

1 Multiple Choice Questions

The exam begins on the next page.

PHYSICS C: MECHANICS

SECTION I

Time—45 minutes

35 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding circle on the answer sheet.

Note: To simplify calculations, you may use $g = 10 \text{ m/s}^2$ in all problems.

1. A person exerts a horizontal force on a crate that is at rest on a horizontal surface, and the crate remains at rest. Which of the following is the best explanation for why the crate remains at rest?

(A) The mass of the crate is greater than the mass of the person.

(B) The force of static friction exerted on the crate is greater than the horizontal force exerted on the crate.

(C) The force of static friction exerted on the crate is equal in magnitude to the horizontal force exerted on the crate.

(D) The normal force exerted by the crate on the person is equal in magnitude to the horizontal force that the person exerts on the crate.

(E) The normal force exerted on the crate by the horizontal surface is equal in magnitude to the force of static friction exerted on the crate.

2. A person with a mass of 100 kg is sitting on a raft of mass 150 kg that is floating on water. A rock of mass 5.0 kg is also on the raft. Resistive forces between the raft and the water are negligible. With what horizontal speed must the person throw the rock of mass 5.0 kg in order to cause the person-raft system to move with a speed of 0.3 m/s in the opposite direction?

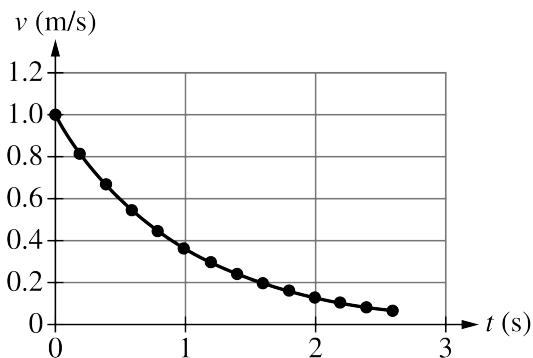
(A) 3.0 m/s

(B) 6.0 m/s

(C) 9.0 m/s

(D) 15 m/s

(E) 21 m/s



3. The velocity v of a toy car as a function of time t as it moves along a horizontal track is represented by the graph shown. Which of the following is a true statement about the car's acceleration during the time interval shown in the graph?

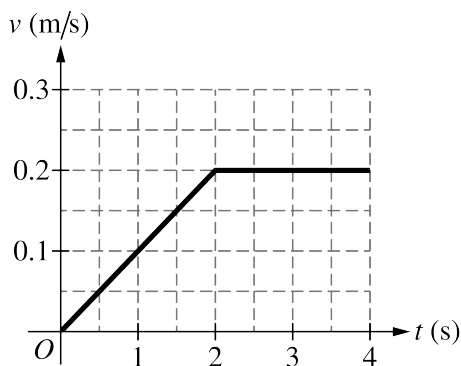
(A) The magnitude of the acceleration is constant.

(B) The magnitude of the acceleration is decreasing and the sign of the acceleration is positive.

(C) The magnitude of the acceleration is increasing and the sign of the acceleration is positive.

(D) The magnitude of the acceleration is decreasing and the sign of the acceleration is negative.

(E) The magnitude of the acceleration is increasing and the sign of the acceleration is negative.



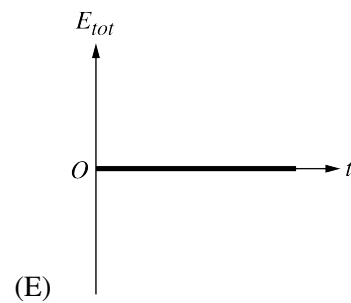
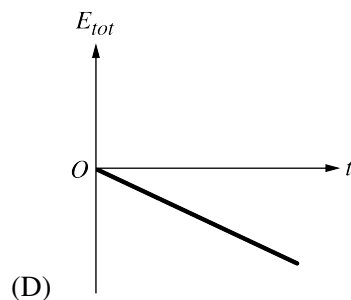
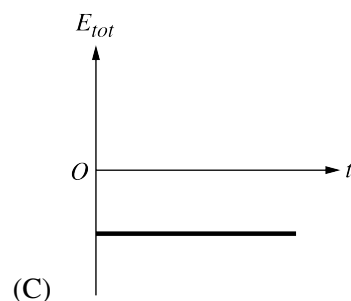
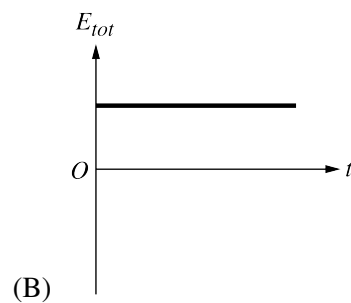
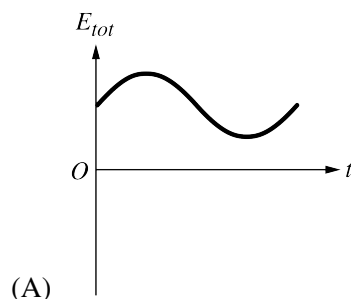
4. The graph shows velocity v as a function of time t for the first 4 seconds of a cart's motion on a horizontal track. Which of the following best describes the acceleration of the cart during the time interval shown?

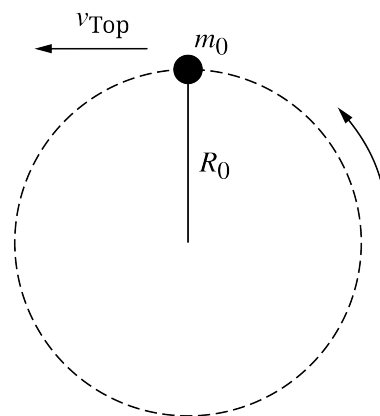
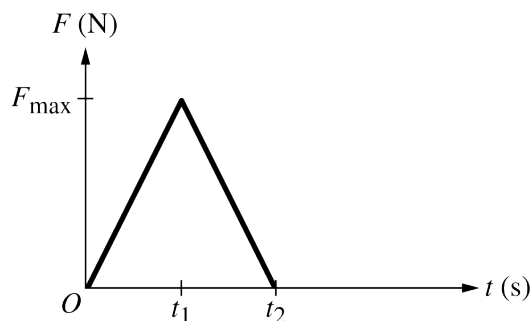
(A) The acceleration is zero during the entire interval.
 (B) The acceleration is nonzero and constant during the entire interval.
 (C) The acceleration is initially nonzero and constant and then is zero.
 (D) The acceleration is initially increasing and then is zero.
 (E) The acceleration is initially increasing and then is nonzero and constant.

5. A solid sphere of mass $M = 2.4$ kg and radius $R = 0.10$ m is released from rest at the top of a ramp of height 1.2 m. The sphere rolls without slipping down the ramp. What is the linear speed of the sphere when it reaches the bottom of the ramp? The rotational inertia of a sphere about its center is $I = \frac{2}{5}MR^2$.

(A) 2.9 m/s
 (B) 3.8 m/s
 (C) 4.1 m/s
 (D) 4.8 m/s
 (E) 5.4 m/s

6. A satellite moves in an elliptical orbit around Earth. Which of the following graphs best represents the total mechanical energy E_{tot} of the satellite-Earth system as a function of time t ?





7. A block is initially at rest on a horizontal surface. There is negligible friction between the block and the surface. A horizontal force F exerted on the block varies as a function of time t , as shown in the graph. At time $t = t_2$, the block has a known speed of v . Which feature of the graph can be used to determine the mass m of the block?
- (A) Using the slope of the graph from $t = 0$ to $t = t_1$ and from $t = t_1$ to $t = t_2$ to determine the change in momentum of the block
- (B) Using the slope of the graph from $t = 0$ to $t = t_1$ and from $t = t_1$ to $t = t_2$ to determine the change in speed for the block
- (C) Using the area of the graph from $t = 0$ to $t = t_1$ to determine the work done on the block
- (D) Using the area of the graph from $t = t_1$ to $t = t_2$ to determine the work done on the block
- (E) Using the area of the graph from $t = t_1$ to $t = t_2$ to determine the impulse on the block
8. A 1100 kg car is moving with speed 15 m/s along a horizontal track when its brakes are applied such that the car begins to skid. The car skids a distance d_1 before coming to rest. The process is repeated with the car moving at 30 m/s, and it skids a distance d_2 . The coefficients of static and kinetic friction between the tires and the track are 0.90 and 0.60, respectively. The difference in skid distances $d_2 - d_1$ is most nearly
- (A) 12 m
- (B) 38 m
- (C) 50 m
- (D) 56 m
- (E) 75 m
9. A ball of mass m_0 is attached to one end of a string and the ball is swung in a vertical circle of radius R_0 , as shown. When the ball is at the top of the circle, the speed of the ball is $v_{\text{TOP}} = \sqrt{\frac{3gR_0}{2}}$. Air resistance is considered to be negligible. What is the magnitude of the force of tension that is exerted on the ball by the string when the ball is at the top of the circle?
- (A) $\frac{m_0 g}{2}$
- (B) $\frac{5m_0 g}{4}$
- (C) $\frac{3m_0 g}{2}$
- (D) $\frac{5m_0 g}{2}$
- (E) $\frac{13m_0 g}{4}$
10. An object's motion along the x -axis is described by the differential equation $\frac{d^2x}{dt^2} + 16x = 0$. Which of the following best illustrates possible directions of the displacement and acceleration vectors of the object at some time t ?
- | | Displacement | Acceleration |
|-----|---------------|---------------|
| I | $\rightarrow$ | $\rightarrow$ |
| II | $\rightarrow$ | $\leftarrow$ |
| III | $\leftarrow$ | $\rightarrow$ |
- (A) I only
- (B) II only
- (C) III only
- (D) I and III
- (E) II and III

11. An object with mass 2.0 kg moves on a horizontal surface. At time $t = 0$, the object's velocity is 0.50 m/s to the north. At time $t = 1$ s, the object's velocity is 0.50 m/s to the east. Which of the following best describes why the average net force F exerted on the object between $t = 0$ and $t = 1$ s can or cannot be determined?

- (A) F cannot be determined because there may be more than one force exerted on the object.
- (B) F cannot be determined because the displacement of the object between $t = 0$ and $t = 1$ s is not known.
- (C) F cannot be determined because the velocity of the object is not known for every instant between $t = 0$ and $t = 1$ s.
- (D) F can be determined because the magnitudes of the initial and final velocities are the same.
- (E) F can be determined because the mass, initial velocity, final velocity, and time interval are known.

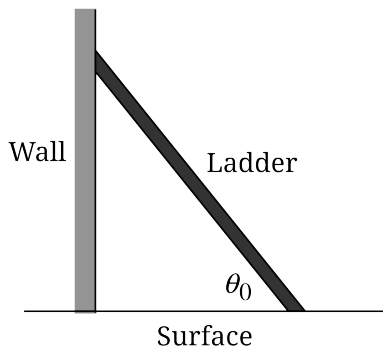
12. The force F needed to stretch a spring is given by the equation $F(x) = \frac{1}{2}bx^2$, where x is the distance the spring is stretched and b is a constant with appropriate units. Which of the following is a correct expression for the elastic potential energy stored when the spring is stretched a distance x ?

- (A) bx
- (B) $\frac{1}{2}bx^2$
- (C) $\frac{1}{3}bx^3$
- (D) $\frac{1}{6}bx^3$
- (E) $\frac{1}{2}bx^3$

Time in Orbit (days)	Period of Oscillation (s)
10	35.02
20	34.80
30	33.59
40	32.10
60	31.82
80	31.56

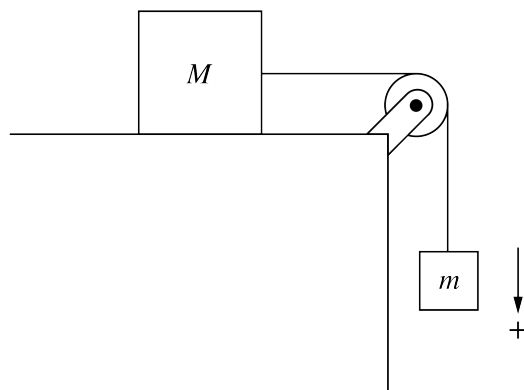
13. An astronaut orbits in a space station above Earth's surface. The astronaut's mass is determined by having the astronaut oscillate on a chair that is attached to an ideal spring and measuring the period of oscillation. The period of oscillation is measured on different days while the astronaut is in the space station, as shown in the data table. Which of the following claims is supported by the data across the 80-day duration?

- (A) The mass of the astronaut increases because the period increases.
- (B) The mass of the astronaut increases because the period decreases.
- (C) The mass of the astronaut decreases because the period increases.
- (D) The mass of the astronaut decreases because the period decreases.
- (E) A change in mass cannot be determined because the astronaut is weightless in space.



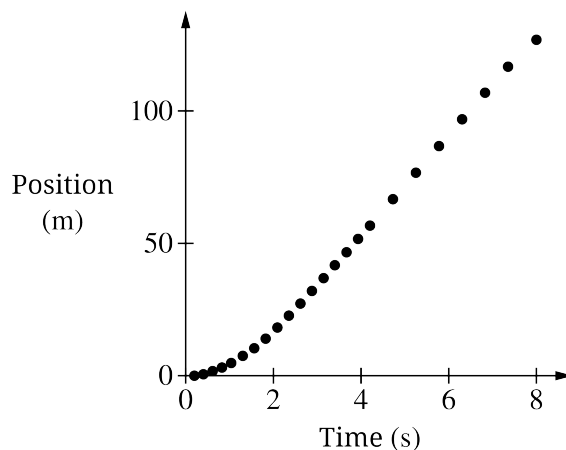
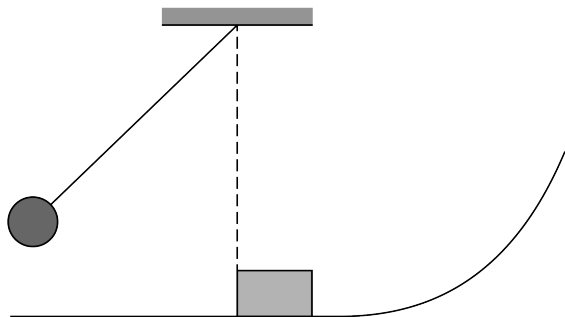
14. A ladder of uniform mass density and mass m_0 rests against a vertical wall, as shown. The coefficient of static friction between the ladder and the horizontal surface is μ_0 . The frictional force between the wall and the ladder is considered to be negligible. What is the minimum angle θ_0 at which the ladder remains at rest?

- (A) $\tan^{-1} \left(\frac{1}{4\mu_0} \right)$
- (B) $\tan^{-1} \left(\frac{1}{2\mu_0} \right)$
- (C) $\tan^{-1} \left(\frac{1}{\mu_0} \right)$
- (D) $\tan^{-1} (2\mu_0)$
- (E) $\tan^{-1} (4\mu_0)$



15. A block of mass M on a horizontal table is attached by a string that passes over a pulley to a suspended block of mass m , as shown. All frictional forces are negligible, and the string and pulley have negligible mass. A student investigates the acceleration a of the suspended block for different mass ratios $\frac{M}{m}$ when the system is released from rest, where the positive direction is defined as shown. Which of the following statements is a correct prediction for the relationship between a and $\frac{M}{m}$?

- (A) $a < g$ for all values of $\frac{M}{m}$
- (B) $a = 0$ if $\frac{M}{m} > 1$
- (C) $a < 0$ if $\frac{M}{m} > 1$
- (D) $a = g$ if $\frac{M}{m} = 1$
- (E) $a > g$ if $\frac{M}{m} < 1$



16. A simple pendulum is made of a sphere of unknown mass tied to a long string of negligible mass, as shown. The sphere is released from rest from an initial height and allowed to swing down freely. The sphere strikes a stationary block of known mass at the bottom of the sphere's swing. After the collision, the sphere immediately comes to rest and the block moves to the right along a track of negligible friction that curves upward. The block travels along the track before momentarily coming to rest. Which of the following sets of measurements can be used to determine the mass of the sphere?

