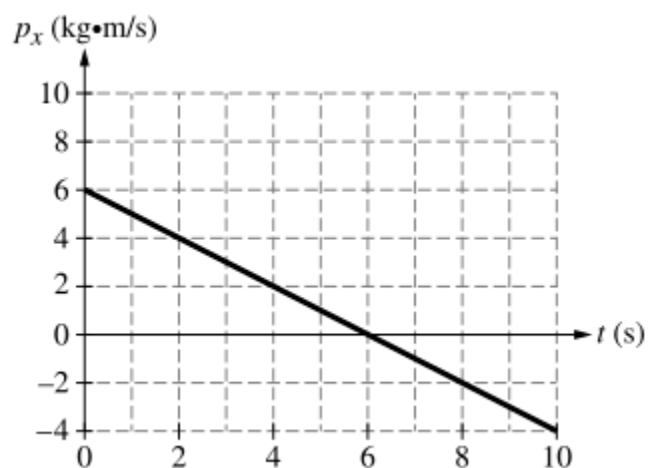
7. At a later time, the angle θ is slowly decreased without changing the magnitude of the two forces. The angle θ continues to decrease until the force $\vec{F}_2$ is horizontal, after which time the force $\vec{F}_2$ remains horizontal. Which of the following best describes the changes in the normal force and the force of friction on the box while the angle is changing?

- (A) The normal force increases and the force of friction increases.
(B) The normal force increases and the force of friction decreases.
(C) The normal force increases and the force of friction remains constant.
(D) The normal force decreases and the force of friction remains constant.
(E) The normal force decreases and the force of friction decreases.



2022 Form I

8.

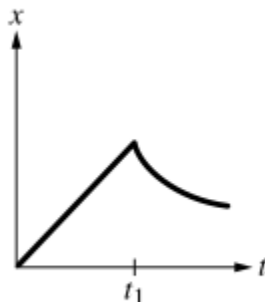


An object is moving in a straight line along the x -axis. The momentum of the object p_x as a function of time t is shown in the graph above. Which statement below correctly describes the net force that the object experiences during the time period shown?

- (A) The net force is decreasing over time.
- (B) The net force is increasing over time.
- (C) The net force is constant and positive.
- (D) The net force is constant and negative. ✓
- (E) The net force is positive for $0 < t < 6$ s and negative for $6 \text{ s} < t < 10$ s.

2022 Form I

9.



An object is moving in the x direction as shown in the graph. At time $t = t_1$, the motion changes. Which of the following describes the velocity v before and acceleration a after $t = t_1$?

	Velocity before $t = t_1$	Acceleration after $t = t_1$
A	Zero	Positive
B	Positive	Negative
C	Positive	Positive
D	Negative	Positive
E	Positive	Zero

(A) A

(B) B

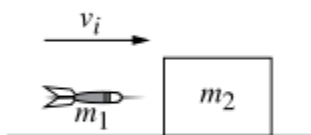
(C) C

(D) D

(E) E



2022 Form I



A projectile of mass m_1 is traveling with a horizontal speed v_i to the right toward a block of mass m_2 that is at rest on a horizontal surface, as shown. The projectile becomes embedded in the block, and the block and projectile move together to the right with speed v_f . There is negligible friction between the block and the surface.

10. Students measure v_f using a motion detector and compare the measured v_f to the expected theoretical value. What could contribute to the experimental value of v_f being greater than the theoretical value?

(A) Greater-than-expected friction between the block and the surface
(B) Deformation of the block as a result of the projectile's impact
(C) Friction between the projectile and the block as the projectile becomes embedded in the block
(D) An increase in temperature of the block as a result of the impact
(E) A slight downward incline of the surface the block moves on



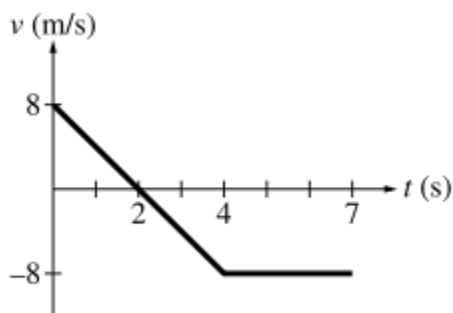
11. The duration of the collision between the projectile and the block is Δt . Which of the following expressions is equal to the average force exerted on the block by the projectile during the collision?

(A) $\frac{m_1 v_i}{2\Delta t}$
(B) $\frac{m_2 v_i}{2\Delta t}$
(C) $\frac{m_1 v_f}{m_2 \Delta t}$
(D) $\frac{m_1 v_f}{\Delta t}$
(E) $\frac{m_2 v_f}{\Delta t}$



2022 Form I

12.



The graph shows the velocity of an object v as a function of time t . What is the **distance** traveled by the object from $t = 0 \text{ s}$ to $t = 7.0 \text{ s}$?

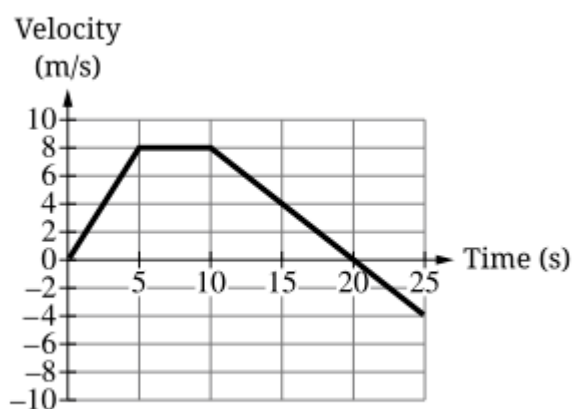
- (A) -24 m
- (B) -16 m
- (C) 24 m
- (D) 40 m
- (E) 56 m



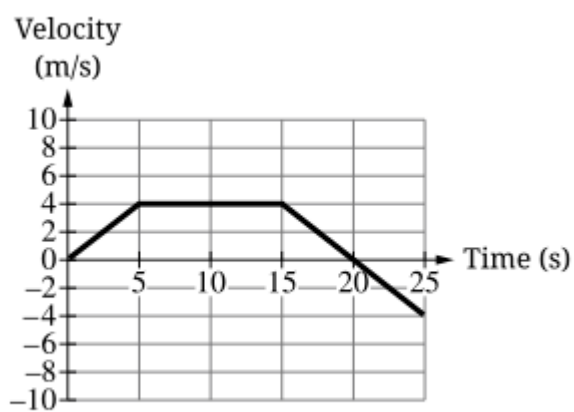
13. Five toy cars are being raced by students. The students plot the velocity as a function of time for each car. Which graph indicates the car that has the greatest displacement at the end of the race?

2022 Form I

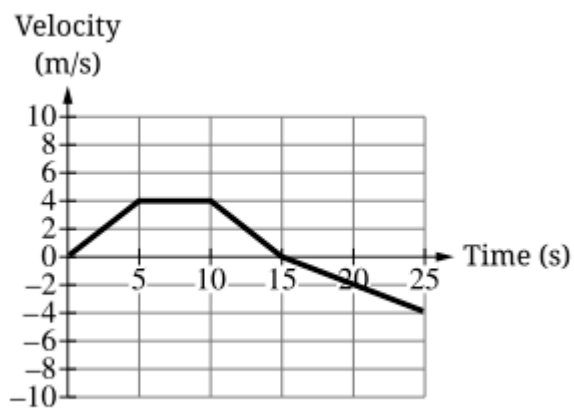
(A)



(B)

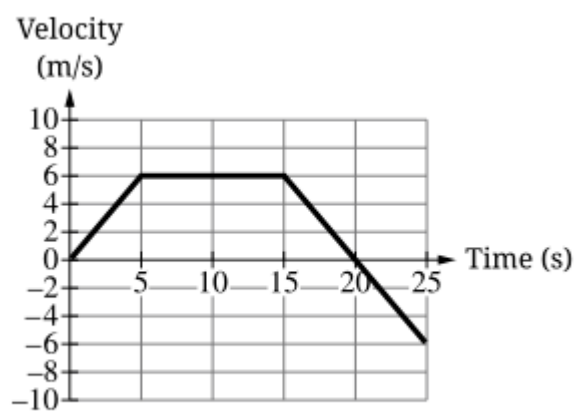


(C)

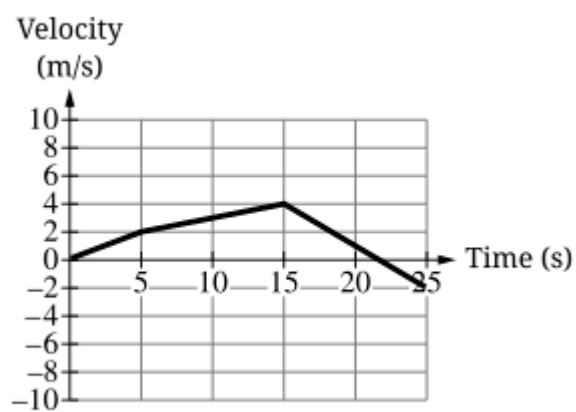


2022 Form I

(D)

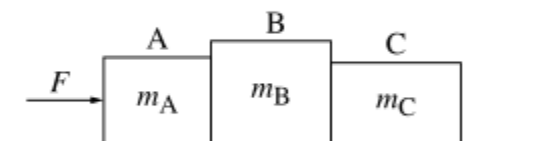


(E)



2022 Form I

14.



