

2023 AP Calculus BC

Section I, Part A (1 hour No calculator 30 questions)

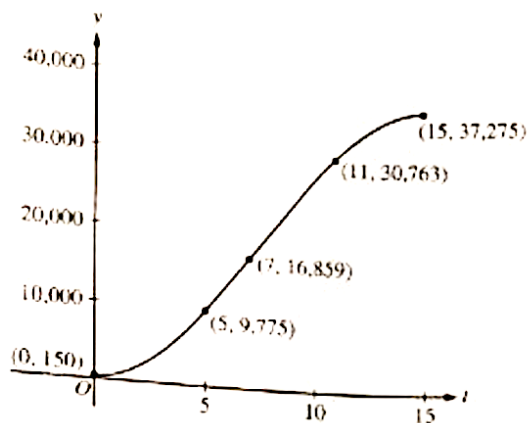
1. If $g(x) = x^2\sqrt{x+1}$, then $g'(3) =$

- A. $3/2$
- B. $39/4$
- C. $57/4$
- D. 36

2. $\int_0^4 100e^{\frac{x}{4}} dx =$

- A. $25(e - 1)$
- B. $100(e - 1)$
- C. $400(e - 1)$
- D. $400(e^4 - 1)$

3.



An airplane's height above sea level is modeled by the differentiable function $y = h(t)$, where t is the number of minutes after takeoff and $h(t)$ is measured in feet. The graph of h is shown in the figure for $0 \leq t \leq 15$. Of the following, which is the best estimate of the rate at which the plane is rising above sea level, in feet per minute, at time $t = 9$?

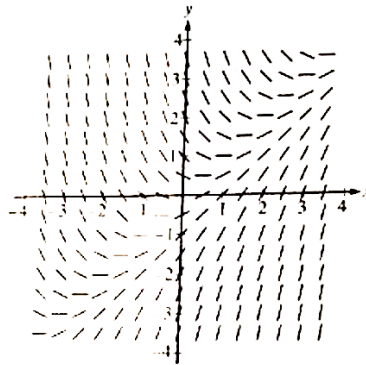
- A. $\frac{37,275-150}{15-0}$
B. $\frac{37,275-9,775}{15-5}$
C. $\frac{37,275-16,859}{15-7}$
D. $\frac{30,763-16,859}{11-7}$
-

4. Let f be a vector-valued function with $f(0) = \langle -2, 3 \rangle$. If the instantaneous rate of change of f with respect to t is given by $\langle 3t^2 + 4t + 1, 2t^3 + t - 2 \rangle$, what is $f(1)$?
- A. $\langle 2, 2 \rangle$
B. $\langle 4, -1 \rangle$
C. $\langle 8, 10 \rangle$
D. $\langle 10, 7 \rangle$
-

5. $\lim_{x \rightarrow 4} \frac{2-\sqrt{x}}{x-4}$
- A. $-\frac{1}{4}$
B. 0
C. $\frac{1}{4}$
D. Nonexistent
-

6. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y^2$ with initial condition $f(1) = 2$. What is the approximation for $f(1.4)$, if Euler's method is used, starting at $x = 1$ with two steps of equal size?
- A. 2.4
B. 3.04
C. 4
D. 5.04
-

7.



A portion of the slope field for the differential equation $\frac{dy}{dx} = x - y$ is shown in the figure. If the linear function $y = f(x)$ is the solution to the differential equation such that $f(-1) = -2$, then $f(3) =$

- A. 0
- B. 1
- C. 2
- D. 3

8. If $f(x) = \sin(e^{3x})$, then $f'(x) =$

- A. $\cos(e^{3x})$
- B. $\cos(3e^{3x})$
- C. $e^{3x} \cos(e^{3x})$
- D. $3e^{3x} \cos(e^{3x})$

9. If the series converges, where $A \neq 0$ and $B \neq 0$, what is the sum of the series?

$$\sum_{n=0}^{\infty} (-1)^n \frac{B^{n+1}}{A^n}$$

- A. $\frac{AB}{A-B}$
- B. $\frac{AB}{A+B}$

C. $\frac{A}{A-B}$

D. $\frac{A}{A+B}$

10. Of the following series, which can be used with the comparison test to show that diverges?

$$\sum_{n=3}^{\infty} \frac{\ln n}{n}$$

A. $\sum_{n=3}^{\infty} \frac{1}{n}$

B. $\sum_{n=3}^{\infty} \frac{1}{n\sqrt{n}}$

C. $\sum_{n=3}^{\infty} \frac{1}{n^2}$

D. $\sum_{n=3}^{\infty} \ln n$

11. A particle moves in the xy -plane so that its position at any time t is given by the parametric equations $x(t) = t^2 - t^3$ and $y(t) = 9 - 6t$. What is the speed of the particle at time $t = 2$?