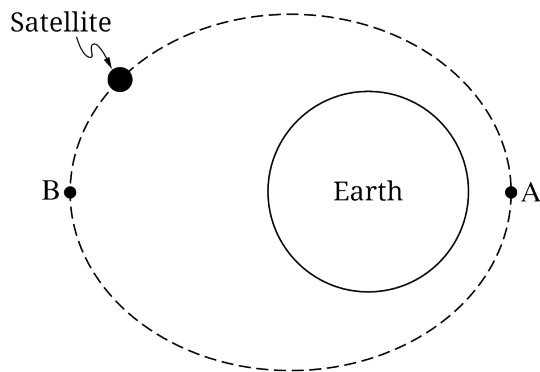
- (A) The change in height of the sphere and the speed of the sphere just before the sphere strikes the block
- (B) The speed of the block immediately after the collision and the change in height of the block when the block momentarily comes to rest
- (C) The change in height of the sphere and the change in height of the block when the block momentarily comes to rest
- (D) The length of the pendulum and the speed of the block immediately after the collision
- (E) The length of the pendulum and the speed of the sphere just before the sphere strikes the block

17. The position of a falling object is plotted as a function of time in the graph shown. Which of the following features of the graph can be used to support the claim that the object reaches terminal velocity?

- (A) The slope of the curve during the time interval $t = 0$ to $t = 8$ s
- (B) The slope of the curve during the time interval $t = 0$ to $t = 4$ s
- (C) The slope of the curve during the time interval $t = 4$ to $t = 8$ s
- (D) The area under the curve during the time interval $t = 0$ to $t = 4$ s
- (E) The area under the curve during the time interval $t = 4$ s and $t = 8$ s

18. Two objects, A and B, move along a horizontal line. Object A has mass 2 kg and initial velocity v_A . Object B has mass 3 kg and initial velocity v_B . Object collides A head-on with Object B. The objects stick together and move with final velocity v_{AB} . Which of the following are possible values for v_A , v_B , and v_{AB} ?

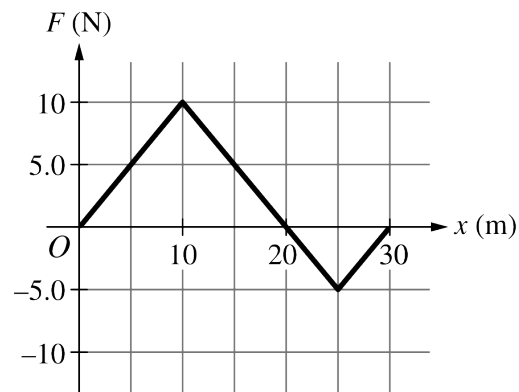
	v_A (m/s)	v_B (m/s)	v_{AB} (m/s)
(A)	4	-3	2
(B)	-5	1	3
(C)	4	-1	0
(D)	1	-4	-2
(E)	-2	3	4



Note: Figure not drawn to scale.

19. A satellite of mass m_0 is in an elliptical orbit around Earth. Earth's mass is M and its radius is R . The satellite's distance r from the center of Earth varies. At Point A, $r = r_A$, and at Point B, $r = r_B$. Which of the following integrals can be used to determine the work done by Earth on the satellite as it moves from Point A to Point B?

- (A) $\int_{r_A}^{r_B} m_0 g r dr$
 (B) $\int_{r_A}^{r_B} -\frac{GMm_0}{r^2} dr$
 (C) $\int_{r_A}^{r_B} -\frac{GMm_0}{r} dr$
 (D) $\int_R^{r_A} -\frac{GMm_0}{r^2} dr + \int_{r_A}^{r_B} -\frac{GMm_0}{r^2} dr$
 (E) $\int_R^{r_A} -\frac{GMm_0}{r} dr + \int_{r_A}^{r_B} -\frac{GMm_0}{r} dr$



20. The graph of a single conservative force F exerted on an object as a function of the position x of the object is shown. What is the work done on the object when it moves from $x = 0$ m to $x = 30$ m under the influence of only this force?

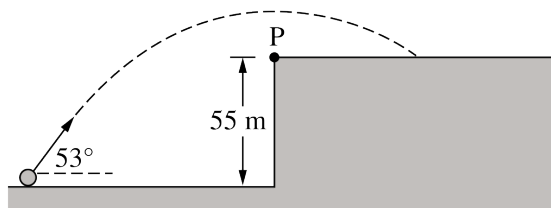
- (A) -125 J
 (B) -75 J
 (C) Zero
 (D) 75 J
 (E) 125 J

Questions 21-22

A small disk is initially at rest. At time $t = 0$, the disk starts to spin around a fixed axis perpendicular to and through the center of the disk. The disk has uniform angular acceleration and reaches an angular speed of 20π rad/s at time $t = 0$.

21. What is the magnitude of the angular acceleration of the disk during the time interval from $t = 0$ to $t = 2.0$ m/s?
- (A) 2π rad/s²
(B) 5π rad/s²
(C) 10π rad/s²
(D) 20π rad/s²
(E) 40π rad/s²
22. What is the linear speed of a point that is 0.030 m from the axis of rotation of the disk at time $t = 2.0$ m/s?
- (A) 0.0015π m/s
(B) 0.30π m/s
(C) 0.6π m/s
(D) 1.2π m/s
(E) 10π m/s

Questions 23-25

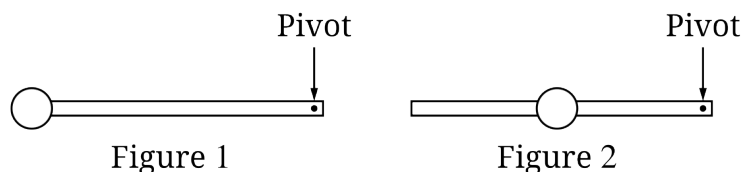


Note: Figure not drawn to scale.

An object is launched with an initial speed of 60 m/s at an angle of 53° above the horizontal. The object lands on top of a cliff that has a height of 55 m, as shown. The object reaches its maximum height when the object is directly above Point P at the edge of the cliff. Air resistance is considered to be negligible.

23. Which of the following is most nearly the distance above Point P at which the object reaches its maximum height?
- (A) 11 m
 - (B) 48 m
 - (C) 55 m
 - (D) 62 m
 - (E) 120 m
24. How long does it take for the object to travel from its initial position to its landing position on top of the cliff?
- (A) 15.5 s
 - (B) 8.5 s
 - (C) 5.2 s
 - (D) 1.3 s
 - (E) 1.0 s
25. A second, identical object is launched with an initial speed of 70 m/s at an angle of 53° above the horizontal at the same time that the original object is launched. Which of the following is a correct statement about the motion of the objects after the objects are launched?
- (A) Both objects have the same speed when each object is at its maximum height.
 - (B) The second object reaches its maximum height in less time than it takes for the original object to reach its maximum height.
 - (C) The horizontal displacement from the launch position to the landing position is smaller for the second object than it is for the original object.
 - (D) The vertical displacement from the launch position to the landing position is greater for the second object than it is for the original object.
 - (E) The total distance traveled from the launch position to the landing position is greater for the second object than it is for the original object.

Questions 26-28

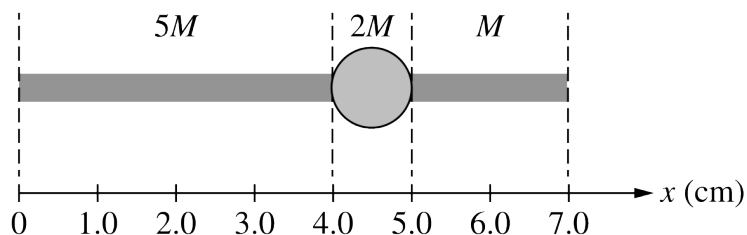


Side View

Note: Figure not drawn to scale.

The figures show two identical thin rods of negligible mass and length d that are able to freely rotate in a vertical plane about pivot points at the right ends of the rods. The rod in Figure 1 has a small sphere attached at its left end. The rod in Figure 2 has an identical sphere attached at its center. The rods are released from rest and begin to rotate under the influence of gravity.

26. Which of the following is a true statement about the linear speed of the spheres as each sphere swings through its lowest point?
- (A) The sphere in Figure 1 will have a greater linear speed than the sphere in Figure 2 will have.
 - (B) The sphere in Figure 1 will have a smaller linear speed than the sphere in Figure 2 will have.
 - (C) The sphere in Figure 1 will have the same linear speed as the sphere in Figure 2 will have.
 - (D) The linear speeds cannot be compared without knowing the masses of the spheres.
 - (E) The linear speeds cannot be compared without knowing the lengths of the rods.
27. For the system consisting of the thin rod and small sphere shown in Figure 1, which of the following is an expression for the angular momentum of the system as the sphere swings through its lowest point?
- (A) $\sqrt{m^2 g d^2}$
 - (B) $\sqrt{m^2 g d^3}$
 - (C) $\sqrt{2 m^2 g d}$
 - (D) $\sqrt{2 m^2 g d^2}$
 - (E) $\sqrt{2 m^2 g d^3}$
28. The initial angular acceleration of the rod-sphere system in Figure 1 is α_1 . The initial angular acceleration of the rod-sphere system in Figure 2 is α_2 . Which of the following correctly compares α_1 and α_2 ?
- (A) $\alpha_1 > \alpha_2$
 - (B) $\alpha_1 < \alpha_2$
 - (C) $\alpha_1 = \alpha_2$
 - (D) The initial angular accelerations cannot be compared without knowing the masses of the spheres.
 - (E) The initial angular accelerations cannot be compared without knowing the lengths of the rods.



29. Two uniform thin rods are connected to opposite sides of a uniform sphere, as shown. The rod on the left has mass $5M$ and length 4.0 cm, the rod on the right has mass M and length 2.0 cm, and the sphere has mass $2M$ and diameter 1.0 cm. The center of the sphere and the axis of both rods lie on the same straight horizontal line. What is the x -coordinate of the center of mass of the three-object system?

- (A) 2.0 cm
- (B) 3.1 cm
- (C) 3.5 cm
- (D) 4.2 cm
- (E) 4.5 cm



Figure 1

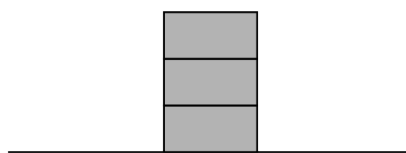
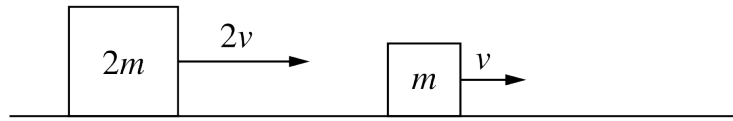


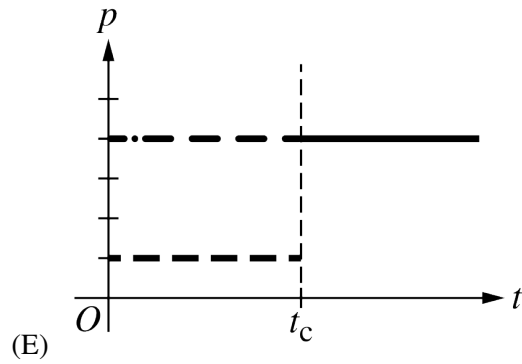
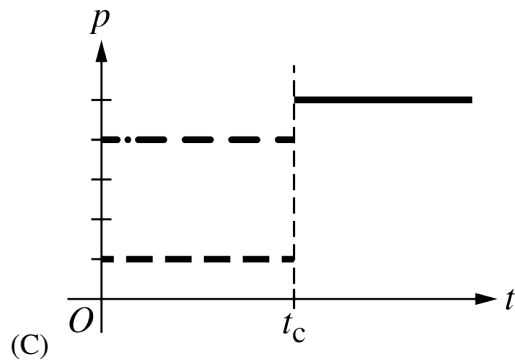
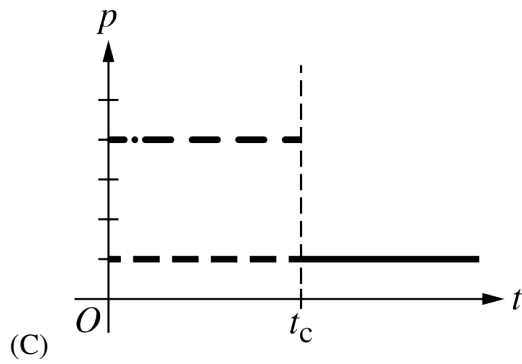
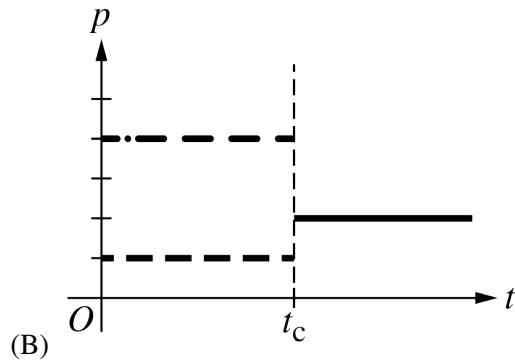
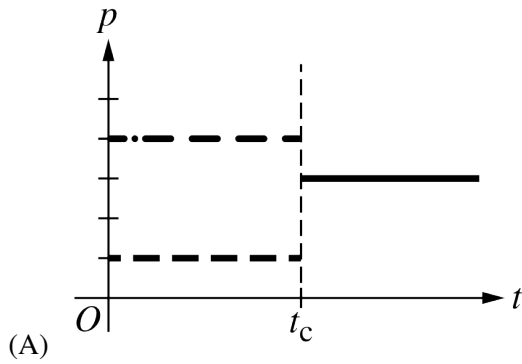
Figure 2

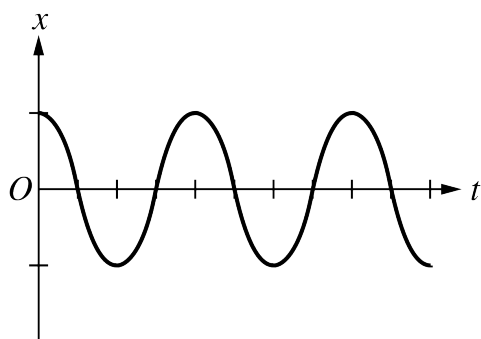
30. Three bricks rest on a level, horizontal surface, as shown in Figure 1. The mass of each brick is 2 kg. Each brick is 5 cm tall and 10 cm wide. Which of the following is the minimum amount of work that must be done to make a stack of the three bricks as shown in Figure 2?

- (A) 1 J
- (B) 2 J
- (C) 3 J
- (D) 4 J
- (E) 5 J

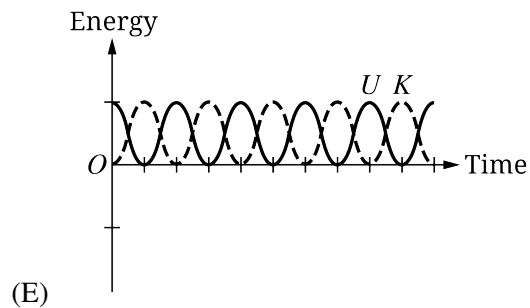
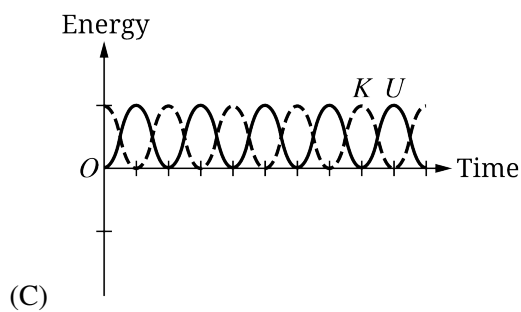
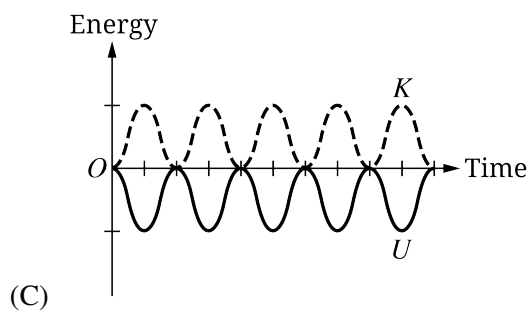
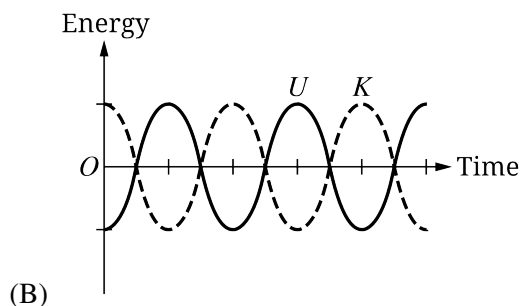
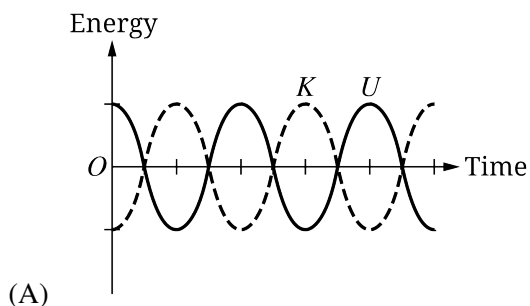


31. A block of mass $2m$ moving with velocity $+2v$ collides with and sticks to a block of mass m that is initially moving with velocity $+v$, as shown. Given that the collision takes place at time $t = t_c$, which of the following graphs most accurately illustrates the momentum p of the two blocks as a function of time? Each dashed horizontal line represents the momentum of an individual block, and the solid horizontal lines represent the combined momentum of both blocks after they collide.