Boxes A, B, and C of masses m_A , m_B , and m_C , respectively, are on a horizontal surface with negligible friction. A horizontal force F is exerted on Box A, as shown. The boxes remain in contact with each other as they move along the surface. The magnitude of the force that Box B exerts on Box C is

(A) F

(B) $\left(\frac{m_C}{m_A+m_B}\right)F$

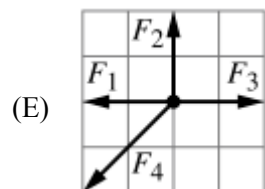
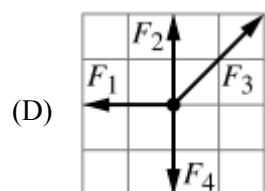
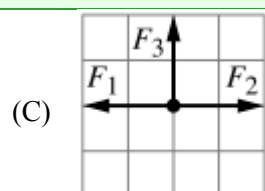
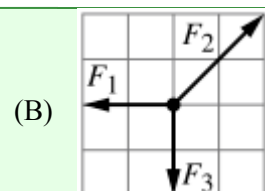
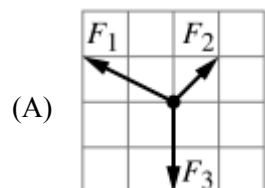
(C) $\left(\frac{m_A+m_B}{m_C}\right)F$

(D) $\left(\frac{m_C}{m_A+m_B+m_C}\right)F$ ✓

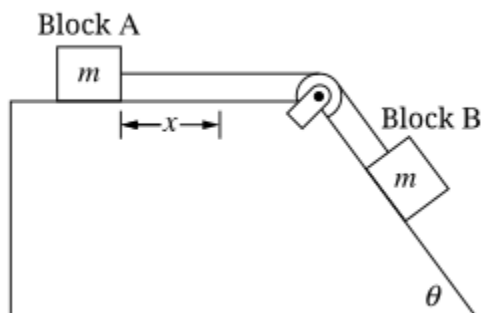
(E) $\left(\frac{m_A+m_B+m_C}{m_C}\right)F$

15. Which of the following free-body diagrams shows an object in equilibrium?

2022 Form I



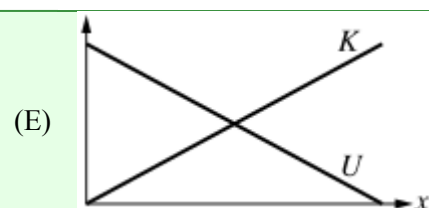
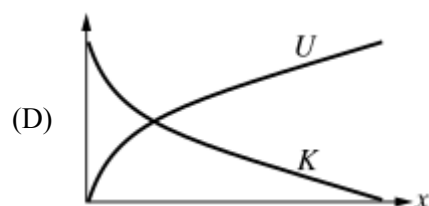
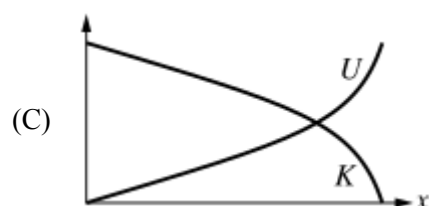
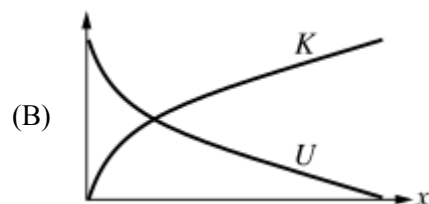
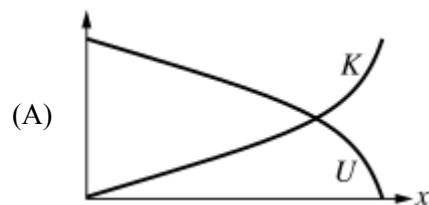
2022 Form I



Blocks **A** and **B**, each of mass m , are connected by a string that extends over a pulley. Block **A** slides on a horizontal surface, and Block **B** is on a surface that is inclined at an angle θ above the horizontal, as shown. Both surfaces have negligible friction. The masses of the string and pulley are negligible, and the pulley rotates with negligible friction.

16. Which of the following graphs best shows the sum of the kinetic energy K of the two blocks and the potential energy U of the two-block-Earth system, as a function of the distance x Block **A** has moved along the horizontal surface before Block **A** reaches the pulley?

2022 Form I



17. The system is released from rest. Which of the following is a correct expression for the speed of Block B when it has moved a distance x ?

(A) $\sqrt{gx}$

(B) $\sqrt{gx \sin \theta}$

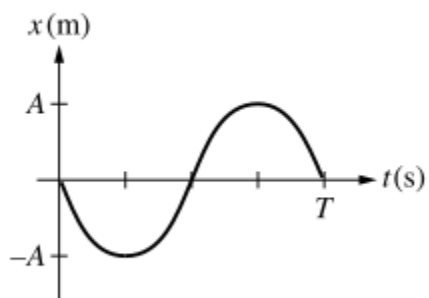
(C) $\sqrt{2gx \sin \theta}$

(D) $\sqrt{gx(1 + \sin \theta)}$

(E) $\sqrt{2gx(1 + \sin \theta)}$

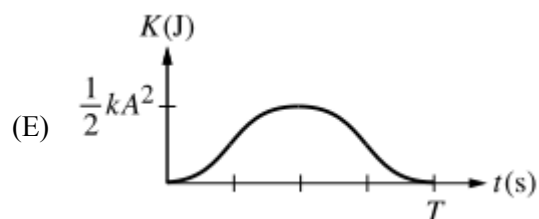
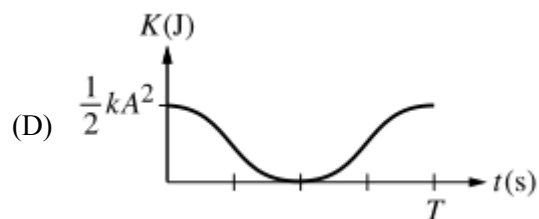
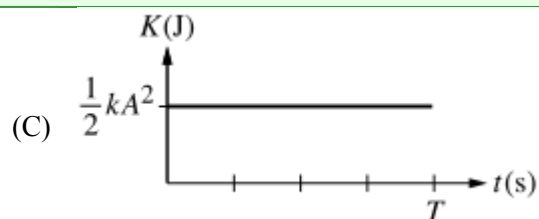
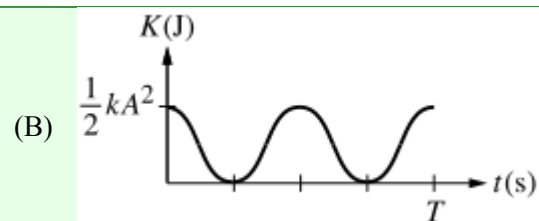
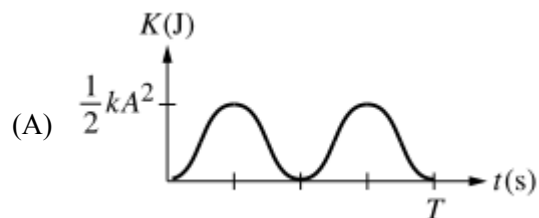
2022 Form I

18.