A. $\frac{3}{4}$

B. 5

C. 10

D. 14

12. Which of the following is the solution to the differential equation $\frac{dy}{dx} = -3y$ with the initial condition $y(0) = 7$?

A. $y = 7e^{-3x}$

B. $y = e^{-3x} + 6$

C. $y = -\frac{3}{2}x^2 + 7$

D. $y = \sqrt{49 - 6x}$

13. Consider the series. Which of the following statements is true?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\cos\left(\frac{1}{n}\right)}$$

- A. The series converges absolutely.
 - B. The series converges conditionally.
 - C. The series diverges.
 - D. It cannot be determined whether the series converges or diverges from the information given.
-

14. If f is a function such that $\lim_{x \rightarrow \infty} f(x) = 0$, which of the following could be an expression for $f(x)$?

- A. $\frac{x^2}{x^2+4}$
 - B. $\frac{\sin x}{x}$
 - C. $\cos x$
 - D. e^x
-

15. $\int \frac{x+1}{x(x+3)} dx =$

- A. $\frac{1}{3} \ln|x| + \frac{2}{3} \ln|x+3| + C$
 - B. $\frac{1}{3} \ln|x| + \frac{2}{9} \ln|x+3| + C$
 - C. $\frac{2}{3} \ln|x| + \frac{1}{3} \ln|x+3| + C$
 - D. $\frac{2}{3} \ln|x| + \frac{1}{9} \ln|x+3| + C$
-

16. $f(x) = \begin{cases} x^2 - 2x, & x \leq 4 \\ g(x), & x > 4 \end{cases}$

Let f be the function defined as shown. If f is differentiable, which of the following could define $g(x)$?

- A. $6x$
- B. $x^{3/2}$
- C. $x^2 - 8$
- D. $\frac{1}{8}x^3$

17. The function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = 2xy$ with the initial condition $f(0) = 5$. What is the value of $f(2)$?

- A. $\frac{5}{21}$
 - B. $\sqrt{33}$
 - C. $4 + e^4$
 - D. $5e^4$
-

18. Which of the following gives the length of the curve $y = \tan x$ from $x = -\frac{\pi}{4}$ to $x = \frac{\pi}{4}$?

- A. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 - \tan x} \, dx$
 - B. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x} \, dx$
 - C. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \sec^2 x} \, dx$
 - D. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \sec^4 x} \, dx$
-

19. If $y = f(x)$ is a function such that $\frac{dy}{dx} = \frac{2x-y}{x}$, then $\frac{d^2y}{dx^2} =$

- A. $\frac{2y-2x}{x^2}$
 - B. $\frac{y-xy}{x^2}$
 - C. $\frac{y-x}{x^2}$
 - D. $\frac{y}{x^2}$
-

20. Which of the following series converges to $\frac{1}{e}$?

- A. $\sum_{n=0}^{\infty} \frac{1}{n!}$
- B. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$
- C. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$
- D. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!}$

21. The function g has a continuous second derivative. If $g(0) = 2$, $g'(0) = 4$, $g(3) = 5$ and $g'(3) = 6$, what is the value of $\int_0^3 x \cdot g''(x) dx$?

- A. 9
 - B. 15
 - C. 21
 - D. 27
-

22. The Maclaurin series for the function f is given below. What is the value of $f^{(4)}(0)$?

$$\sum_{n=1}^{\infty} \frac{(-2)^n x^{2n}}{n}$$

- A. 2
 - B. 4
 - C. 8
 - D. 48
-

23. Let f be a differentiable function such that $f(\pi) = 2$ and $f'(\pi) = -3$. What is the slope of the line tangent to the polar curve $r = f(\theta)$ at $\theta = \pi$?