32. The graph shows the displacement x from equilibrium as a function of time t for a block oscillating on the end of a spring on a smooth horizontal surface, starting at time $t = 0$. Which of the following graphs correctly represents the potential energy U (represented by a solid line) and the kinetic energy K (represented by a dashed line) of the block-spring system as a function of time, starting at $t = 0$?



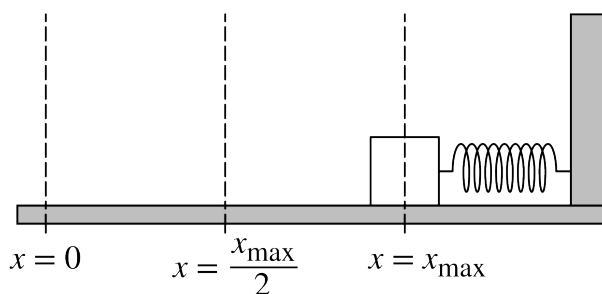


Figure 1

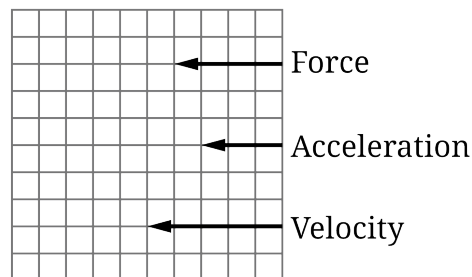
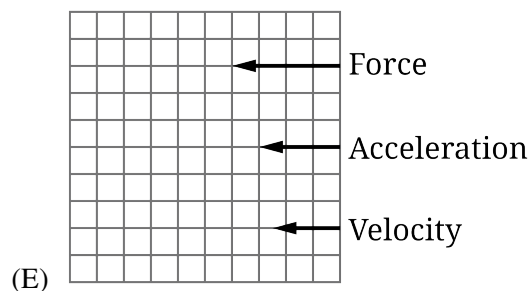
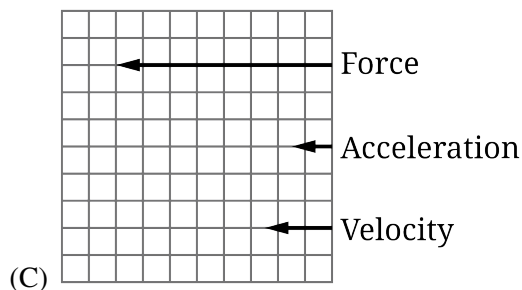
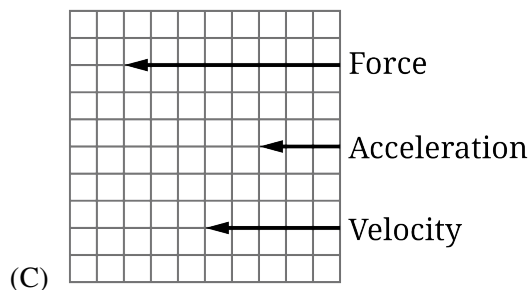
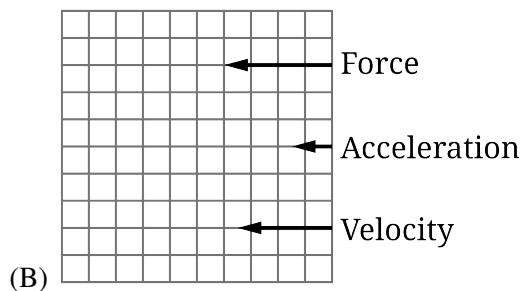
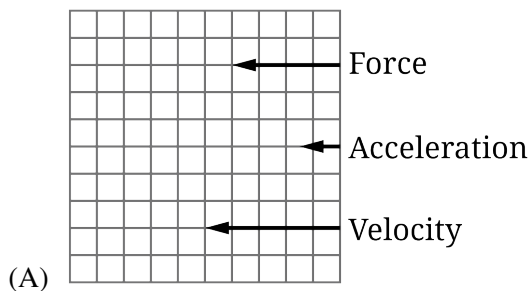
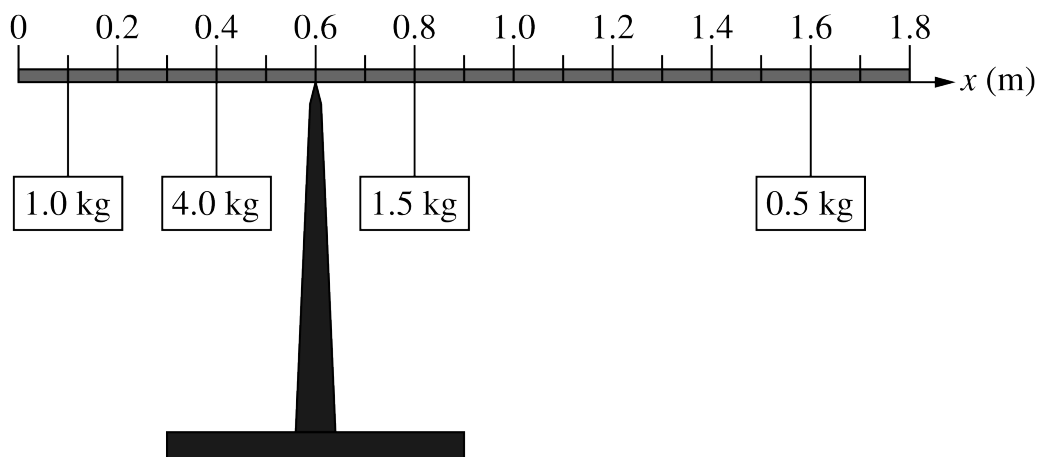


Figure 2

33. A block of mass m is attached to an ideal spring and displaced a distance $x_{\max}$ from the equilibrium position at $x = 0$, as shown in Figure 1. There is negligible friction between the block and the horizontal surface. The block is released from rest, and the relative magnitudes of the force, acceleration, and velocity vectors for the block when the block is located at $x = \frac{x_{\max}}{2}$ are shown in Figure 2. The scenario is repeated with a new block of mass $2m$. Which of the following diagrams correctly shows the relative magnitudes of the force, acceleration, and velocity vectors when the new block is located at $x = \frac{x_{\max}}{2}$?





Note: Figure not drawn to scale.

34. A rod of length 1.8 m has negligible mass and is pivoted at a point 0.6 m from the left end of the rod, as shown. Four different objects are hung at the locations shown. Which object, if any, should be removed so that the rod will be in equilibrium?
- (A) The 0.5 kg object
 - (B) The 1.0 kg object
 - (C) The 1.5 kg object
 - (D) The 4.5 kg object
 - (E) None of the objects should be removed.

Item #35 was not scored.

**STOP
END OF SECTION I.**

2 Free-Response Questions

The exam begins on the next page.

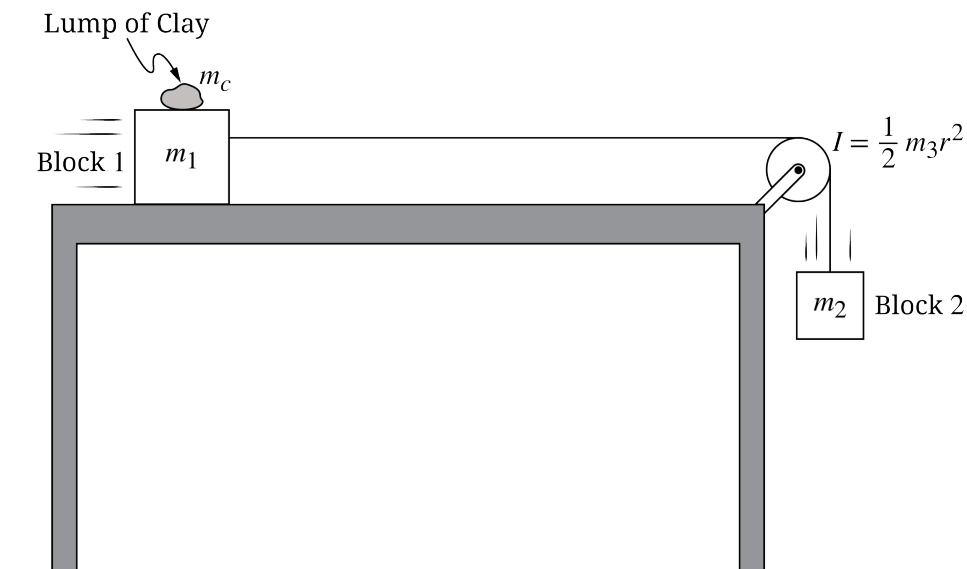
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



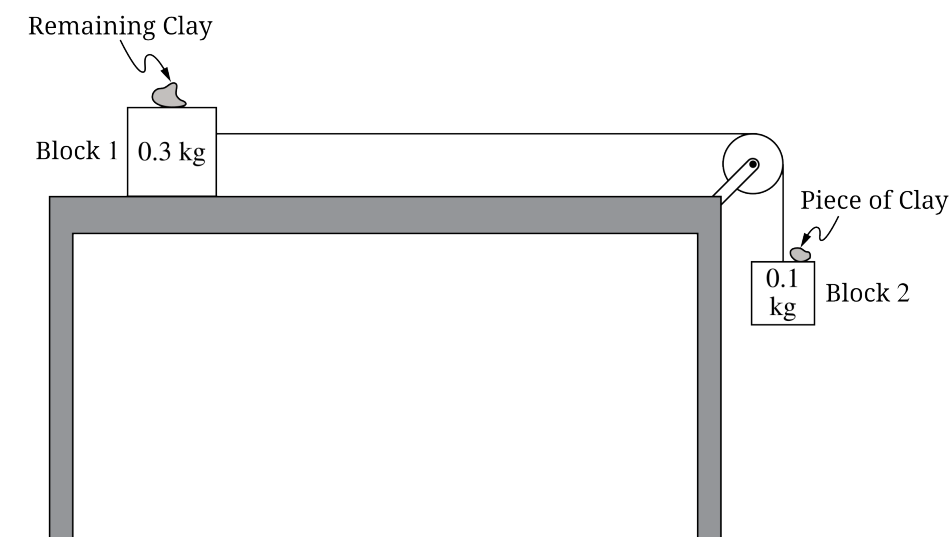
1. A string connects Block 1 of mass m_1 and Block 2 of mass m_2 . The string passes over a pulley so that Block 2 hangs vertically, as shown. The pulley is a disk with mass m_3 and radius r ; the rotational inertia of the pulley is $I = \frac{1}{2}m_3r^2$. A lump of clay of mass m_c is attached to the top of Block 1. The system consisting of the two blocks, pulley, and clay is released from rest and accelerates. Frictional forces are negligible at the axle of the pulley and between Block 1 and the surface. The string has negligible mass.
 - (a) The two dots in the following figure represent Block 1 and Block 2. Draw and label the forces (not components) that act on Block 1 and Block 2 once the blocks-pulley-clay system has been released and is accelerating. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



Block 1

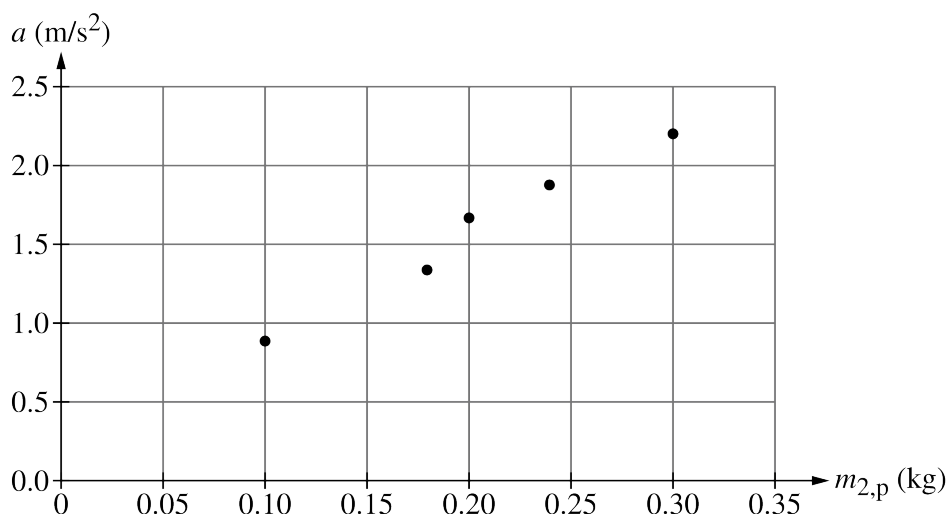
Block 2

- (b) Derive an expression for the magnitude of the acceleration of the blocks-pulley-clay system immediately after the system is released from rest. Express your answer in terms of m_1 , m_2 , m_c , m_3 , r , and fundamental constants, as appropriate.



- (c) A student uses the setup from part (a) to determine an experimental value for the mass m_3 of the pulley. The student uses the following procedure.
- The student measures the masses of the blocks and clay: $m_1 = 0.3$ kg, $m_2 = 0.1$ kg, and $m_c = 0.25$ kg.
 - The student removes a small piece of clay from the lump of clay on the top of Block 1 and attaches the piece of clay to the top of Block 2, as shown. The student measures the mass $m_{2,p}$ of the system consisting of Block 2 and the attached piece of clay.
 - The student releases the blocks-pulley-clay system from rest and measures the acceleration a of this system.
 - The student conducts multiple trials with different values for $m_{2,p}$.

The student plots the data for the acceleration a of the blocks-pulley-clay system as a function of the mass $m_{2,p}$ in the following graph.

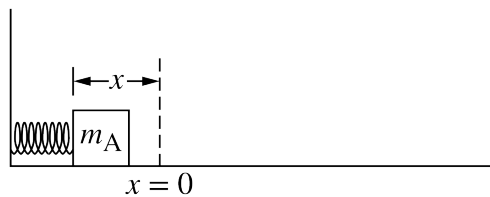


- Draw the best-fit line for the data.
- Use the best-fit line to determine an experimental value for the mass m_3 of the pulley.

- (d) The student repeats the experiment described in part (c), but now the student removes the pieces of clay from the system instead of attaching the pieces to Block 2. The student measures the mass $m_{1,r}$ of the system consisting of Block 1 and the remaining clay. On the following axes, sketch the magnitude of the acceleration of the system consisting of the blocks, pulley and remaining clay as a function of $m_{1,r}$.
- (e) The student repeats the experiment described in part (c), using a different pulley with the same mass, radius, and rotational inertia but with nonnegligible friction at the axle. The student's analysis neglects to account for the friction at the axle of the pulley. Will the new experimental value for the mass of the pulley m_3 be greater than, less than, or equal to the value for the mass of the pulley determined in part (c)?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

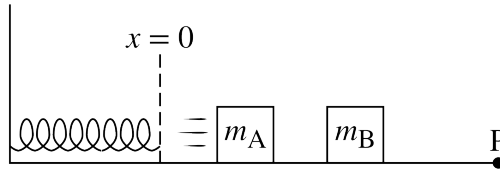


Note: Figure not drawn to scale.

Figure 1

2. Block A has a mass of $m_A = 0.020$ kg and sits at rest on a horizontal surface. Friction between the block and the surface is negligible. The block is pushed against a horizontal spring that is attached to a wall. The spring is compressed a distance of $x = 0.040$ m, as shown in Figure 1. The force exerted on the block by the spring has magnitude $F(x) = \beta x^{\frac{1}{2}}$, where $\beta = 15$ N/m $^{\frac{1}{2}}$.
 - (a) On the following axes, sketch a graph of the force exerted on the block by the spring as a function of horizontal position while the spring is being compressed. Explicitly label the magnitude of the force when the spring is compressed a distance of 0.040 m.