A mass on a spring of spring constant k oscillates as shown. Which of the following graphs would correctly depict the kinetic energy versus time for the oscillating mass?

2022 Form I



19. A particle has a potential energy given as a function of position x by $U(x) = 9x^2 - 4x$. What is the force acting on the particle at $x = a$?

(A) $-18a + 4$

(B) $-18a - 4$

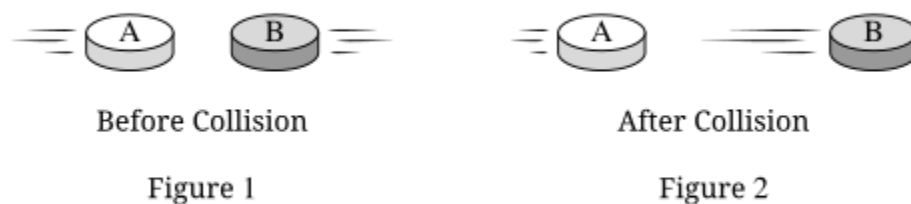
(C) $18a - 4$

(D) $3a^3 - 2a^2$

(E) $-3a^3 + 2a^2$

2022 Form I

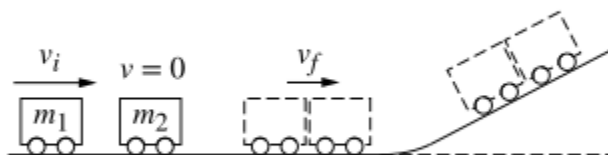
20.



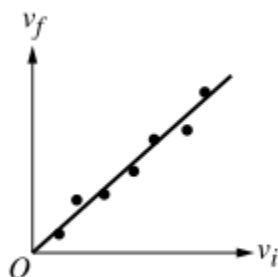
Two pucks, A and B, with unequal masses, slide with equal speeds in opposite directions on a horizontal surface with negligible friction, as shown in Figure 1. The pucks undergo a head-on collision. After the collision, both pucks move to the right. The speed of Puck A has decreased, and the speed of Puck B has increased, as shown in Figure 2. Which of the following is a plausible explanation for the observed behavior of the pucks?

- (A) The magnitude of change in momentum is the same for each puck, but Puck A has more momentum initially. ✓
- (B) The magnitude of change in momentum is the same for each puck, but Puck B has more momentum initially.
- (C) The magnitude of change of momentum is the same for each puck because each puck has the same momentum initially.
- (D) The magnitude of change in momentum of Puck A is greater than that of Puck B because Puck A has a greater initial momentum.
- (E) The magnitude of change of momentum of Puck A is less than that of Puck B because Puck A has greater mass.

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A student conducts an experiment in which a cart of mass m_1 is moving on a level horizontal surface of negligible friction with speed v_i toward a second cart of mass m_2 that is at rest. The carts collide, stick together, and move with speed v_f . The carts then travel up a ramp, momentarily come to rest, and then roll back down the ramp. The student repeats the experiment for several different values of v_i .



The student plots v_f as a function of v_i and draws a best-fit straight line to represent the data.

21. Which of the following expressions can be determined using the slope of the best-fit line?

(A) $m_1 m_2$

(B) $\frac{m_1}{m_2}$

(C) $\frac{m_1}{(m_1 + m_2)}$

(D) $\frac{m_2}{(m_1 + m_2)}$

(E) $\frac{(m_1 - m_2)}{(m_1 + m_2)}$



2022 Form I

22. The student repeats the whole experiment but doubles the initial speed v_i of the cart with mass m_1 for each data point collected. When the new data points are plotted and a best-fit line is drawn, how would the slope of this best-fit line compare to the slope on the original graph?

- (A) The slope would stay the same. ✓
- (B) The slope would be one-half as great.
- (C) The slope would be one-quarter as great.
- (D) The slope would be twice as great.
- (E) The slope would be four times as great.

23.

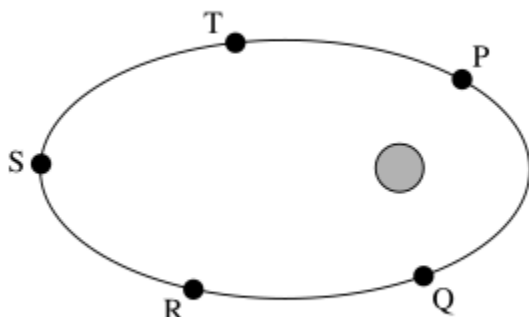


A rod with a nonuniform mass density given by $\lambda = Ax^2$, where $A = 5.0 \text{ kg/m}^3$, is aligned on the x -axis with one end at the origin and the other end at $x = 2.0 \text{ m}$, as shown. The rotational inertia of the rod about the origin is most nearly

- (A) $8 \text{ kg}\cdot\text{m}^2$
- (B) $13 \text{ kg}\cdot\text{m}^2$
- (C) $32 \text{ kg}\cdot\text{m}^2$ ✓
- (D) $53 \text{ kg}\cdot\text{m}^2$
- (E) $80 \text{ kg}\cdot\text{m}^2$

2022 Form I

24.



A satellite is orbiting a planet in an elliptical orbit. Five locations in the orbit are P, Q, R, S, and T, as shown. The distance traveled by the satellite between each point along the path is the same. Which of the following accurately represents the angular momentum about the planet and the kinetic energy of the satellite at the different locations?

	Angular momentum	Kinetic energy
A	$L_P > L_T$	$K_P = K_T$
B	$L_Q > L_S$	$K_Q > K_S$
C	$L_Q = L_R$	$K_Q > K_R$
D	$L_P = L_T$	$K_P < K_T$
E	$L_R = L_T$	$K_R > K_T$

(A) A

(B) B

(C) C

(D) D

(E) E

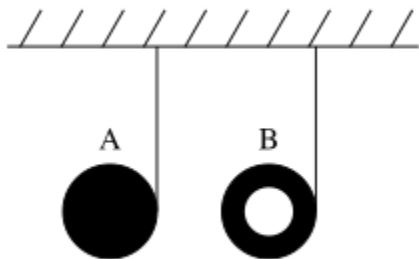
25. A sled that weighs 5.0×10^3 N is pulled across level snow by dogs exerting a horizontal force on it. The coefficient of kinetic friction between the sled and snow is 0.050. How much work is done by the dogs on the sled when the sled moves 1.0×10^3 m at constant speed?

(A) Zero

(B) 2.5×10^5 J(C) 5.0×10^5 J(D) 2.5×10^6 J(E) 5.0×10^6 J

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26.

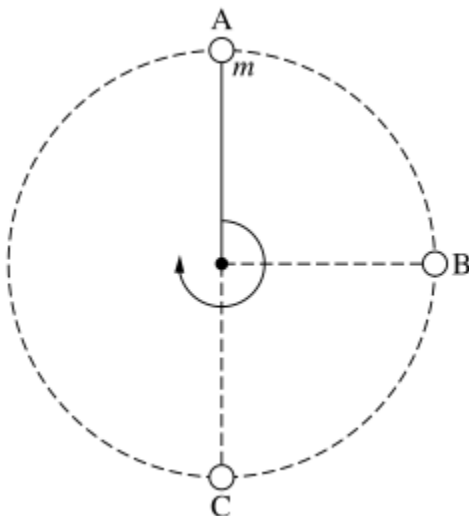


Two disks have equal masses and radii. Disk A is solid, and Disk B is hollow. Each disk has a thin string wrapped around it. Both strings are attached to the ceiling as shown. The disks are simultaneously released from rest from the same height. Which of the following accurately describes which disk will hit the ground first and why?

- (A) Disk A will hit the ground first. Disk A has a greater rotational inertia; therefore, the tension in the string for Disk A will be greater than the tension in the string for Disk B.
- (B) Disk B will hit the ground first. Disk B has a smaller rotational inertia; therefore, it will have a greater angular acceleration.
- (C) Disk A will hit the ground first. Disk A has a smaller rotational inertia; therefore, it will have a greater linear acceleration. ✓
- (D) Disk B will hit the ground first. Disk B has a greater rotational inertia; therefore, it will have a greater angular acceleration.
- (E) Both disks will hit the ground at the same time. Since both disks have the same mass, they will have the same linear acceleration.