- A. -3
 - B. -2
 - C. $-\frac{3}{2}$
 - D. $-\frac{2}{3}$
-

24. $\int_{-\infty}^1 x^2 2^{x^3} dx$ is

- A. $\frac{6}{\ln 2}$
 - B. $\frac{2}{3 \ln 2}$
 - C. $\frac{2 \ln 2}{3}$
 - D. divergent
-

25. $\int \frac{dx}{x^2+4x+5} =$

A. $\tan^{-1}(x+2) + C$

B. $\ln|x^2+4x+5| + C$

C. $\frac{1}{2x+4} \ln|x^2+4x+5| + C$

D. $\frac{1}{\frac{1}{3}x^3+2x^2+5x} + C$

26. Let f be a function defined by $f(x) = x^3 + x$. If g is the reverse function of f , what is the slope of the line tangent to the graph of g at the point $(2,1)$?

A. $-\frac{1}{4}$

B. $-\frac{1}{13}$

C. $\frac{1}{13}$

D. $\frac{1}{4}$

x	0	2	3	5
$f(x)$	2	3	6	12

27. The function f is continuous on the closed interval $[0,5]$ and differentiable on the open interval $(0,5)$. Selected values of f are given in the table shown. The value $c = 4$ satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[0,5]$. What is the slope of the line tangent to graph of f at $x = 4$?

A. 2

B. 2.8

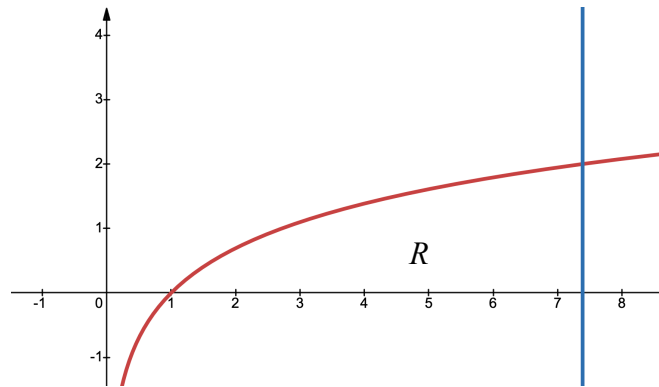
C. 4.5

D. 10

28. Let f be a function with $f(0) = 0$ and derivative given by $f'(x) = \cos(x^2)$.

Which of the following is the Maclaurin series for f ?

- A. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
- B. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$
- C. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}$
- D. $\sum_{n=0}^{\infty} \frac{(-1)^n (4n)x^{4n-1}}{(2n)!}$



29. Let R be the region bounded by the graph of $y = \ln x$, the vertical line $x = e^2$, and the x -axis, as shown in the figure. Which of the following integrals are equal to the area of R ?

- I. $\int_1^{e^2} \ln x \, dx$
- II. $\int_0^2 e^y \, dy$
- III. $\int_0^2 (e^2 - e^y) \, dy$

- A. I only
- B. III only
- C. I and II only
- D. I and III only

30. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{3}{n}k\right)^2 \left(\frac{3}{n}\right)$ is

A. 1

B. 3

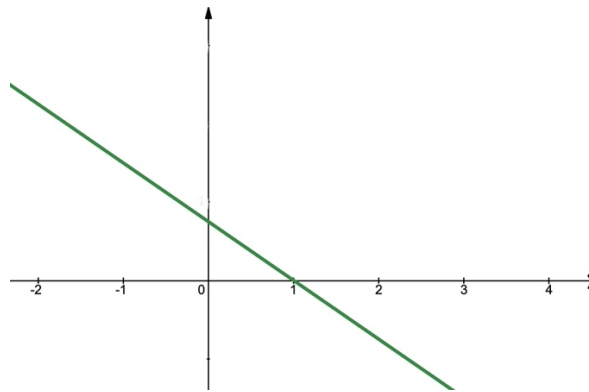
C. 9

D. nonexistent

Section I, Part B (45 minutes calculator 15 questions)