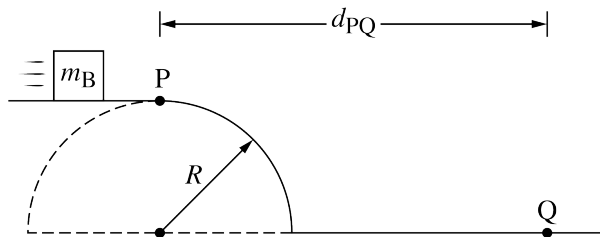




Note: Figure not drawn to scale.

Figure 2

- (b) Block A is released from rest and slides horizontally toward an identical Block B ($m_A = m_B$), which is initially at rest, as shown in Figure 2. Frictional forces between each block and the horizontal surface are negligible. Block A collides elastically with Block B, causing Block B to slide toward Point P.
- Calculate the speed v_A of Block A immediately before the collision.
 - Calculate the speed v_B of Block B immediately before the collision.



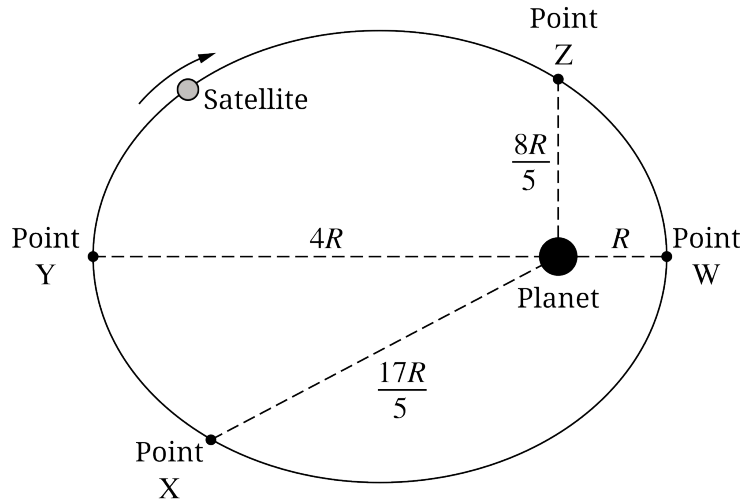
Note: Figure not drawn to scale.

Figure 3

- (c) At Point P the surface begins to follow a circular arc of radius R . A smooth horizontal surface is located a vertical distance R below Point P, as shown in Figure 3.
- At Point P, Block B loses contact with the surface. Calculate the maximum value of the radius $R_{\max}$ for which Block B loses contact with the surface.
 - When $R = R_{\max}$, Block B loses contact with the surface at Point P and lands at Point Q. The horizontal distance between Point P and Point Q is d_{PQ} . Calculate d_{PQ} .
- (d) Block B is removed and replaced with Block C, which has a mass less than that of Block B. Block C is placed before Point P, and Block A is again released from rest after compressing the spring the same distance as before. Block A collides elastically with Block C. Block C then loses contact with the surface and lands at Point S on the horizontal surface. The horizontal distance between Point P and Point S is d_{PS} . Frictional forces are negligible. Which of the following correctly compares d_{PQ} and d_{PS} ?

_____ $d_{PQ} > d_{PS}$ _____ $d_{PQ} = d_{PS}$ _____ $d_{PQ} < d_{PS}$

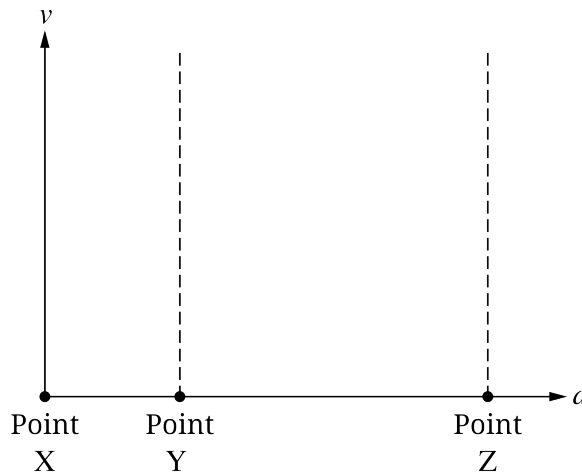
Justify your answer.



Note: Figure not drawn to scale.

3. A satellite of mass m_0 orbits a planet of mass M . The orbit is elliptical in shape and passes through points W, X, Y, and Z, located at distances R , $\frac{17R}{5}$, $4R$, and $\frac{8R}{5}$, respectively, from the center of the planet, as shown. The satellite travels in a clockwise direction in the orbit shown. The distance traveled by the satellite in a complete orbit is D . The distance traveled between points X and Z is $\frac{D}{2}$. The time it takes for the satellite to travel from Point X to Z Point is Δt_1 , and the time it takes for the satellite to travel from Point Z to Point X is Δt_2 .

- (a) i. On the following set of axes, sketch the linear speed v of the satellite as a function of distance d traveled by the satellite as it travels from Point X through Point Y to Point Z.

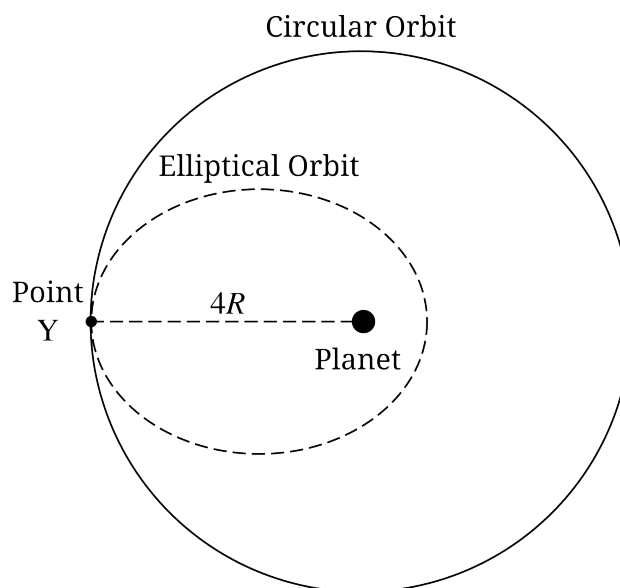


- ii. Is Δt_2 greater than, less than, or equal to Δt_1 ?

_____ $\Delta t_2 > \Delta t_1$ _____ $\Delta t_2 < \Delta t_1$ _____ $\Delta t_2 = \Delta t_1$

Justify your answer.

- (b) The linear speed of the satellite at points W and Y are v_W and v_Y , respectively. Derive an expression for the linear speed v_W . Express your answer in terms of v_Y , m_0 , M , R , and physical constants, as appropriate.



(c) The satellite's propulsion system exerts a force of magnitude F on the satellite for a short time Δt_0 to change the orbit of the satellite at Point Y from the elliptical orbit to the circular orbit shown.

- i. Derive an expression for the linear speed of the satellite $v_{Y,c}$ at Point Y in the circular orbit in terms of m_0 , M , R , and physical constants, as appropriate.
- ii. Derive an expression for the linear speed of the satellite $v_{Y,e}$ at Point Y in the elliptical orbit in terms of m_0 , M , R , F , Δt_0 , and physical constants, as appropriate.
- iii. The magnitude of the angular momentum of the satellite at Point Y is L_c in the circular orbit and L_e in the elliptical orbit. How does L_c compare with L_e ?

_____ $L_c > L_e$ _____ $L_c = L_e$ _____ $L_c < L_e$

Justify your answer.

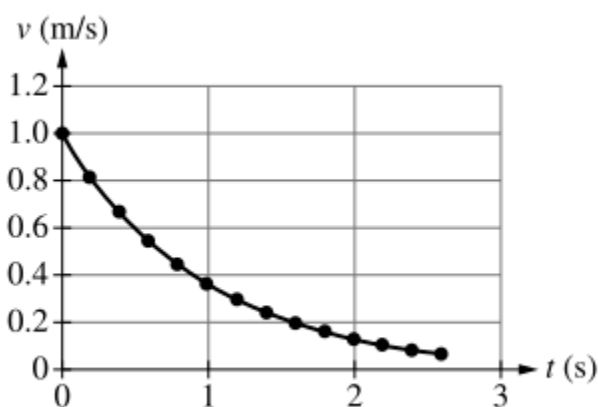
STOP
END OF EXAM

3 Answer and Scoring Guidelines

2023 Form I

1. A person exerts a horizontal force on a crate that is at rest on a horizontal surface, and the crate remains at rest. Which of the following is the best explanation for why the crate remains at rest?
- (A) The mass of the crate is greater than the mass of the person.
 - (B) The force of static friction exerted on the crate is greater than the horizontal force exerted on the crate.
 - (C) The force of static friction exerted on the crate is equal in magnitude to the horizontal force exerted on the crate. ✓
 - (D) The normal force exerted by the crate on the person is equal in magnitude to the horizontal force that the person exerts on the crate.
 - (E) The normal force exerted on the crate by the horizontal surface is equal in magnitude to the force of static friction exerted on the crate.
2. A person with a mass of 100 kg is sitting on a raft of mass 150 kg that is floating on water. A rock of mass 5.0 kg is also on the raft. Resistive forces between the raft and the water are negligible. With what horizontal speed must the person throw the rock of mass 5.0 kg in order to cause the person-raft system to move with a speed of 0.3 m/s in the opposite direction?
- (A) 3.0 m/s
 - (B) 6.0 m/s
 - (C) 9.0 m/s
 - (D) 15 m/s ✓
 - (E) 21 m/s

3.

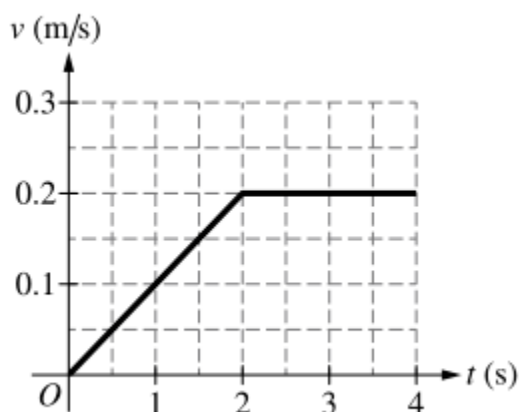


The velocity v of a toy car as a function of time t as it moves along a horizontal track is represented by the graph shown. Which of the following is a true statement about the car's acceleration during the time interval shown in the graph?

2023 Form I

- (A) The magnitude of the acceleration is constant.
- (B) The magnitude of the acceleration is decreasing and the sign of the acceleration is positive.
- (C) The magnitude of the acceleration is increasing and the sign of the acceleration is positive.
- (D) The magnitude of the acceleration is decreasing and the sign of the acceleration is negative. ✓
- (E) The magnitude of the acceleration is increasing and the sign of the acceleration is negative.

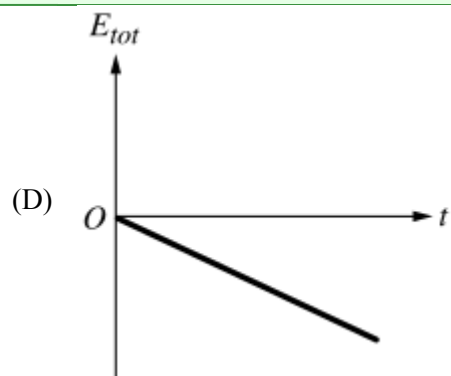
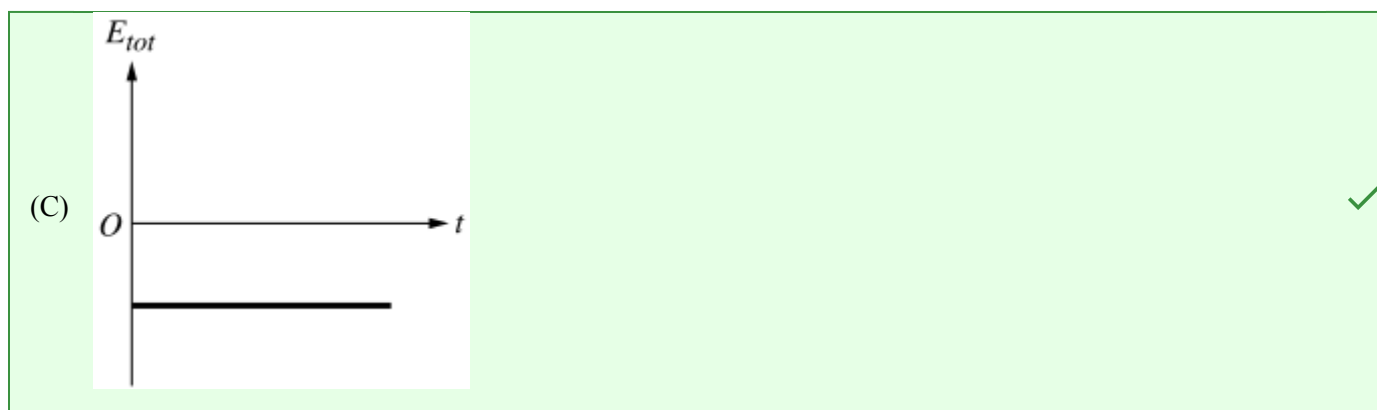
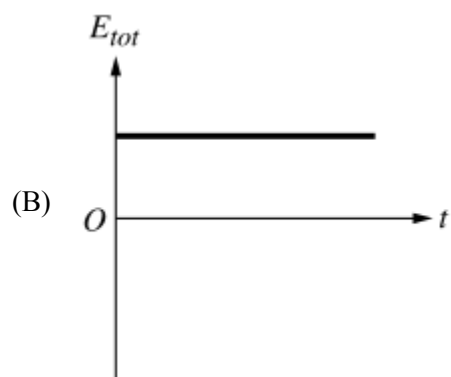
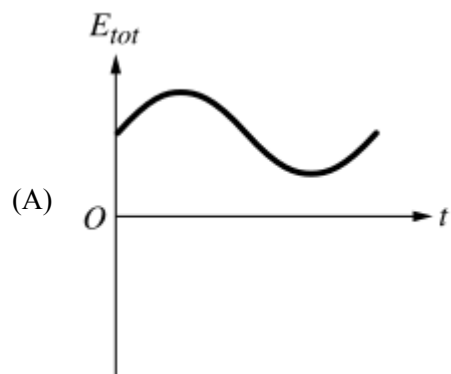
4.



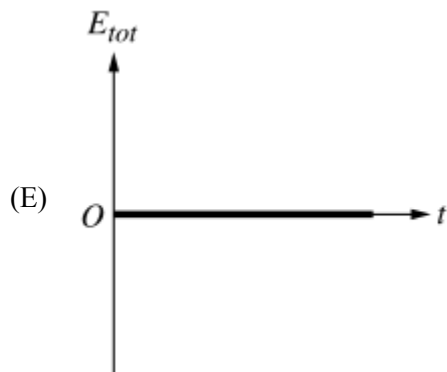
The graph shows velocity v as a function of time t for the first 4 seconds of a cart's motion on a horizontal track. Which of the following best describes the acceleration of the cart during the time interval shown?

- (A) The acceleration is zero during the entire interval.
- (B) The acceleration is nonzero and constant during the entire interval.
- (C) The acceleration is initially nonzero and constant and then is zero. ✓
- (D) The acceleration is initially increasing and then is zero.
- (E) The acceleration is initially increasing and then is nonzero and constant.
5. A solid sphere of mass $M = 2.4 \text{ kg}$ and radius $R = 0.10 \text{ m}$ is released from rest at the top of a ramp of height 1.2 m . The sphere rolls without slipping down the ramp. What is the linear speed of the sphere when it reaches the bottom of the ramp? The rotational inertia of a sphere about its center is $I = \frac{2}{5}MR^2$.
- (A) 2.9 m/s
- (B) 3.8 m/s
- (C) 4.1 m/s ✓
- (D) 4.8 m/s
- (E) 5.4 m/s
6. A satellite moves in an elliptical orbit around Earth. Which of the following graphs best represents the total mechanical energy E_{tot} of the satellite-Earth system as a function of time t ?