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27.

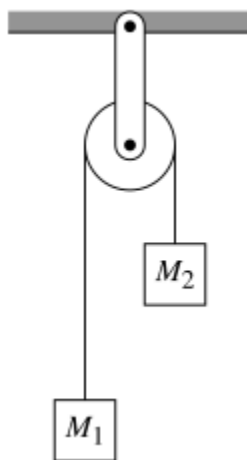


A sphere of mass m is connected to a lightweight rod of length L and moves at a constant speed greater than $\sqrt{Lg}$ in a vertical circle, as shown. The magnitudes of the forces the rod exerts on the sphere at positions A, B, and C are F_A , F_B , and F_C , respectively. Which of the following expressions is equal to $2mg$?

- (A) $F_A + F_B$
- (B) $F_B + F_C$
- (C) $F_C + F_A$
- (D) $F_B - F_A$
- (E) $F_C - F_A$



28.

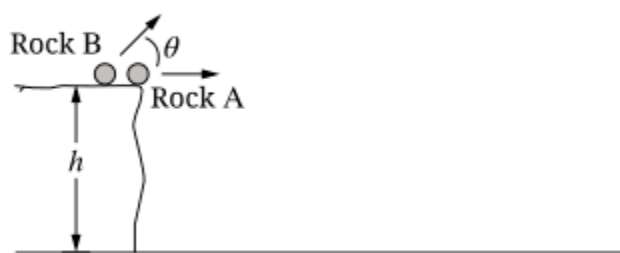


Two objects of mass M_1 and M_2 are connected by a string that passes over a pulley with negligible mass and negligible friction, as shown. A group of students wants to use this setup to measure the acceleration due to gravity g . Which of the following procedures could the students use?

2022 Form I

- (A) Use equal masses, collect velocity data for M_2 , and calculate its acceleration.
- (B) Use equal masses, collect position data for M_2 , and calculate its average velocity.
- (C) Use unequal masses, collect velocity data for M_2 , and calculate its acceleration. ✓
- (D) Use unequal masses, collect position data for M_2 , and calculate its average velocity.
- (E) Use different sets of equal masses, collect velocity data for each value of M_2 , and calculate the average acceleration.

29.



Two rocks are thrown simultaneously from a height h above the ground, as shown. Rock A has a mass m and is thrown horizontally with a speed v_0 . Rock B has a mass $2m$ and is thrown with the same speed v_0 at an angle θ above the horizontal. Which of the following statements is true?

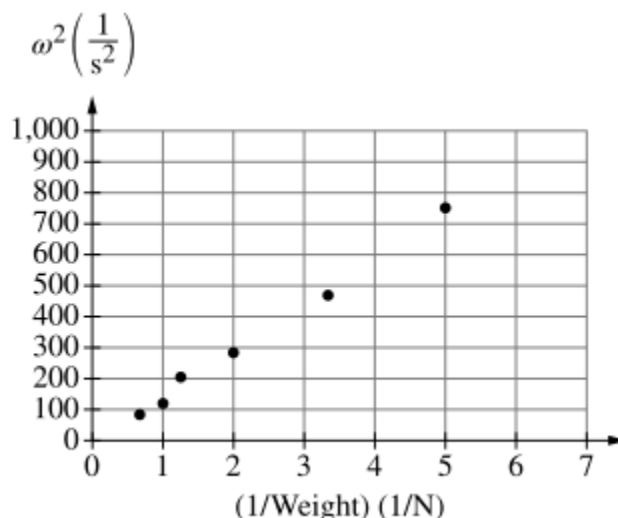
- (A) Rock A hits the ground with a greater speed than Rock B.
- (B) Rock B hits the ground with a greater acceleration than Rock A.
- (C) Rock A and Rock B hit the ground with the same speed. ✓
- (D) Rock A and Rock B hit the ground simultaneously.
- (E) Rock B's acceleration changes direction during the flight.

30. A block of mass m is pulled from rest along a horizontal surface by a horizontal force whose magnitude is given by the equation $F(t) = At + B$, where A and B are constants with appropriate units. The coefficient of friction between the block and the surface is μ_k . Which expression correctly represents the velocity of the block as a function of time?

- (A) $\frac{At^2}{m} + \frac{Bt}{m} + g\mu_k t$
- (B) $\frac{At^2}{2m} + \frac{Bt}{m} + g\mu_k t$
- (C) $\frac{At^2}{2m} + \frac{Bt}{m} - g\mu_k t$ ✓
- (D) $\frac{At^2}{2m} - \frac{Bt}{m} - g\mu_k t$
- (E) $\frac{At^2}{m} - \frac{Bt}{m} + g\mu_k t$

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31.



Cylinders of various masses are individually hung on a spring of spring constant k and set into simple harmonic motion. The angular frequency ω is determined. Then, the square of ω is plotted as a function of the inverse of the cylinder weight, as shown in the graph. Which of the following correctly determines the spring constant k and provides correct justification?

(A) $k = 150 \text{ N/m}$, because the slope of the best-fit line is equal to k .

(B) $k = 15 \text{ N/m}$, because the slope of the best-fit line is equal to $g \times k$. ✓

(C) $k = 6.7 \text{ N/m}$, because the slope of the best fit line is equal to $\frac{1}{k}$.

(D) $k = 6.7 \times 10^{-2} \text{ N/m}$, because the slope of the best fit line is equal to $\frac{g}{k}$.

(E) k cannot be determined from the graph, because ω does not depend on k .

32. A rock weighs 2.0 N on the surface of Earth. How far away from the center of a star with mass $2.0 \times 10^{30} \text{ kg}$ must the same rock be in order to exert a gravitational force on the star with a magnitude of 2.0 N ?

(A) $3.7 \times 10^9 \text{ m}$ ✓

(B) $8.1 \times 10^9 \text{ m}$

(C) $1.2 \times 10^{10} \text{ m}$

(D) $1.3 \times 10^{19} \text{ m}$

(E) $6.7 \times 10^{19} \text{ m}$

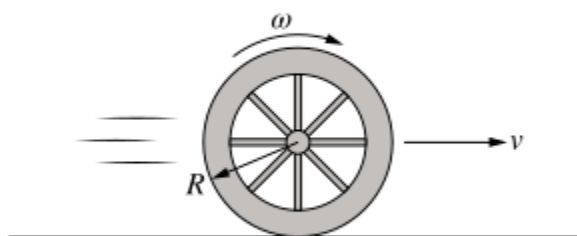
33. A satellite is orbiting Earth at an altitude of one-half Earth's radius. If the weight of the satellite while in orbit is $9,000 \text{ N}$, what is the weight of the satellite on Earth's surface?

2022 Form I

- (A) 4,000 N
- (B) 6,000 N
- (C) 9,000 N
- (D) 13,500 N
- (E) 20,250 N



34.



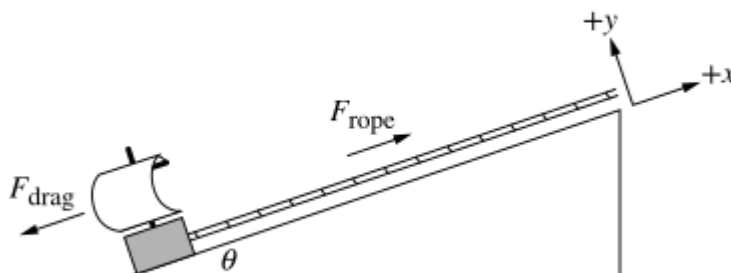
At time $t = 0$, the wheel shown in the diagram has clockwise rotation with an angular speed of 20 rad/s . The center of mass of the wheel is initially moving to the right at 2 m/s . The wheel rolls on a flat rough surface and has a radius of 0.2 m . Which of the following is correct about the type of motion, the angular speed, and the linear speed of the wheel at time $t = 0$?