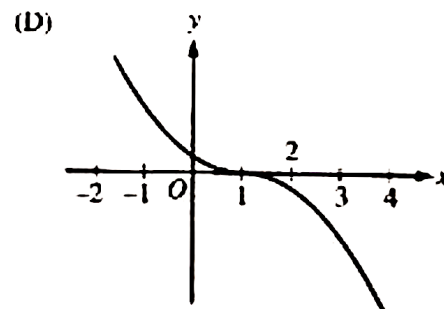
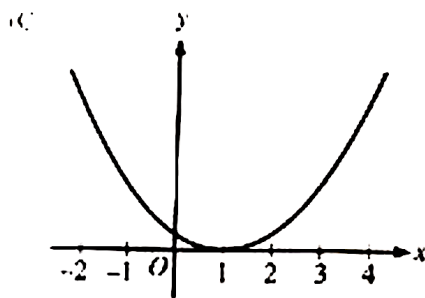
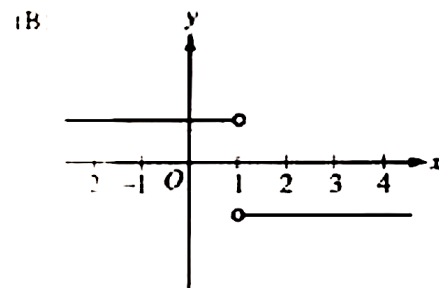
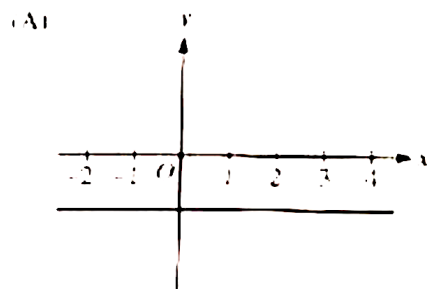
76. If $f'(x) = 1 + 2x - x^3 - \sqrt{x^4 + 1}$, then f has a local minimum at $x =$

- A. -1.836
- B. -1.117
- C. 0
- D. 1.181



Graph of g

77. The preceding figure shows the graph of g . If $g(x) = \int_1^x f(t)dt$, which of the following could be the graph of $y = f(x)$?



x	-1	1
$f(x)$	10	0
$f'(x)$	-1	-9
$f''(x)$	23	-13
$f'''(x)$	-42	6

78. The function f has derivatives of all orders for all real numbers. The table shown gives values of f and its first three derivatives at selected values of x . What is the value of $\int_{-1}^1 f''(x)dx$?

- A. -36
- B. -10
- C. -8
- D. 48

79. On a certain day, an industrial plant manufactures steel at a rate modeled by $R(t) = 10 + 30e^{-0.6t}$ tons per hour, where t is measured in hours since the plant opened that day. What is the average rate at which the plant manufactures steel, in tons per hour, over the time interval $0 \leq t \leq 5$?

- A. 11.494
- B. 19.502
- C. 25.747
- D. 97.511

80. The power series $\sum_{n=0}^{\infty} a_n(x+1)^n$ converges at $x = 3$. Which of the following must be true?

- A. $\sum_{n=0}^{\infty} a_n(x+1)^n$ converges at $x = -5$
- B. $\sum_{n=0}^{\infty} a_n(x+1)^n$ diverges at $x = -5$
- C. $\sum_{n=0}^{\infty} a_n(x+1)^n$ converges at $x = -4$
- D. $\sum_{n=0}^{\infty} a_n(x+1)^n$ diverges at $x = 4$

81. A thin rectangular sheet of metal expands in a hot oven. At the instant when the width of the rectangular sheet of metal is 200 millimeters and the length is 300 millimeters, the width is increasing at a rate of 2 millimeters per minute, and the length is increasing at a rate of 3 millimeters per minute. What is the rate of change of the area, in square millimeters per minute, of the rectangular sheet of metal at that instant?

- A. 5
 - B. 6
 - C. 600
 - D. 1200
-

82. A particle travels along x - axis so that its velocity at time $t \geq 0$ is given by $\frac{dy}{dx} = \sin(t^2)$. What is the total distance the particle travels from $t = 0$ to $t = 2.7$?

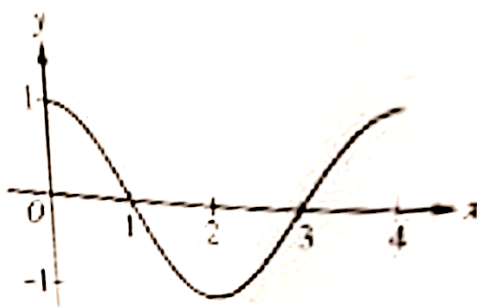
- A. 0.519
 - B. 0.845
 - C. 1.448
 - D. 2.887
-

83. The function f is continuous on the closed interval $[a, b]$, where a and b are constants and $a < b$. If $f(a) < 0 < f(b)$, which of the following statements must be true?

- A. There is at least one value of x in the interval (a, b) such that $f(x) = 0$
 - B. There is at least one value of x in the interval (a, b) such that $f'(x) = \frac{f(b)-f(a)}{b-a}$
 - C. $f'(x) > 0$ for all x in the interval (a, b)
 - D. $f(a) < f(x) < f(b)$ for all x in the interval (a, b)
-

84. The alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+2}$ converges to S . If the sum of the first six terms of the series is used to approximate S . What is the alternating series error bound?