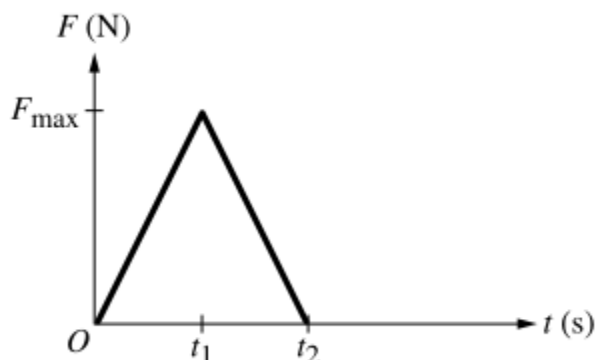
2023 Form I



2023 Form I



7.



A block is initially at rest on a horizontal surface. There is negligible friction between the block and the surface. A horizontal force F exerted on the block varies as a function of time t , as shown in the graph. At time $t = t_2$, the block has a known speed of v . Which feature of the graph can be used to determine the mass m of the block?

- (A) Using the slope of the graph from $t = 0$ to $t = t_1$ and from $t = t_1$ to $t = t_2$ to determine the change in momentum of the block
- (B) Using the slope of the graph from $t = 0$ to $t = t_1$ and from $t = t_1$ to $t = t_2$ to determine the change in speed for the block
- (C) Using the area of the graph from $t = 0$ to $t = t_1$ to determine the work done on the block
- (D) Using the area of the graph from $t = t_1$ to $t = t_2$ to determine the work done on the block
- (E) Using the area of the graph from $t = 0$ to $t = t_2$ to determine the impulse on the block ✓

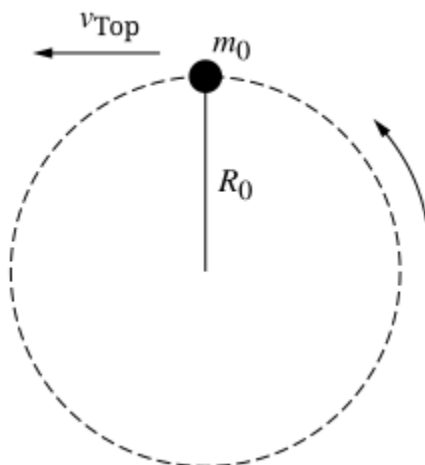
8. A 1100 kg car is moving with speed 15 m/s along a horizontal track when its brakes are applied such that the car begins to skid. The car skids a distance d_1 before coming to rest. The process is repeated with the car moving at 30 m/s, and it skids a distance d_2 . The coefficients of static and kinetic friction between the tires and the track are 0.90 and 0.60, respectively. The difference in skid distances $d_2 - d_1$ is most nearly

2023 Form I

- (A) 12 m
 (B) 38 m
 (C) 50 m
 (D) 56 m
 (E) 75 m



9.



A ball of mass m_0 is attached to one end of a string and the ball is swung in a vertical circle of radius R_0 , as shown. When the ball is at the top of the circle, the speed of the ball is $v_{\text{Top}} = \sqrt{\frac{3gR_0}{2}}$. Air resistance is considered to be negligible. What is the magnitude of the force of tension that is exerted on the ball by the string when the ball is at the top of the circle?

- (A) $\frac{m_0 g}{2}$
 (B) $\frac{5m_0 g}{4}$
 (C) $\frac{3m_0 g}{2}$
 (D) $\frac{5m_0 g}{2}$
 (E) $\frac{13m_0 g}{4}$



10. An object's motion along the x -axis is described by the differential equation $\frac{d^2x}{dt^2} + 16x = 0$. Which of the following best illustrates possible directions of the displacement and acceleration vectors of the object at some time t ?

	Displacement	Acceleration
I	$\longrightarrow$	$\longrightarrow$
II	$\longrightarrow$	$\longleftarrow$
III	$\longleftarrow$	$\longrightarrow$

2023 Form I

- (A) I only
- (B) II only
- (C) III only
- (D) I and III

(E) II and III



11. An object with mass 2.0 kg moves on a horizontal surface. At time $t = 0$, the object's velocity is 0.50 m/s to the north. At time $t = 1$ s, the object's velocity is 0.50 m/s to the east. Which of the following best describes why the average net force F exerted on the object between $t = 0$ and $t = 1$ s can or cannot be determined?

- (A) F cannot be determined because there may be more than one force exerted on the object.
- (B) F cannot be determined because the displacement of the object between $t = 0$ and $t = 1$ s is not known.
- (C) F cannot be determined because the velocity of the object is not known for every instant between $t = 0$ and $t = 1$ s.
- (D) F can be determined because the magnitudes of the initial and final velocities are the same.

(E) F can be determined because the mass, initial velocity, final velocity, and time interval are known.



12. The force F needed to stretch a spring is given by the equation $F(x) = \frac{1}{2}bx^2$, where x is the distance the spring is stretched and b is a constant with appropriate units. Which of the following is a correct expression for the elastic potential energy stored when the spring is stretched a distance x ?

- (A) bx
- (B) $\frac{1}{2}bx^2$
- (C) $\frac{1}{3}bx^3$

(D) $\frac{1}{6}bx^3$



(E) $\frac{1}{2}bx^3$

13.

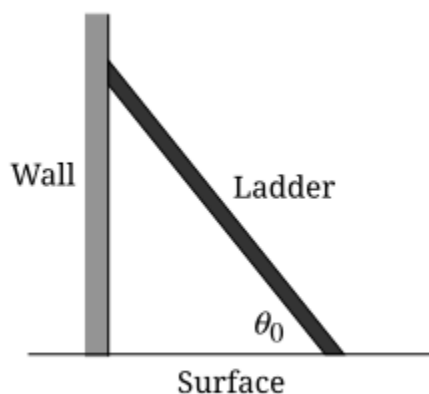
<u>Time in Orbit (days)</u>	<u>Period of Oscillation (s)</u>
10	35.02
20	34.80
30	33.59
40	32.10
60	31.82
80	31.56

An astronaut orbits in a space station above Earth's surface. The astronaut's mass is determined by having the astronaut oscillate on a chair that is attached to an ideal spring and measuring the period of oscillation. The period of oscillation is measured on different days while the astronaut is in the space station, as shown in the data table. Which of the following claims is supported by the data across the 80-day duration?

2023 Form I

- (A) The mass of the astronaut increases because the period increases.
- (B) The mass of the astronaut increases because the period decreases.
- (C) The mass of the astronaut decreases because the period increases.
- (D) The mass of the astronaut decreases because the period decreases. ✓
- (E) A change in mass cannot be determined because the astronaut is weightless in space.

14.

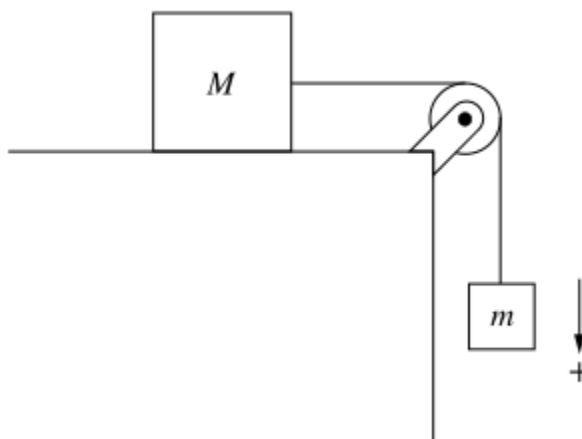


A ladder of uniform mass density and mass m_0 rests against a vertical wall, as shown. The coefficient of static friction between the ladder and the horizontal surface is μ_0 . The frictional force between the wall and the ladder is considered to be negligible. What is the minimum angle θ_0 at which the ladder remains at rest?

- (A) $\tan^{-1} \left(\frac{1}{4\mu_0} \right)$
- (B) $\tan^{-1} \left(\frac{1}{2\mu_0} \right)$ ✓
- (C) $\tan^{-1} \left(\frac{1}{\mu_0} \right)$
- (D) $\tan^{-1} (2\mu_0)$
- (E) $\tan^{-1} (4\mu_0)$

2023 Form I

15.



A block of mass M on a horizontal table is attached by a string that passes over a pulley to a suspended block of mass m , as shown. All frictional forces are negligible, and the string and pulley have negligible mass. A student investigates the acceleration a of the suspended block for different mass ratios $\frac{M}{m}$ when the system is released from rest, where the positive direction is defined as shown. Which of the following statements is a correct prediction for the relationship between a and $\frac{M}{m}$?

(A) $a < g$ for all values of $\frac{M}{m}$



(B) $a = 0$ if $\frac{M}{m} > 1$

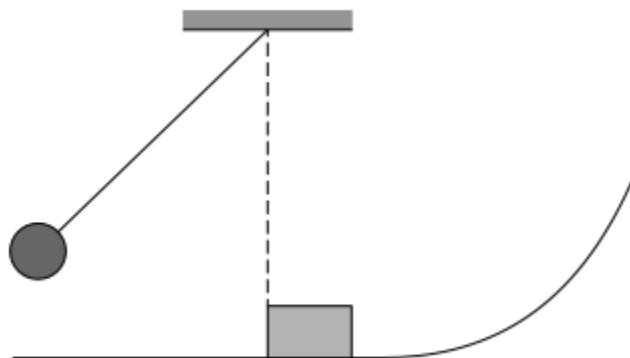
(C) $a < 0$ if $\frac{M}{m} > 1$

(D) $a = g$ if $\frac{M}{m} = 1$

(E) $a > g$ if $\frac{M}{m} < 1$

2023 Form I

16.



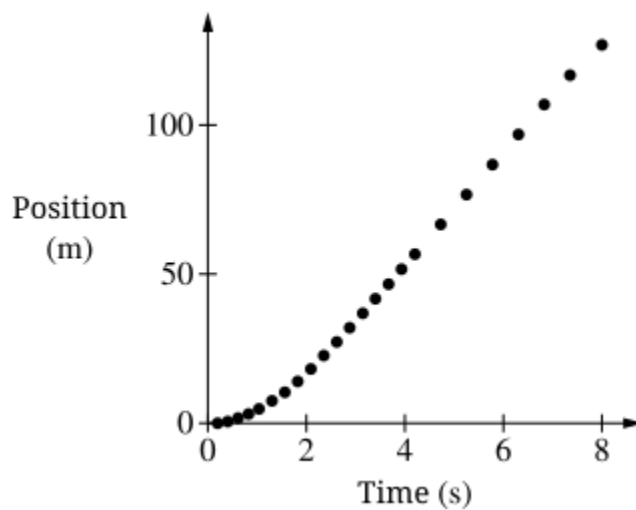
A simple pendulum is made of a sphere of unknown mass tied to a long string of negligible mass, as shown. The sphere is released from rest from an initial height and allowed to swing down freely. The sphere strikes a stationary block of known mass at the bottom of the sphere's swing. After the collision, the sphere immediately comes to rest and the block moves to the right along a track of negligible friction that curves upward. The block travels along the track before momentarily coming to rest. Which of the following sets of measurements can be used to determine the mass of the sphere?

- (A) The change in height of the sphere and the speed of the sphere just before the sphere strikes the block
- (B) The speed of the block immediately after the collision and the change in height of the block when the block momentarily comes to rest
- (C) The change in height of the sphere and the change in height of the block when the block momentarily comes to rest
- (D) The length of the pendulum and the speed of the block immediately after the collision
- (E) The length of the pendulum and the speed of the sphere just before the sphere strikes the block



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17.



The position of a falling object is plotted as a function of time in the graph shown. Which of the following features of the graph can be used to support the claim that the object reaches terminal velocity?

(A) The slope of the curve during the time interval $t = 0$ to $t = 8$ s

(B) The slope of the curve during the time interval $t = 0$ to $t = 4$ s

(C) The slope of the curve during the time interval $t = 4$ s to $t = 8$ s



(D) The area under the curve during the time interval $t = 0$ to $t = 4$ s

(E) The area under the curve during the time interval $t = 4$ s and $t = 8$ s

18. Two objects, A and B, move along a horizontal line. Object A has mass 2 kg and initial velocity v_A . Object B has mass 3 kg and initial velocity v_B . Object A collides head-on with Object B. The objects stick together and move with final velocity v_{AB} . Which of the following are possible values for v_A , v_B , and v_{AB} ?

	v_A (m/s)	v_B (m/s)	v_{AB} (m/s)
A	4	-3	2
B	-5	1	3
C	4	-1	0
D	1	-4	-2
E	-2	3	4

(A) A

(B) B

(C) C

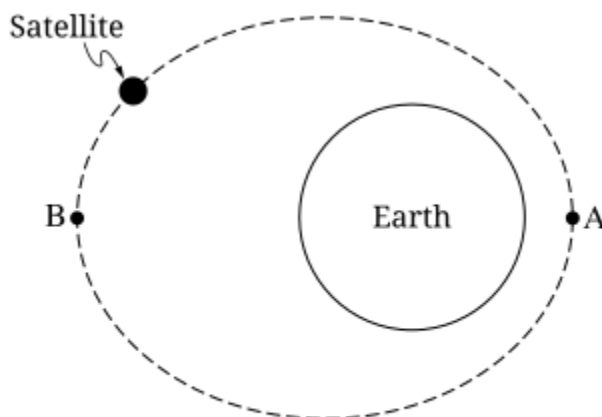
(D) D



(E) E

2023 Form I

19.



Note: Figure not drawn to scale.

A satellite of mass m_0 is in an elliptical orbit around Earth. Earth's mass is M and its radius is R . The satellite's distance r from the center of Earth varies. At Point A, $r = r_A$, and at Point B, $r = r_B$. Which of the following integrals can be used to determine the work done by Earth on the satellite as it moves from Point A to Point B?

(A) $\int_{r_A}^{r_B} m_0 g r dr$

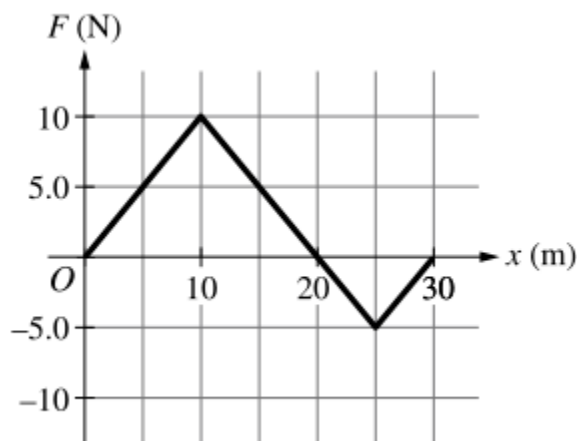
(B) $\int_{r_A}^{r_B} -\frac{GMm_0}{r^2} dr$

(C) $\int_{r_A}^{r_B} -\frac{GMm_0}{r} dr$

(D) $\int_R^{r_A} -\frac{GMm_0}{r^2} dr + \int_{r_A}^{r_B} -\frac{GMm_0}{r^2} dr$

(E) $\int_R^{r_A} -\frac{GMm_0}{r} dr + \int_{r_A}^{r_B} -\frac{GMm_0}{r} dr$

20.



The graph of a single conservative force F exerted on an object as a function of the position x of the object is shown. What is the work done on the object when it moves from $x = 0$ m to $x = 30$ m under the influence of only this force?

2023 Form I

- (A) -125 J
- (B) -75 J
- (C) Zero
- (D) 75 J
- (E) 125 J



A small disk is initially at rest. At time $t = 0$, the disk starts to spin around a fixed axis perpendicular to and through the center of the disk. The disk has uniform angular acceleration and reaches an angular speed of $20\pi \text{ rad/s}$ at time $t = 2.0 \text{ s}$.

21. What is the magnitude of the angular acceleration of the disk during the time interval from $t = 0$ to $t = 2.0 \text{ s}$?

- (A) $2\pi \text{ rad/s}^2$
- (B) $5\pi \text{ rad/s}^2$
- (C) $10\pi \text{ rad/s}^2$
- (D) $20\pi \text{ rad/s}^2$
- (E) $40\pi \text{ rad/s}^2$

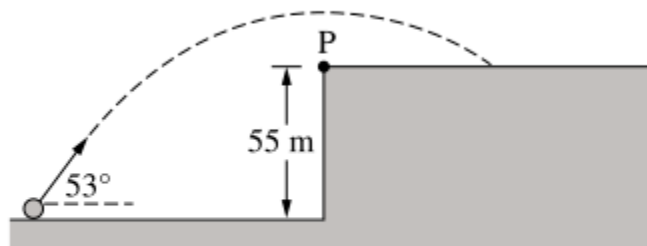


22. What is the linear speed of a point that is 0.030 m from the axis of rotation of the disk at time $t = 2.0 \text{ s}$?