- (A) The type of motion is rolling with slipping, the angular speed remains the same, and the linear speed is decreasing.
- (B) The type of motion is rolling with slipping, the angular speed is decreasing, and the linear speed is increasing.
- (C) The type of motion is rolling with slipping, the angular speed is decreasing, and the linear speed is decreasing.
- (D) The type of motion is rolling without slipping, the angular speed is increasing, and the linear speed is increasing.
- (E) The type of motion is rolling without slipping, the angular speed is decreasing, and the linear speed is decreasing.



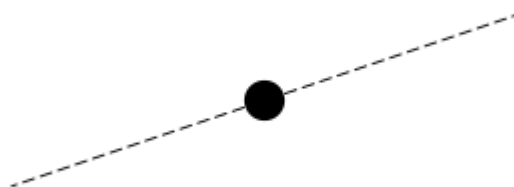
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35. Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.



A block-sail system of mass m is pulled up a ramp by a rope. There is negligible friction between the system and the ramp, which is inclined at an angle θ from the horizontal, as shown. The rope exerts a constant force of magnitude $F_{\text{rope}} = 2mg$ on the system. Air exerts a drag force of magnitude $F_{\text{drag}} = bv$ on the system, where v is the speed of the system and b is a positive constant with units of kg/s . The system is at position $x = 0$ at rest at time $t = 0$.

- (a) On the dot below, which represents the block-sail system a short time after time $t = 0$, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



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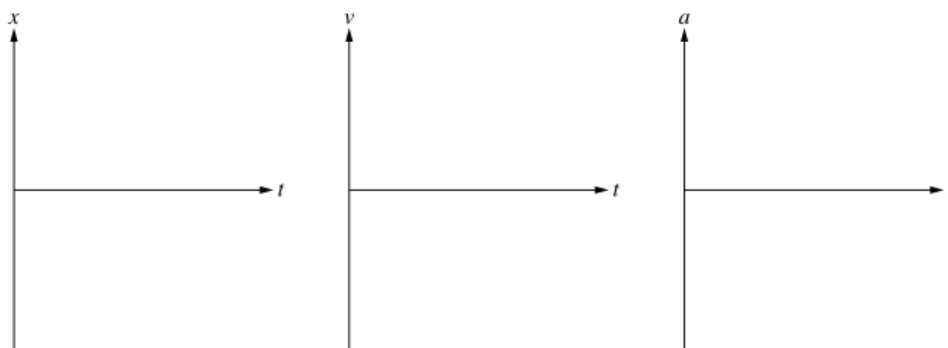
(b) Write, but do NOT solve, a differential equation that could be used to solve for the speed $v(t)$ of the block-sail system as a function of time t . Express your answer in terms of m , θ , b , v , and physical constants as appropriate.

(c) The velocity of the block-sail system can be modeled by the equation $v(t) = v_{\text{terminal}} \left(1 - e^{-\frac{bt}{m}} \right)$.

i. Derive an expression for the position of the block-sail system as a function of time t in terms of θ , m , and b . Do not include v_{terminal} in your final expression.

ii. Derive an expression for the change in kinetic energy of the block-sail system as a function of time t .

(d) On the axes below, sketch the position x , velocity v , and acceleration a of the block-sail system as functions of time t .



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



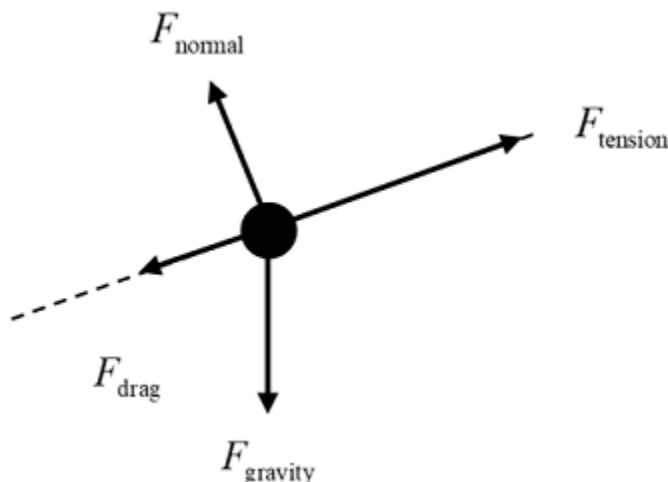
0	1	2	3
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The response accurately includes **all** of the following criteria.

- ☐ One point is earned for correctly drawing and labeling the gravitational force
- ☐ One point is earned for correctly drawing and labeling the normal force perpendicular to the ramp
- ☐ One point is earned for correctly drawing and labeling the rope force and the drag force parallel to the ramp and in the correct directions relative to the ramp

Example Response:

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Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “F Earth on block,” “F on block by Earth,” $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n , F_N , N , “normal force,” “ground force,” or similar labels may be used for the normal force. F_{rope} , F_R , F_T , F_{Tension} , T , “rope force,” “tension force” or similar labels may be used for the tension force exerted by the string.

Scoring Note: The forces do not have to be drawn to scale. If extraneous forces are present, a maximum of 2 points can be earned.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3
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The response accurately includes **all** of the following criteria.

- ☐ One point is earned for writing an equation of motion in accordance with Newton’s second law that includes at least two of the three forces F_{rope} , F_{drag} , F_{gravity} (has to end in $= ma$)
- ☐ One point is earned for substituting $\frac{dv}{dt}$ for a in the Newton’s second law equation above
- ☐ One point is earned for a correct differential equation including $\frac{dv}{dt}$, and expressions for the drag force, rope force, and the component of weight parallel to the ramp in terms of the variables given and appropriate physical constants

Example Response:

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$$\sum F = ma$$

$$F_{\text{rope}} - F_{g,\parallel} - F_{\text{drag}} = ma$$

$$F_{\text{rope}} - F_{g,\parallel} - F_{\text{drag}} = m \frac{dv}{dt}$$

$$2mg - mg \sin \theta - bv = m \frac{dv}{dt}$$

$$2g - g \sin \theta - \frac{bv}{m} = \frac{dv}{dt}$$

$$g(2 - \sin \theta) - \frac{bv}{m} = \frac{dv}{dt}$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3	4	5	6
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The response accurately includes **all** of the following criteria.

- ☐ One point is earned for correctly relating the position to the velocity, with an integral of velocity as a function of time
- ☐ One point is earned for providing the correct limits or constants of integration
- ☐ One point is earned for finding an expression for the terminal velocity in terms of m , b , and θ

Example Response:

$$F_{\text{drag}} + F_{g,\parallel} = F_{\text{rope}}$$

$$bv_{\text{terminal}} = 2mg - mg \sin \theta$$

$$v_{\text{terminal}} = \frac{mg}{b}(2 - \sin \theta)$$

- ☐ For deriving the correct expression of the integral

Example Response:

$$x(t) = \int v(t) dt$$

$$x(t) = \int_{t'=0}^{t'=t} v_{\text{terminal}} (1 - e^{-bt'/m}) dt'$$

$$x(t) = \left[v_{\text{terminal}} t' + \frac{mv_{\text{terminal}}}{b} e^{-bt'/m} \right]_{t'=0}^{t'=t}$$

$$x(t) = \left[v_{\text{terminal}} t + \frac{mv_{\text{terminal}}}{b} e^{-bt/m} \right] - \left[\frac{mv_{\text{terminal}}}{b} \right]$$

$$x(t) = v_{\text{terminal}} t + \frac{mv_{\text{terminal}}}{b} e^{-bt/m} - \frac{mv_{\text{terminal}}}{b}$$

$$x(t) = v_{\text{terminal}} \left[t + \frac{m}{b} (e^{-bt/m} - 1) \right]$$

$$x(t) = \frac{mg}{b} (2 - \sin \theta) \left[t + \frac{m}{b} (e^{-bt/m} - 1) \right]$$

- ☐ For a correct equation for the change in kinetic energy

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- ☐ For a correct substitution of both velocities in the equation from part (c)(i)

Example Response:

$$\Delta K = \frac{1}{2}m \left[(v(t))^2 - v_0^2 \right]$$

$$\Delta K = \frac{1}{2}m \left[\left(v_{\text{terminal}} \left(1 - e^{-\frac{bt}{m}} \right) \right)^2 - 0 \right]$$

$$\Delta K = \frac{1}{2}m \left[v_{\text{terminal}} \left(1 - e^{-\frac{bt}{m}} \right) \right]^2$$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

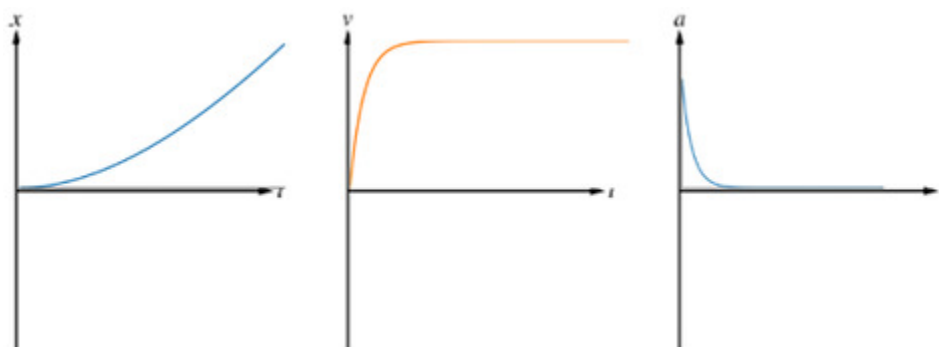


0	1	2	3
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The response accurately includes **all** of the following criteria.