- A. 1/6
- B. 1/7
- C. 1/14
- D. 1/16



85. The graph of a twice-differentiable function f is shown for $0 < x < 4$. The graph of f has a horizontal tangent at $x = 2$ and points of inflection at $x = 1$ and $x = 3$. On what intervals do $f'(x)$ and $f''(x)$ have the same sign?

- A. $0 < x < 1$ only
- B. $2 < x < 4$
- C. $0 < x < 1$ and $2 < x < 3$
- D. $1 < x < 2$ and $3 < x < 4$

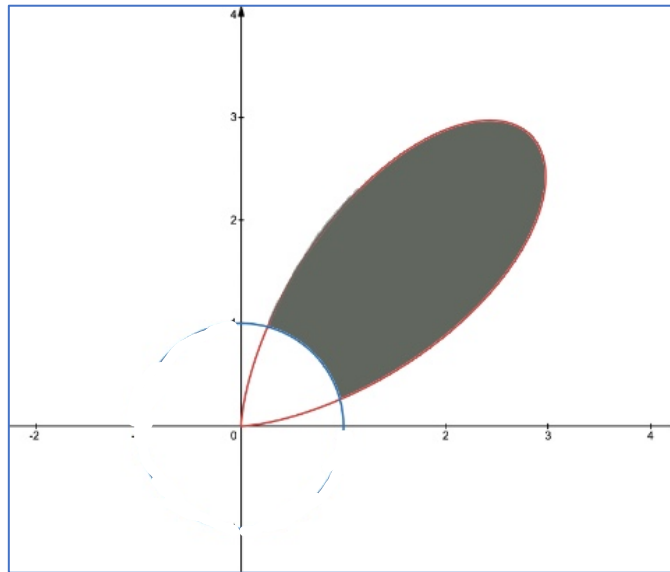
x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	-6	7	4	-1
4	5	6	0	5

86. The table shown gives values of the twice-differentiable function f and g and their derivatives f' and g' at selected values of x . What is $\lim_{x \rightarrow 2} \frac{g(x^2)}{3x + f(x)}$?

- A. $\frac{1}{2}$
- B. $\frac{20}{13}$
- C. 2
- D. The limit does not exist.

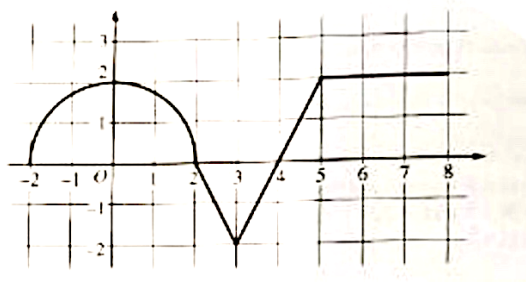
87. The velocity of a particle moving along the x - axis is given by $v(t) = \frac{1-t^3}{1+t}$ for time $t \geq 0$. Which of the following statements is true about the particle at time $t = 1.5$?

- A. The acceleration is negative, and the speed is decreasing
- B. The acceleration is negative, and the speed is increasing
- C. The acceleration is positive, and the speed is decreasing
- D. The acceleration is positive, and the speed is increasing



88. The figure shows the graphs of the polar curve $r = 4(\sin(2\theta))^2$ and $r = 1$ for $0 \leq \theta \leq \frac{\pi}{2}$. What is the area of the shaded region?

- A. 8.267
- B. 4.134
- C. 3.927
- D. 1.913



Graph of f

89. The graph of the function f shown consists of a semicircle centered at the origin and three line segments. If $g(x) = \int_{-2}^x (f(t) + 1) dt$, then $g(8) =$

- A. $2\pi + 6$
- B. $2\pi + 10$
- C. $2\pi + 15$
- D. $2\pi + 19$

90. The position of a particle moving in the xy -plane is given by the parametric equation $x(t) = 5 \ln(t + 5)$ and $y(t) = 5 \sin(\pi t)$. At any point on the curve, the slope of the line tangent to the path of the particle is given by $\frac{dy}{dx} = \pi \cos(\pi t) (t + 5)$.

What is the value of $\frac{d^2y}{dx^2}$ at $t = 1$?

- A. -18.850
- B. -3.770
- C. -3.142
- D. 0.200