- (A) $0.0015\pi \text{ m/s}$
- (B) $0.30\pi \text{ m/s}$
- (C) $0.6\pi \text{ m/s}$
- (D) $1.2\pi \text{ m/s}$
- (E) $10\pi \text{ m/s}$



2023 Form I



Note: Figure not drawn to scale.

An object is launched with an initial speed of 60 m/s at an angle of 53° above the horizontal. The object lands on top of a cliff that has a height of 55 m , as shown. The object reaches its maximum height when the object is directly above Point P at the edge of the cliff. Air resistance is considered to be negligible.

23. Which of the following is most nearly the distance above Point P at which the object reaches its maximum height?

- (A) 11 m
- (B) 48 m
- (C) 55 m

(D) 62 m

(E) 120 m



24. How long does it take for the object to travel from its initial position to its landing position on top of the cliff?

(A) 15.5 s

(B) 8.5 s

(C) 5.2 s

(D) 1.3 s

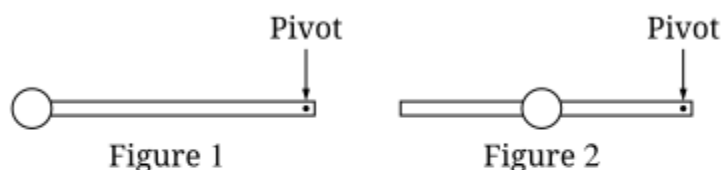
(E) 1.0 s



25. A second, identical object is launched with an initial speed of 70 m/s at an angle of 53° above the horizontal at the same time that the original object is launched. Which of the following is a correct statement about the motion of the objects after the objects are launched?

2023 Form I

- (A) Both objects have the same speed when each object is at its maximum height.
- (B) The second object reaches its maximum height in less time than it takes for the original object to reach its maximum height.
- (C) The horizontal displacement from the launch position to the landing position is smaller for the second object than it is for the original object.
- (D) The vertical displacement from the launch position to the landing position is greater for the second object than it is for the original object.
- (E) The total distance traveled from the launch position to the landing position is greater for the second object than it is for the original object. ✓



Side View

Note: Figure not drawn to scale.

The figures show two identical thin rods of negligible mass and length d that are able to freely rotate in a vertical plane about pivot points at the right ends of the rods. The rod in Figure 1 has a small sphere attached at its left end. The rod in Figure 2 has an identical sphere attached at its center. The rods are released from rest and begin to rotate under the influence of gravity.

26. Which of the following is a true statement about the linear speed of the spheres as each sphere swings through its lowest point?
- (A) The sphere in Figure 1 will have a greater linear speed than the sphere in Figure 2 will have. ✓
- (B) The sphere in Figure 1 will have a smaller linear speed than the sphere in Figure 2 will have.
- (C) The sphere in Figure 1 will have the same linear speed as the sphere in Figure 2 will have.
- (D) The linear speeds cannot be compared without knowing the masses of the spheres.
- (E) The linear speeds cannot be compared without knowing the lengths of the rods.
27. For the system consisting of the thin rod and small sphere shown in Figure 1, which of the following is an expression for the angular momentum of the system as the sphere swings through its lowest point?

2023 Form I

- (A) $\sqrt{m^2gd^2}$
(B) $\sqrt{m^2gd^3}$
(C) $\sqrt{2m^2gd}$
(D) $\sqrt{2m^2gd^2}$
(E) $\sqrt{2m^2gd^3}$

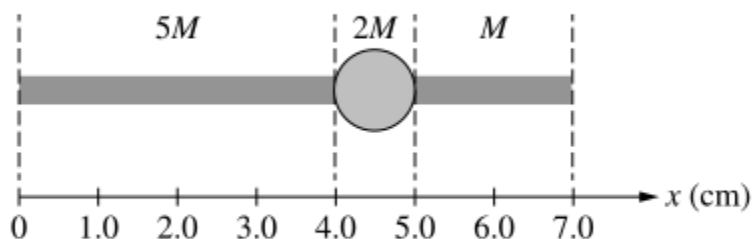


28. The initial angular acceleration of the rod-sphere system in Figure 1 is α_1 . The initial angular acceleration of the rod-sphere system in Figure 2 is α_2 . Which of the following correctly compares α_1 and α_2 ?

- (A) $\alpha_1 > \alpha_2$
(B) $\alpha_1 < \alpha_2$
(C) $\alpha_1 = \alpha_2$
(D) The initial angular accelerations cannot be compared without knowing the masses of the spheres.
(E) The initial angular accelerations cannot be compared without knowing the lengths of the rods.



29.



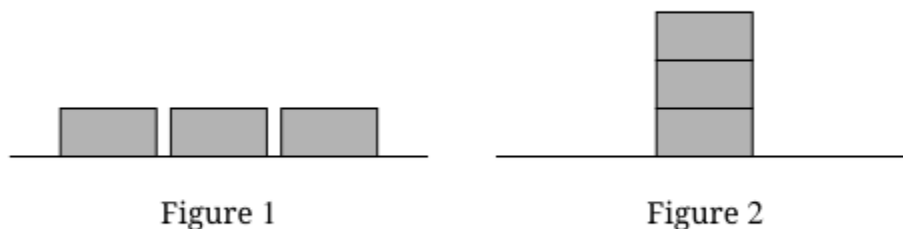
Two uniform thin rods are connected to opposite sides of a uniform sphere, as shown. The rod on the left has mass $5M$ and length 4.0 cm, the rod on the right has mass M and length 2.0 cm, and the sphere has mass $2M$ and diameter 1.0 cm. The center of the sphere and the axis of both rods lie on the same straight horizontal line. What is the x -coordinate of the center of mass of the three-object system?

- (A) 2.0 cm
(B) 3.1 cm
(C) 3.5 cm
(D) 4.2 cm
(E) 4.5 cm



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30.

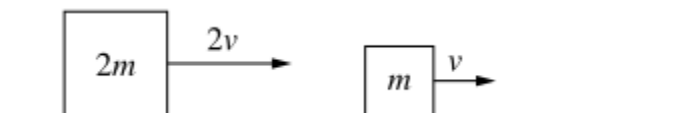


Three bricks rest on a level, horizontal surface, as shown in Figure 1. The mass of each brick is 2 kg. Each brick is 5 cm tall and 10 cm wide. Which of the following is the minimum amount of work that must be done to make a stack of the three bricks as shown in Figure 2?

- (A) 1 J
- (B) 2 J
- (C) 3 J
- (D) 4 J
- (E) 5 J

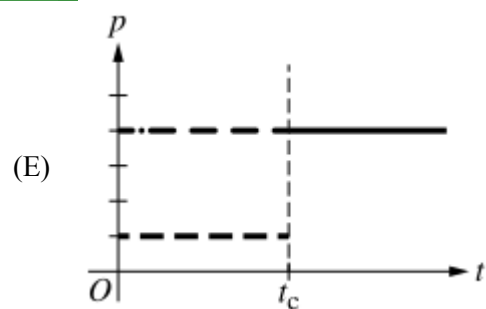
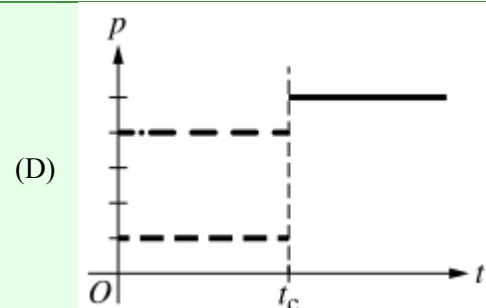
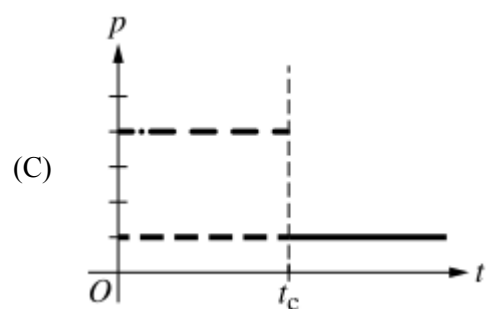
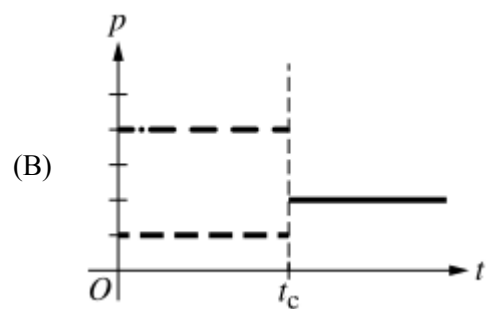
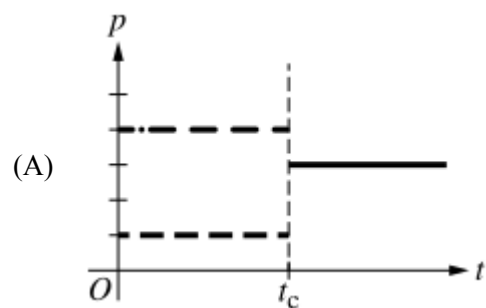


31.



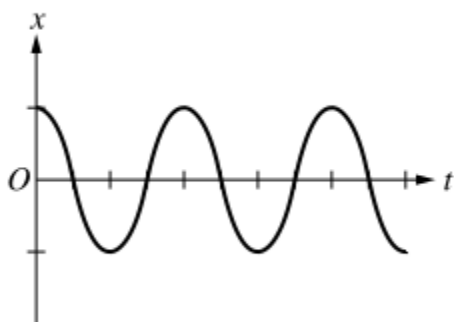
A block of mass $2m$ moving with velocity $+2v$ collides with and sticks to a block of mass m that is initially moving with velocity $+v$, as shown. Given that the collision takes place at time $t = t_c$, which of the following graphs most accurately illustrates the momentum p of the two blocks as a function of time? Each dashed horizontal line represents the momentum of an individual block, and the solid horizontal lines represent the combined momentum of both blocks after they collide.

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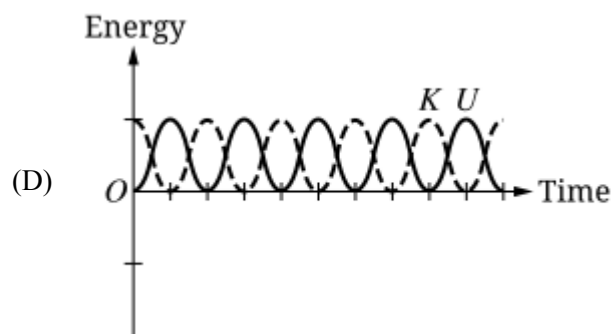
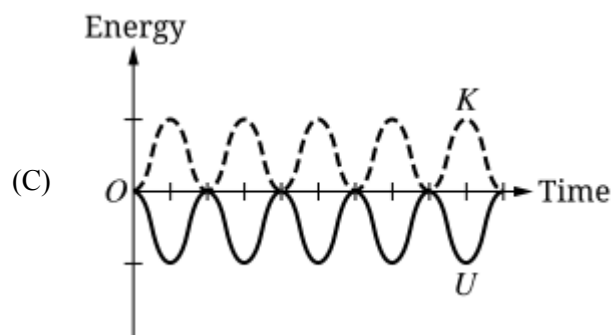
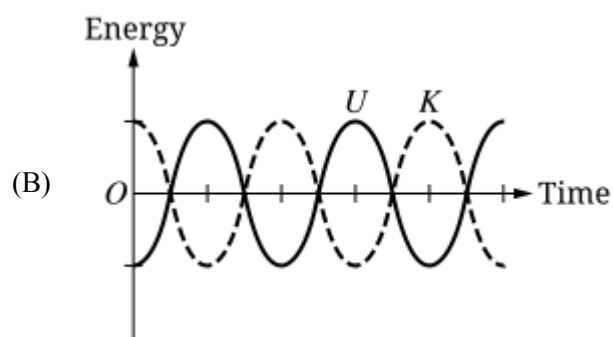
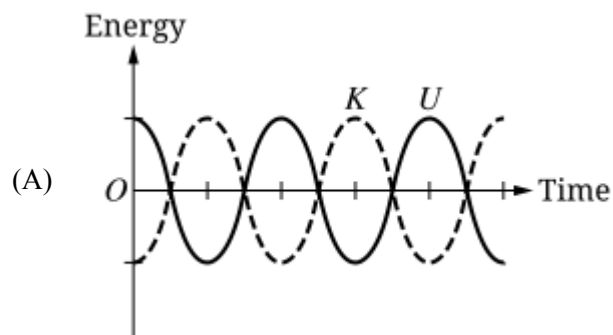
2023 Form I

32.

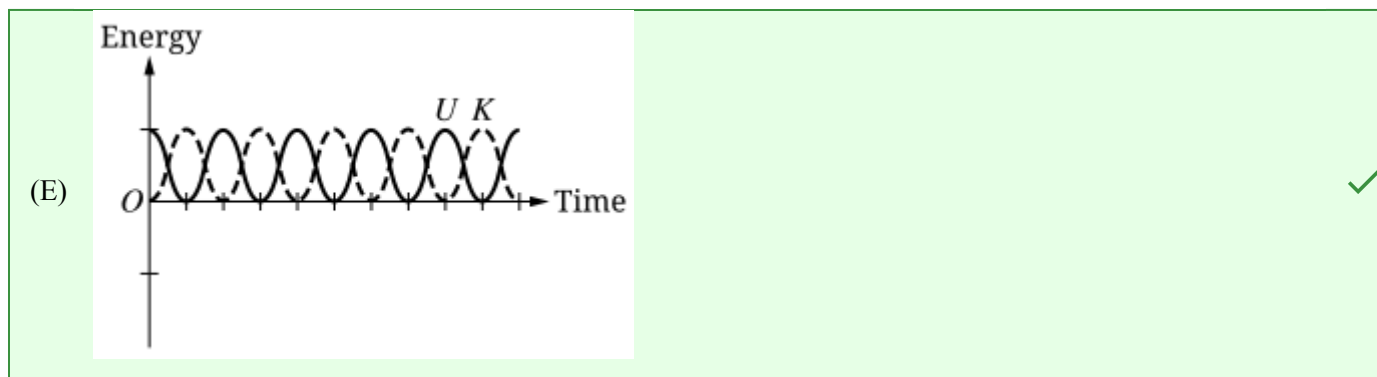


The graph shows the displacement x from equilibrium as a function of time t for a block oscillating on the end of a spring on a smooth horizontal surface, starting at time $t = 0$. Which of the following graphs correctly represents the potential energy U (represented by a solid line) and the kinetic energy K (represented by a dashed line) of the block-spring system as a function of time, starting at $t = 0$?

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2023 Form I



33.