- ☐ One point is earned for a concave up curve with positive values that approaches a diagonal asymptote for the graph of position as a function of time
- ☐ One point is earned for a concave down curve with positive values that approaches a positive horizontal asymptote for the graph of speed as a function of time
- ☐ One point is earned for a concave up curve with positive values that approaches an asymptote at the horizontal axis for the graph of acceleration as a function of time

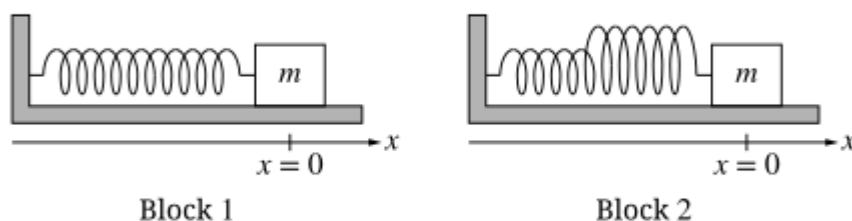
Example Response:



Scoring Note: Graphs that are reflected over the horizontal axis can earn full credit.

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36. Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.



Two identical blocks of mass $m = 0.65$ kg are initially at rest on horizontal surfaces at position $x = 0$. There is negligible friction between the blocks and the surfaces. One end of a spring is attached to each block, and the other end of the spring is attached to a fixed vertical support. The force exerted by the spring on each block is given as a function of the position x of the block, as indicated by the following equations.

Block 1: $F = -k_1x$ where $k_1 = 10.0$ N/m

Block 2:
$$\begin{cases} F = -k_1x & \text{for } |x| \leq 0.50 \text{ m where } k_1 = 10.0 \text{ N/m} \\ F = -k_2x^3 & \text{for } |x| > 0.50 \text{ m where } k_2 = 40.0 \text{ N/m}^3 \end{cases}$$

Both blocks are pulled 0.80 m to the right and released from rest.

(a) Which block or blocks, if either, exhibits simple harmonic motion?

☐ Block 1 only ☐ Block 2 only ☐ Both ☐ Neither

Briefly justify your answer.

(b)

i. Calculate the amount of work required to pull Block 1 from the equilibrium position to a position $x = 0.80$ m.

ii. Calculate the amount of work required to pull Block 2 from the equilibrium position to a position $x = 0.80$ m.

(c)

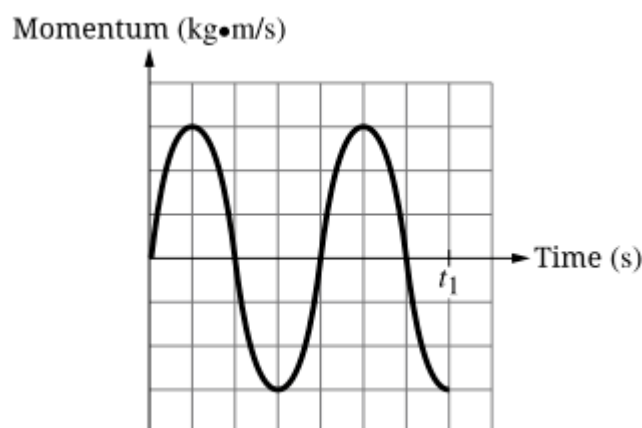
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- i. At time $t = 0$, Block 1 is released from rest at $x = 0.80$ m. Calculate the time t_{eq} at which Block 1 passes through the equilibrium position the first time.
- ii. If both blocks are released simultaneously from rest at $x = 0.80$ m, which block passes through its equilibrium first?

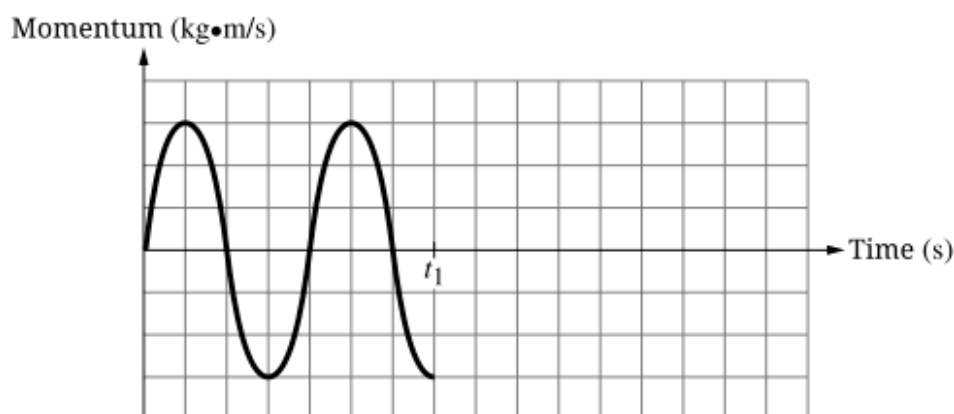
___ Block 1 ___ Block 2 ___ Same time

Briefly justify your answer.

Block 1 is pressed against the spring by 0.40 m and released from rest. The momentum of the block as a function of time is shown below.



- (d) At time t_1 , a second block of mass $3m$ falls straight down and sticks to Block 1. The two blocks then oscillate together. On the grid below, sketch a graph of the momentum of the two-block system as a function of time after time $t = t_1$.

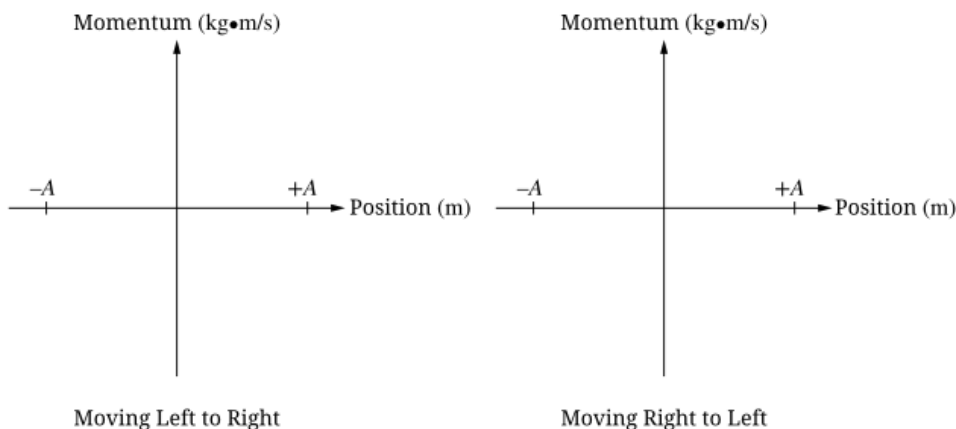


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(e) The second block is removed from the top of Block 1 and Block 1 oscillates with an amplitude A .

Use the axes below on the left to sketch a graph of the momentum of Block 1 as a function of position as Block 1 moves from left to right.

Use the axes below on the right to sketch a graph of the momentum of Block 1 as a function of position as Block 1 moves from right to left.



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2
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The response accurately includes **both** of the following criteria.

- ☐ One point is earned for selecting “Block 1 only” with an attempt at a justification
- ☐ One point is earned for a correct justification that only springs that obey Hooke’s law exhibit simple harmonic motion

Example Response:

The definition of SHM is that the restoring force has to be proportional to the displacement from equilibrium.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3	4
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The response accurately includes **all** of the following criteria.

- ☐ One point is earned for correctly substituting the values for Block 1 into a correct work equation

Example Response:

$$W = \int \vec{F}_{\text{sp}} \cdot d\vec{x}$$

$$W = \int_0^{0.8} (10x) dx$$

$$W = [5x^2]_0^{0.8}$$

$$W = 3.2 \text{ J}$$

- ☐ One point is earned for using a correct expression for work that integrates force over the path traveled

- ☐ One point is earned for indicating that the work is the sum over two intervals

- ☐ One point is earned for stating correct integration limits **and** correctly substituting force for the second interval

Example Response:

$$W = \int \vec{F}_{\text{sp}} \cdot d\vec{x}$$

$$W = \int_0^{0.5} \vec{F}_{\text{sp}} \cdot d\vec{x} + \int_{0.5}^{0.8} \vec{F}_{\text{sp}} \cdot d\vec{x}$$

$$W = \int_0^{0.5} (10x) dx + \int_{0.5}^{0.8} (40x^3) dx$$

$$W = [5x^2]_0^{0.5} + [10x^4]_{0.5}^{0.8}$$

$$W = 4.7 \text{ J}$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3	4
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The response accurately includes **all** of the following criteria.

- ☐ One point is earned for correctly substituting the values for Block 1 into a correct equation to find period of oscillation

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- ☐ One point is earned for indicating that the time it takes Block 1 to reach the equilibrium position is one-quarter of the period of oscillation

Example Response:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{0.65\text{ kg}}{10\text{ N/m}}} = 1.6\text{ s}$$

$$\text{Time to equilibrium} = \frac{T}{4} = 0.4\text{ s}$$

- ☐ One point is earned for selecting “Block 2” with an attempt at a justification
- ☐ One point is earned for a justification that includes correctly relating a larger net force or a larger average acceleration over the same distance to a smaller time for Block 2

Example Response:

Block 2 should reach its equilibrium first because it is also interacting with a spring with a larger spring constant, which would make the average force larger for Block 2. The larger average force will cause the block to have a larger average acceleration and thus reach the equilibrium position in less time than Block 1.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

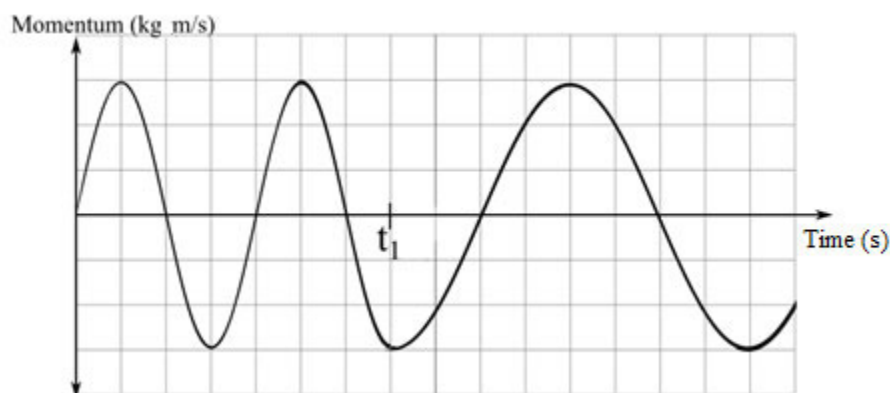


0	1	2
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The response accurately includes **both** of the following criteria.

- ☐ One point is earned for a sinusoidal graph with a period equal to twice the original period for $t > t_1$
- ☐ One point is earned for a continuous function with maximum and minimum momentum that are the same as in the original graph

Example Response:



Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

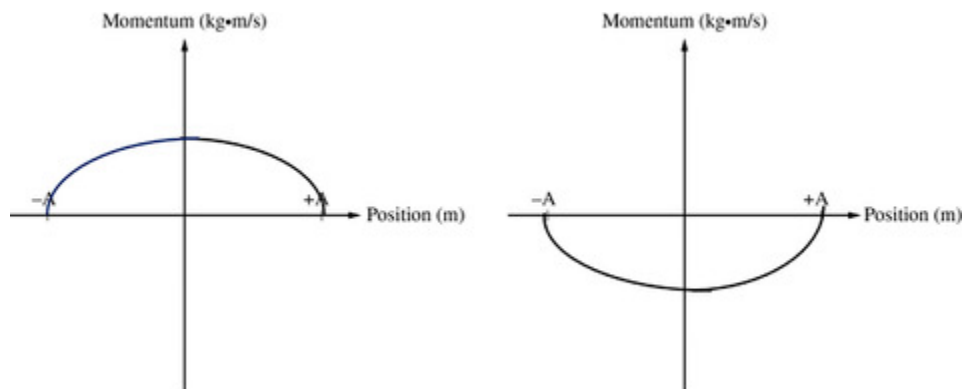
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0	1	2	3
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The response accurately includes **all** of the following.

- ☐ One point is earned for indicating that the momentum is zero at $-A$ and $+A$ on the graph on the left
- ☐ One point is earned for a graph that is concave down and positive with a maximum at $x = 0$ on the graph on the left
- ☐ For a graph on the right that is a reflection, across the x -axis, of the graph on the left

Example Response:


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37. Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.

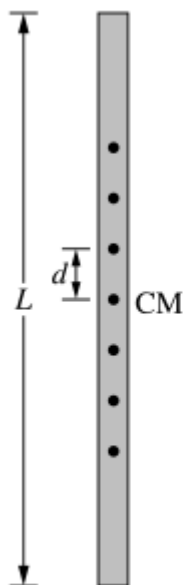


Figure 1

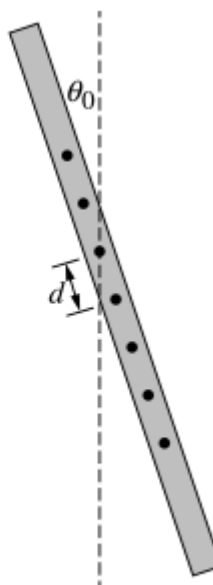


Figure 2

A group of students perform experiments using a long, uniform, thin rod that can be pivoted at various points along its length, as shown in Figure 1. The length of the rod is L and the mass of the rod is M . The point CM is the center of mass of the rod. The moment of inertia of the rod about point CM is $I_{\text{CM}} = \frac{1}{12}ML^2$.

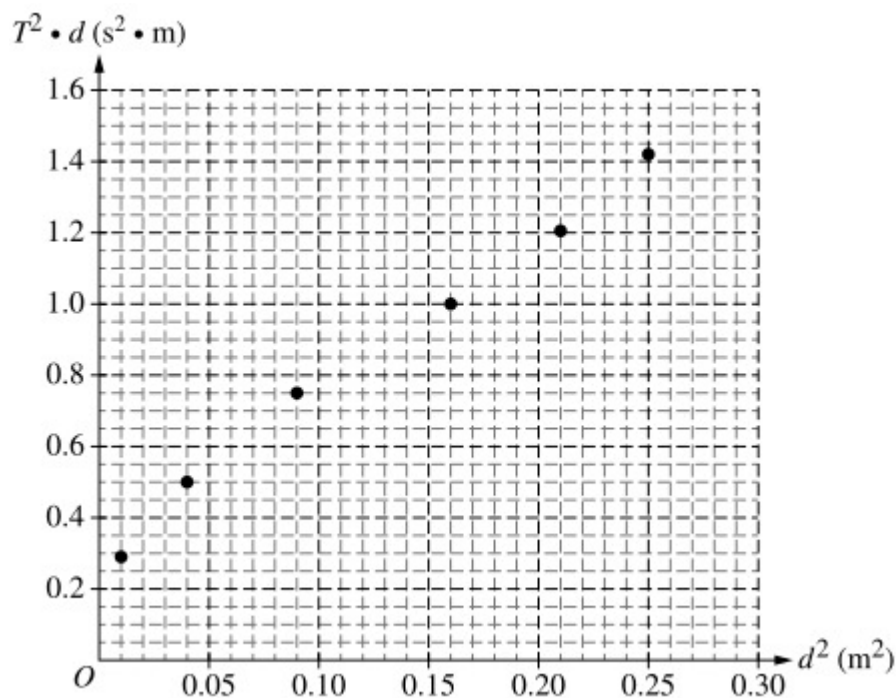
The rod is attached to a pivot that is located a distance d from point CM. The rod is then displaced by a small angle θ_0 from the vertical and released from rest, as shown in Figure 2.