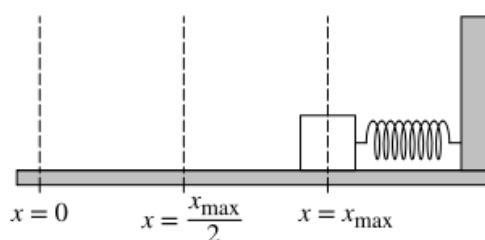


Figure 1

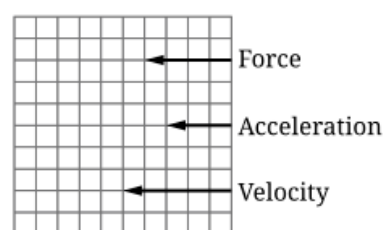
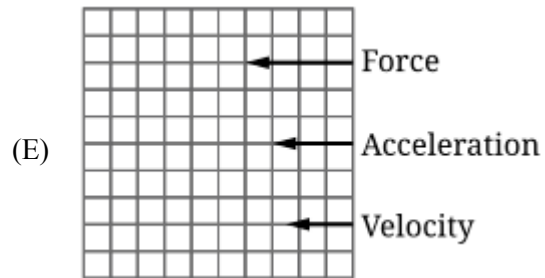
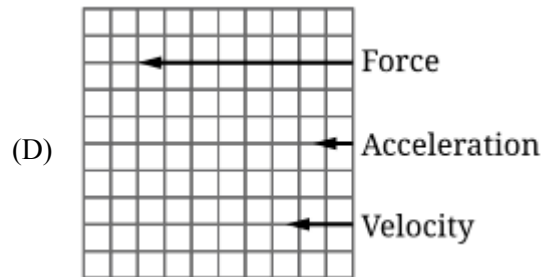
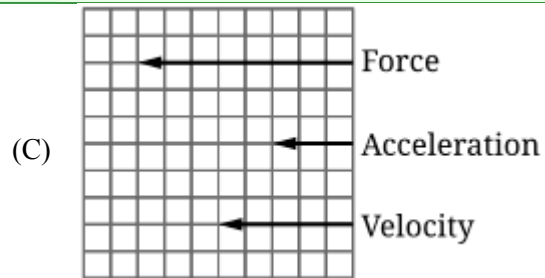
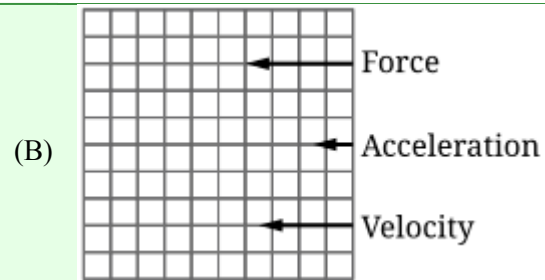
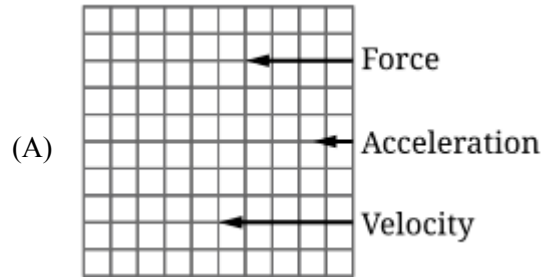


Figure 2

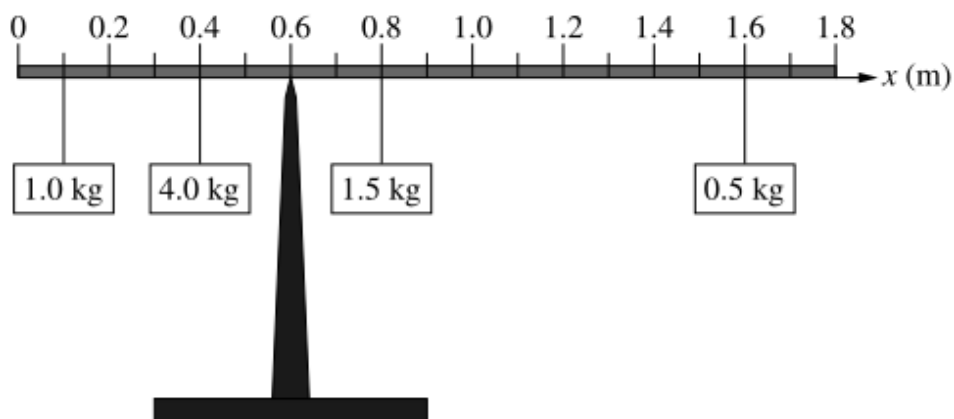
A block of mass m is attached to an ideal spring and displaced a distance $x_{\max}$ from the equilibrium position at $x = 0$, as shown in Figure 1. There is negligible friction between the block and the horizontal surface. The block is released from rest, and the relative magnitudes of the force, acceleration, and velocity vectors for the block when the block is located at $x = \frac{x_{\max}}{2}$ are shown in Figure 2. The scenario is repeated with a new block of mass $2m$. Which of the following diagrams correctly shows the relative magnitudes of the force, acceleration, and velocity vectors when the new block is located at $x = \frac{x_{\max}}{2}$?

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2023 Form I

34.



Note: Figure not drawn to scale.

A rod of length 1.8 m has negligible mass and is pivoted at a point 0.6 m from the left end of the rod, as shown. Four different objects are hung at the locations shown. Which object, if any, should be removed so that the rod will be in equilibrium?

(A) The 0.5 kg object

(B) The 1.0 kg object

(C) The 1.5 kg object

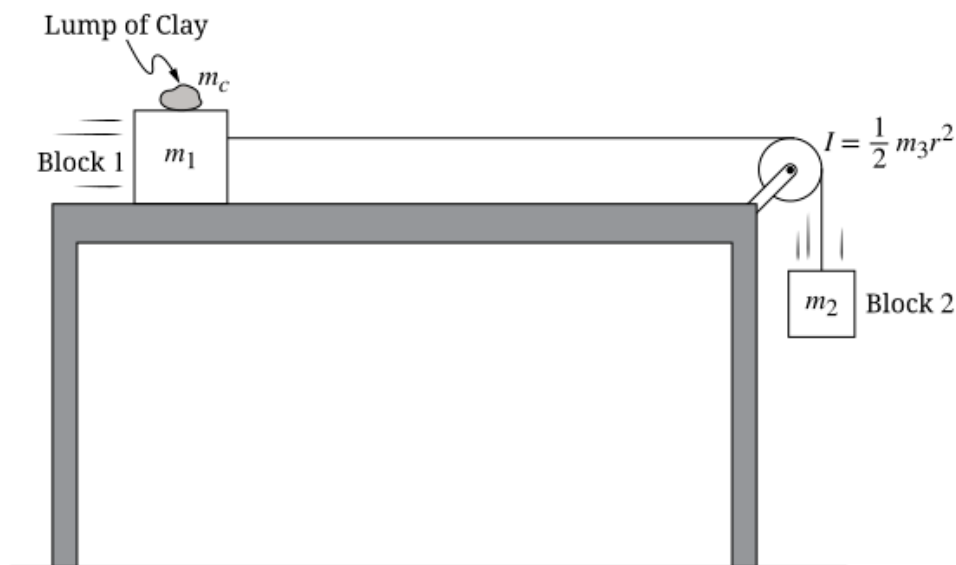
(D) The 4.0 kg object

(E) None of the objects should be removed.



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35. Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.



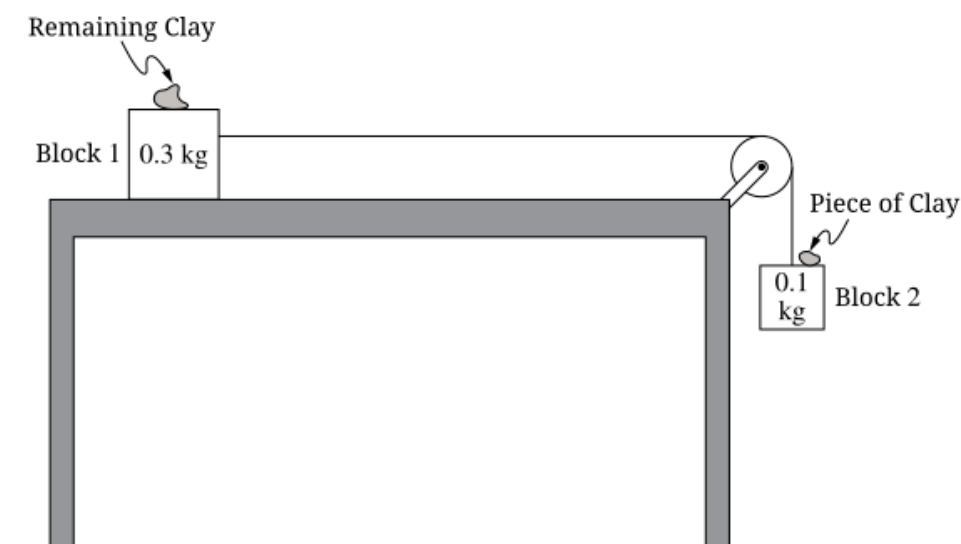
A string connects Block 1 of mass m_1 and Block 2 of mass m_2 . The string passes over a pulley so that Block 2 hangs vertically, as shown. The pulley is a disk with mass m_3 and radius r ; the rotational inertia of the pulley is $I = \frac{1}{2}m_3r^2$. A lump of clay of mass m_c is attached to the top of Block 1. The system consisting of the two blocks, pulley, and clay is released from rest and accelerates. Frictional forces are negligible at the axle of the pulley and between Block 1 and the surface. The string has negligible mass.

- (a) The two dots in the following figure represent Block 1 and Block 2. Draw and label the forces (not components) that act on Block 1 and Block 2 once the blocks-pulley-clay system has been released and is accelerating. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

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(b) Derive an expression for the magnitude of the acceleration of the blocks-pulley-clay system immediately after the system is released from rest. Express your answer in terms of m_1 , m_2 , m_c , m_3 , r , and fundamental constants, as appropriate.



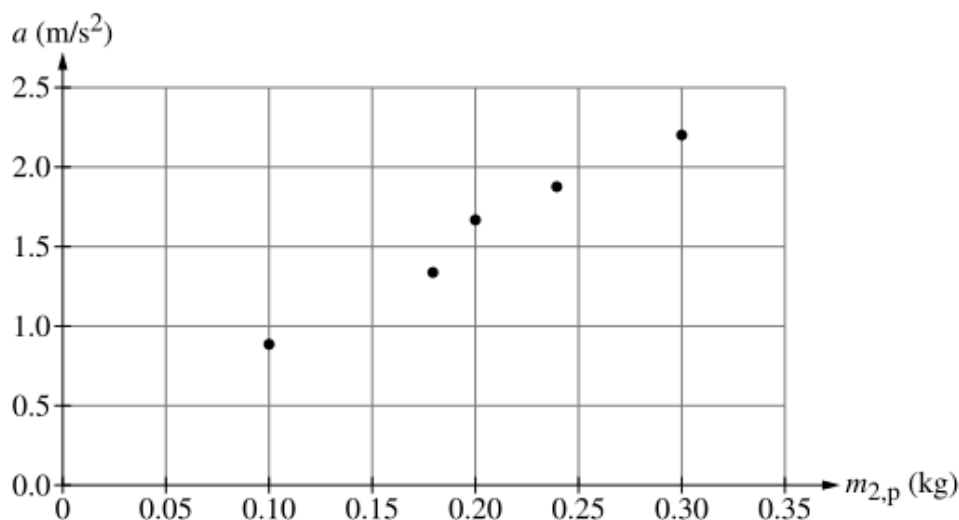
(c) A student uses the setup from part (a) to determine an experimental value for the mass m_3 of the pulley. The student uses the following procedure.

- The student measures the masses of the blocks and clay: $m_1 = 0.3 \text{ kg}$, $m_2 = 0.1 \text{ kg}$, and $m_c = 0.25 \text{ kg}$.
- The student removes a small piece of clay from the lump of clay on the top of Block 1 and attaches the piece of clay to the top of Block 2, as shown. The student measures the mass $m_{2,p}$ of the system consisting of Block 2 and the attached piece of clay.
- The student releases the blocks-pulley-clay system from rest and measures the acceleration a of this system.

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- The student conducts multiple trials with different values for $m_{2,p}$.

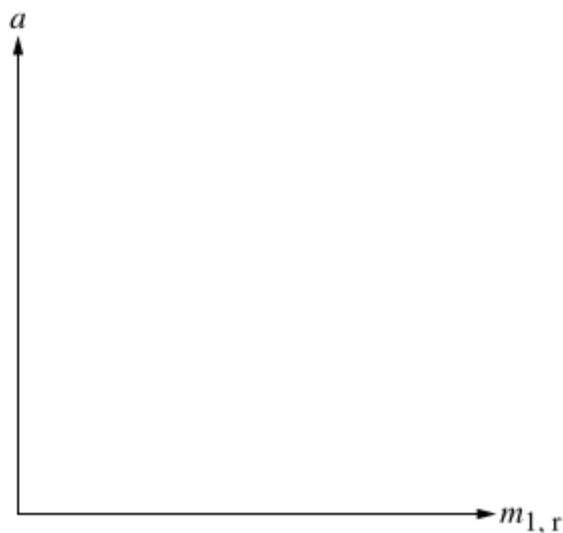
The student plots the data for the acceleration a of the blocks-pulley-clay system as a function of the mass $m_{2,p}$ in the following graph.



i. Draw the best-fit line for the data.

ii. Use the best-fit line to determine an experimental value for the mass m_3 of the pulley.

(d) The student repeats the experiment described in part (c), but now the student removes the pieces of clay from the system instead of attaching the pieces to Block 2. The student measures the mass $m_{1,r}$ of the system consisting of Block 1 and the remaining clay. On the following axes, sketch the magnitude of the acceleration of the system consisting of the blocks, pulley and remaining clay as a function of $m_{1,r}$.



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(e) The student repeats the experiment described in part (c), using a different pulley with the same mass, radius, and rotational inertia but with nonnegligible friction at the axle. The student's analysis neglects to account for the friction at the axle of the pulley. Will the new experimental value for the mass of the pulley m_3 be greater than, less than, or equal to the value for the mass of the pulley determined in part (c)?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



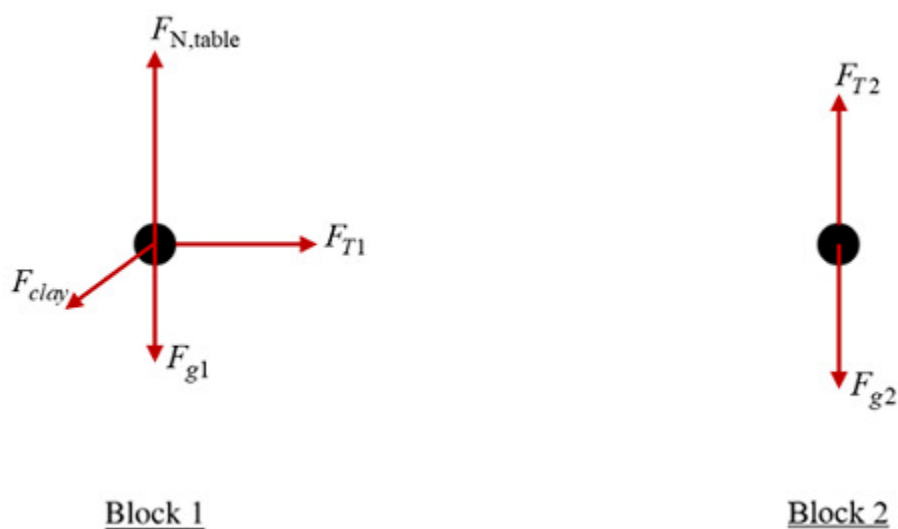
0	1	2	3
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The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for drawing and labeling a gravitational force on each block.
- ☐ 1 point is earned for drawing and labeling a tension force to the right on Block 1 and a tension force up on Block 2.
- ☐ 1 point is earned for drawing and labeling a force from the clay that is down and to the left.

Scoring Note: If the response indicates two separate forces from the clay with one down and one to the left, the response will earn this point.

Example Response



Scoring Note: Examples of appropriate labels for the force due to gravity include F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “F Earth on block,” “F on block by Earth,” $F_{\text{Earth on Block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$. The labels G or g are not appropriate labels for the force due to gravity. F_n , F_N , N , “normal force”, “table force”, or similar labels may be used for

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the normal force. F_{string} , F_s , F_T , F_{Tension} , “string force,” “tension force” or similar labels may be used for the tension force exerted by the string.

Scoring Note: The forces do not have to be drawn to scale. If extraneous forces are present, a maximum of 2 points can be earned.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3
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The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for **one** of the following:
- a correct expression for the net force on each block
 - a correct expression for the net force on the blocks-clay-pulley system

Scoring Note: Students may represent the difference between the tension in the horizontal section of string and the tension in the vertical section of string with a single clearly labeled variable and earn this point.

- ☐ 1 point is earned for correctly using the rotational form of Newton’s second law to relate the angular motion of the pulley to the tensions in the horizontal and vertical sections of the string.
- Scoring Note:** Response does not have to start with $\Sigma\tau = I\alpha$ but must substitute correct expressions for net torque and angular acceleration consistent with the chosen method to analyze the motion of the system.
- ☐ 1 point is earned for an expression for the magnitude of the acceleration of the blocks-clay system in terms of the given variables that is consistent with previous work.