- (a) Derive an expression for the initial angular acceleration α of the rod. Express your answer in terms of M , L , d , θ_0 , and physical constants, as appropriate.

The period of oscillation of the rod is described by the relationship $T^2d = (2\pi)^2 \left(\frac{d^2 + L^2/12}{g} \right)$.

The students now conduct an experiment to determine the value of the gravitational constant g . The students use a rod of mass $M = 0.200$ kg and length $L = 1.0$ m and measure the period T of the oscillations of the rod for several different values of d . The students then create the plot of T^2d as a function of d^2 shown below.

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(b)

i. Draw a best-fit line for the data.

ii. Using the slope of the best-fit line, calculate the acceleration due to gravity g .iii. Using the y -intercept of the best-fit line, calculate the acceleration due to gravity g .

(c) The students measure the length of the rod, and they determine that the length of the rod is not exactly 1.0 m.

i. Did this difference in length affect the value of g calculated using the slope of the line, the y -intercept, both, or neither?

☐ Slope ☐ The y -intercept ☐ Both ☐ Neither

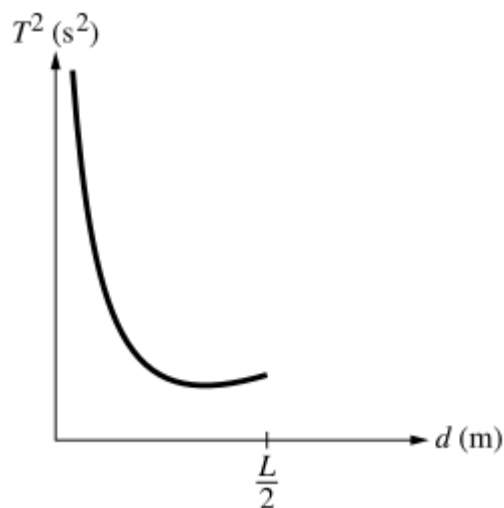
Briefly justify your answer.

ii. Is the actual length of the rod greater than or less than 1.0 m?

☐ Less than ☐ Greater than ☐ Not enough information to determine

Briefly justify your answer.

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(d) Students correctly sketch the graph above, of the square of the period, T^2 , of the rod as a function of d . In terms of torque, explain the physical significance of the vertical asymptote as d approaches zero.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3	4
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The response accurately includes **all** of the following criteria.

- ☐ One point is earned for correctly writing the total torque acting on the rod
- ☐ One point is earned for correctly using an equation that is consistent with the rotational analog of Newton's second law
- ☐ One point is earned for correctly applying the parallel axis theorem
- ☐ One point is earned for a correct answer

Example Response:

$$\sum \tau = Mgd \sin \theta_0$$

$$Mgd \sin \theta_0 = I_d \alpha$$

$$I_d = Mr^2 + I_{\text{CM}} = M(d^2 + \frac{L^2}{12})$$

$$\alpha = \frac{Mgd \sin \theta_0}{M(d^2 + \frac{L^2}{12})} = \frac{12gd \sin \theta_0}{L^2 + 12d^2}$$

Part B

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

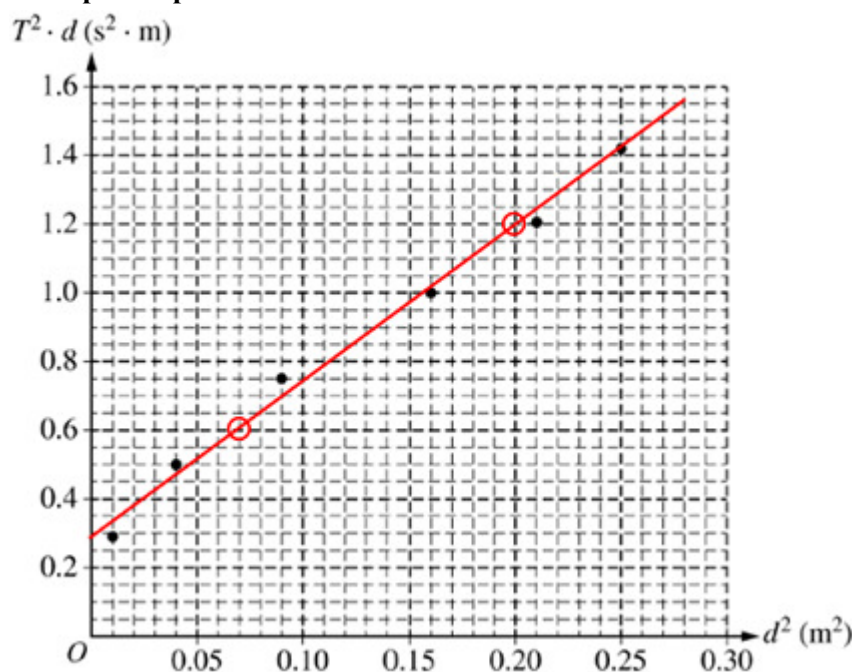


0	1	2	3	4	5
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The response accurately includes **all** of the following criteria.

- ☐ One point is earned for drawing an appropriate best-fit line with approximately half the data points above and below the line

Example Response:



- ☐ One point is earned for calculating the slope of the line from part (b)(i) using two points on the best-fit line that are not data points
- ☐ One point is earned for relating the period and I_d to the slope and getting a value for g consistent with part (a) and the values for the parameters provided
- ☐ One point is earned for determining the expression for the y-intercept of the best-fit line from part (b)(i)
- ☐ One point is earned for getting a value for g using the values for the parameters provided

Example Response:

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$$\text{slope} = \frac{(1.2 - 0.6) \text{ s}^2 \text{ m}}{(0.2 - 0.07) \text{ m}^2}$$

$$T^2 d = \frac{4\pi^2}{g} d^2 + \frac{4\pi^2 L^2}{12g} \rightarrow \text{slope} = \frac{4\pi^2}{g}$$

$$g = \frac{4\pi^2}{4.61 \frac{\text{s}^2}{\text{m}}} = 8.56 \frac{\text{m}}{\text{s}^2}$$

$$y\text{-intercept} = 4\pi^2 \frac{L^2}{12g} = 0.28 \text{ s}^2 \text{ m}$$

$$g = \frac{4\pi^2 (1.0 \text{ m})^2}{12(0.28 \text{ s}^2 \text{ m})}$$

$$= 11.7 \text{ m/s}^2$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3	4
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The response accurately includes **all** of the following criteria.

- ☐ One point is earned for selecting “the y-intercept” with an attempt at a relevant justification
- ☐ One point is earned for a correct justification mentioning the dependence of the y-intercept on L

Example Response:

The slope does not depend on L and the y-intercept does depend on L . As a result, a difference in L would only affect the y-intercept.

- ☐ One point is earned for selecting “Less than” with an attempt at a relevant justification
- ☐ One point is earned for a correct justification

Example Response:

The expression for the slope of the best-fit line has no dependence on L and the value for g obtained in part (b)(ii) is less than the accepted value. However, using the y-intercept resulted in a larger than accepted value for g since L^2 and g are directly proportional. To get a lower calculated value for g , the actual value for L is slightly smaller than 1.0 m.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2
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The response accurately includes **both** of the following criteria.

- ☐ One point is earned for indicating that when $d \rightarrow 0$ the rod will be hung from its center of mass

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One point is earned for indicating that when the object is hung from its center of mass then there is no net torque acting on the rod to drive oscillations

Example Response:

When d goes to zero the pivot point will be the center of mass of the rod. There will be no net torque acting on the rod. When there is no restoring force then there will be no oscillations. Thus, the period will be infinite.