Example Response

$$\Sigma F = ma$$

$$T_1 = (m_1 + m_c)a$$

$$m_2g - T_2 = m_2a$$

OR

$$m_2g - (T_2 - T_1) = (m_1 + m_2 + m_c)a$$

$$\Sigma\tau = I\alpha$$

$$(T_2 - T_1)r = I\alpha$$

$$(T_2 - T_1)r = \frac{1}{2}m_3r^2\left(\frac{a}{r}\right)$$

$$(T_2 - T_1) = \frac{1}{2}m_3a$$

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$$a = \frac{m_2 g}{m_1 + m_c + \frac{1}{2} m_3 + m_2}$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

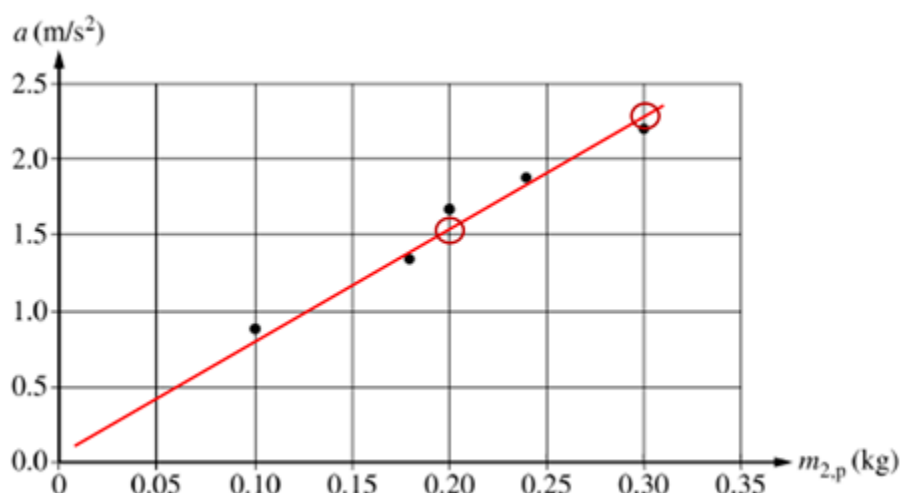


0	1	2	3	4
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The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for drawing an appropriate line or curve of best fit that approximates the data.
- ☐ 1 point is earned for correctly calculating the slope of the best-fit line using two points on the line.
- ☐ 1 point is earned for an indication that the slope of the best-fit line represents the ratio of g and the sum of the mass of the two blocks, the clay, and half the mass of the pulley or a ratio consistent with part (b).
- ☐ 1 point is earned for calculating m_3 using the calculated slope of the best-fit line that is between 1.05 kg and 2.13 kg.

Example Response



$$\text{slope} = \frac{\Delta a}{\Delta m_{2,p}}$$

$$\text{slope} = \frac{2.25 \text{ m/s}^2 - 1.5 \text{ m/s}^2}{0.30 \text{ kg} - 0.20 \text{ kg}}$$

$$\therefore \text{slope} = 7.5 \frac{\text{m}}{\text{kg} \cdot \text{s}^2}$$

$$\text{slope} = \frac{\Delta a}{\Delta m_{2,p}}$$

$$\text{slope} = \frac{g}{m_1 + m_c + \frac{1}{2} m_3 + m_2}$$

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$$m_3 = \frac{2g}{\text{slope}} - 2(m_1 + m_c + m_2)$$

$$m_3 = \frac{2(9.8 \text{ m/s}^2)}{\frac{2.25 \text{ m/s}^2 - 1.5 \text{ m/s}^2}{0.30 - 0.20}} - 2(0.3 \text{ kg} + 0.1 \text{ kg} + 0.25 \text{ kg})$$

$$\therefore m_3 = 1.3 \text{ kg}$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

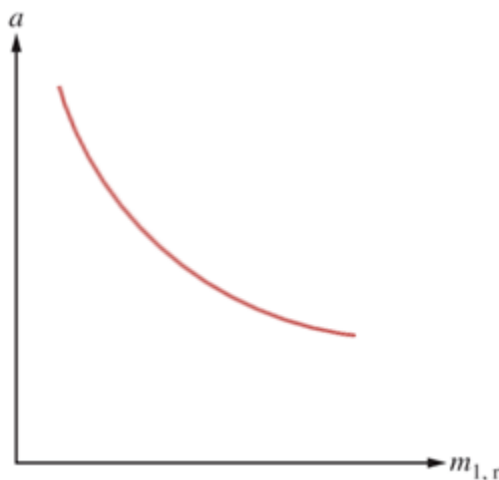


0	1	2
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The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for drawing a sketch that is decreasing.
- ☐ 1 point is earned for drawing a curve that is concave up.

Example Response



Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3
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The response accurately includes **all** of the following criteria.

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- ☐ 1 point is earned for selecting “greater than” with an attempt at a justification.
- ☐ 1 point is earned for a statement that indicates that the friction on the axle decreases the net force exerted on the system.
- ☐ 1 point is earned for correctly relating the net force exerted on the system to the acceleration of the system.

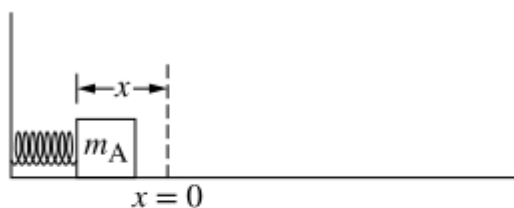
Example Response

Friction between the axle and pulley exerts a torque on the pulley that opposes its motion, reducing the net force accelerating the blocks-clay-pulley system. Since the net force is less, the acceleration of the system is also less. Since the acceleration of the system is less and is inversely related to the mass of the system, the experimental value for the mass of the pulley is greater.

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36. Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.

Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.



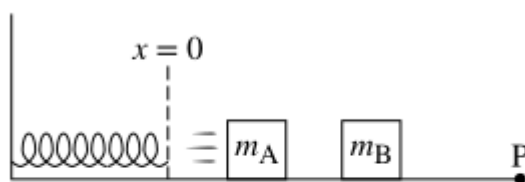
Note: Figure not drawn to scale.

Figure 1

Block A has a mass of $m_A = 0.020$ kg and sits at rest on a horizontal surface. Friction between the block and the surface is negligible. The block is pushed against a horizontal spring that is attached to a wall. The spring is compressed a distance of $x = 0.040$ m, as shown in Figure 1. The force exerted on the block by the spring has magnitude $F(x) = \beta x^{1/2}$, where $\beta = 15$ N/m^{1/2}.

- (a) On the following axes, sketch a graph of the force exerted on the block by the spring as a function of horizontal position while the spring is being compressed. Explicitly label the magnitude of the force when the spring is compressed a distance of 0.040 m.

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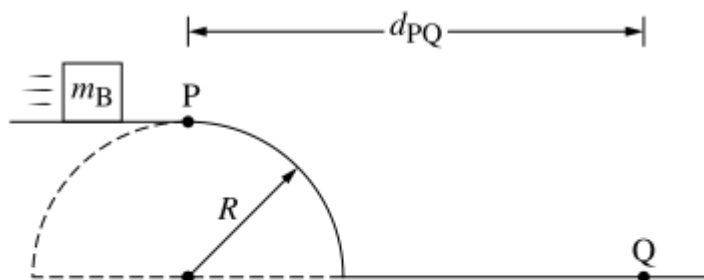
Note: Figure not drawn to scale.

Figure 2

(b) Block A is released from rest and slides horizontally toward an identical Block B ($m_A = m_B$), which is initially at rest, as shown in Figure 2. Frictional forces between each block and the horizontal surface are negligible. Block A collides elastically with Block B, causing Block B to slide toward Point P.

- Calculate the speed v_A of Block A immediately before the collision.
- Determine the speed v_B of Block B immediately after the collision.

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Note: Figure not drawn to scale.

Figure 3

(c) At Point P the surface begins to follow a circular arc of radius R . A smooth horizontal surface is located a vertical distance R below Point P, as shown in Figure 3.

- At Point P, Block B loses contact with the surface. Calculate the maximum value of the radius $R_{\max}$ for which Block B loses contact with the surface.
- When $R = R_{\max}$, Block B loses contact with the surface at Point P and lands at Point Q. The horizontal distance between Point P and Point Q is d_{PQ} . Calculate d_{PQ} .

(d) Block B is removed and replaced with Block C, which has a mass less than that of Block B. Block C is placed before Point P, and Block A is again released from rest after compressing the spring the same distance as before. Block A collides elastically with Block C. Block C then loses contact with the surface and lands at Point S on the horizontal surface. The horizontal distance between Point P and Point S is d_{PS} . Frictional forces are negligible. Which of the following correctly compares d_{PQ} and d_{PS} ?

_____ $d_{PQ} > d_{PS}$ _____ $d_{PQ} = d_{PS}$ _____ $d_{PQ} < d_{PS}$

Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2
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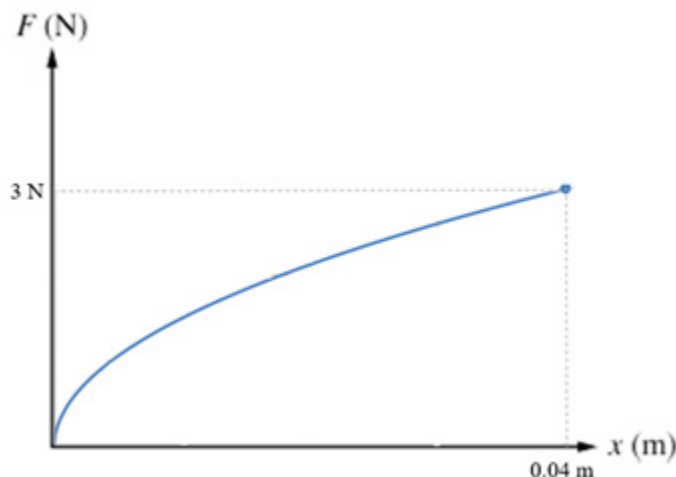
The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a curve that starts at the origin and has decreasing slope.
- ☐ 1 point is earned for indicating that the spring exerts a 3 N force on the block when the spring is

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compressed a distance of 0.040 m.

Example Response



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3	4	5
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The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for a derivation that indicates that work or change in potential energy is the integral of the given force function with respect to position.
- ☐ 1 point is earned for correctly integrating the force function.
- ☐ 1 point is earned for equating work or change in elastic potential energy with change in kinetic energy.
- ☐ 1 point is earned for correctly combining equations and substituting values.
- ☐ 1 point is earned for indicating that speed of Block B after the collision equals speed of Block A before the collision, as calculated in part (b)(i).

Scoring Note: Calculations do not have to be explicitly shown to earn this point.

Example Response

$$W = \int F(x)dx = \int (15 \text{ N/m}^{1/2})x^{1/2}dx$$

$$W = (15 \text{ N/m}^{1/2})\left(\frac{2}{3}\right)x^{3/2}$$

$$W = (10 \text{ N/m}^{1/2})x^{3/2}$$

$$W = \frac{1}{2}mv^2 - 0$$

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$$v = \sqrt{\frac{2W}{m}}$$

$$= \sqrt{\frac{2(10 \text{ N/m}^{1/2})(0.040 \text{ m})^{3/2}}{0.020 \text{ kg}}}$$

$$\therefore v = 2.8 \text{ m/s}$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3	4	5
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The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for correctly applying Newton's second law.
 - ☐ 1 point is earned for correctly concluding that normal force is zero when Block B loses contact with the surface.
- Scoring Note:** Responses that start with $mg = \frac{mv^2}{R}$ earn the first two points.
- ☐ 1 point is earned for correctly substituting values consistent with part (b)(ii).
 - ☐ 1 point is earned for correctly using kinematics equations in the vertical direction to calculate the time Block B is in the air.
 - ☐ 1 point is earned for a correctly using kinematics equations in the horizontal direction to calculate the range of Block B, consistent with calculation for the time Block B is in the air and with the velocity from (b)(ii).

Example Response

$$F_N - mg = -\frac{mv^2}{R}$$

$$0 - mg = -\frac{mv^2}{R}$$

$$R = \frac{v^2}{g}$$

$$R = \frac{(2.8 \text{ m/s})^2}{9.8 \text{ m/s}^2}$$

$$F_N - mg = -\frac{mv^2}{R}$$

$$0 - mg = -\frac{mv^2}{R}$$

$$\rightarrow R = \frac{v^2}{g}$$

$$R = \frac{(2.8 \text{ m/s})^2}{9.8 \text{ m/s}^2}$$

$$\therefore R = 0.80 \text{ m}$$

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$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$R = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2R}{g}} = \sqrt{\frac{2(0.80 \text{ m})}{9.8 \text{ m/s}^2}}$$

$$t = 0.40 \text{ s}$$

$$d_{\text{PQ}} = v_x t$$

$$d_{\text{PQ}} = (2.8 \text{ m/s})(0.4 \text{ s})$$

$$\therefore d_{\text{PQ}} = 1.1 \text{ m}$$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3
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The response accurately includes **all** the following criteria.

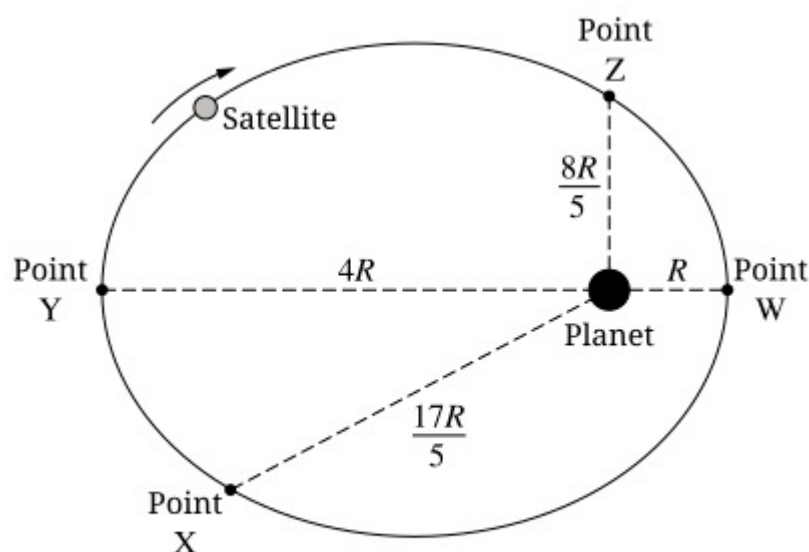
- ☐ 1 point is earned for selecting $d_{\text{PS}} > d_{\text{PQ}}$ with an attempt at a relevant justification.
- ☐ 1 point is earned for indicating the speed of Block C is greater than that of Block B after the collision.
- ☐ 1 point is earned for correctly using kinematics to reason that distance is greater because the velocity is greater for Block C and time in the air is the same for both blocks.

Example Response

After the collision, Block C has a higher speed than Block B did due to Block C's smaller mass. Gravity has the same effect on both blocks, so they spend the same amount of time in the air before landing. Block C has a greater horizontal velocity than Block B did so it covers more horizontal distance than Block B before landing.

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37. Read each question carefully. Show all your work for each part of the question. The parts within the question may not have equal weight.



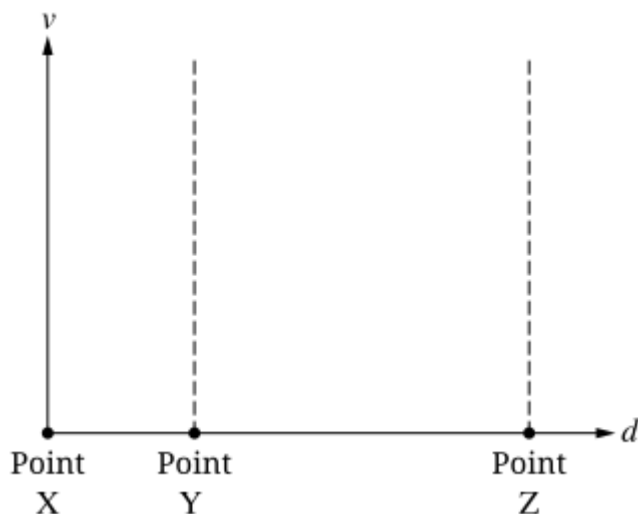
Note: Figure not drawn to scale.

A satellite of mass m_0 orbits a planet of mass M . The orbit is elliptical in shape and passes through points W, X, Y, and Z, located at distances R , $\frac{17R}{5}$, $4R$, and $\frac{8R}{5}$, respectively, from the center of the planet, as shown. The satellite travels in a clockwise direction in the orbit shown. The distance traveled by the satellite in a complete orbit is D . The distance traveled between points X and Z is $\frac{D}{2}$. The time it takes for the satellite to travel from Point X to Point Z is Δt_1 , and the time it takes for the satellite to travel from Point Z to Point X is Δt_2 .

(a)

- i. On the following set of axes, sketch the linear speed v of the satellite as a function of distance d traveled by the satellite as it travels from Point X through Point Y to Point Z.

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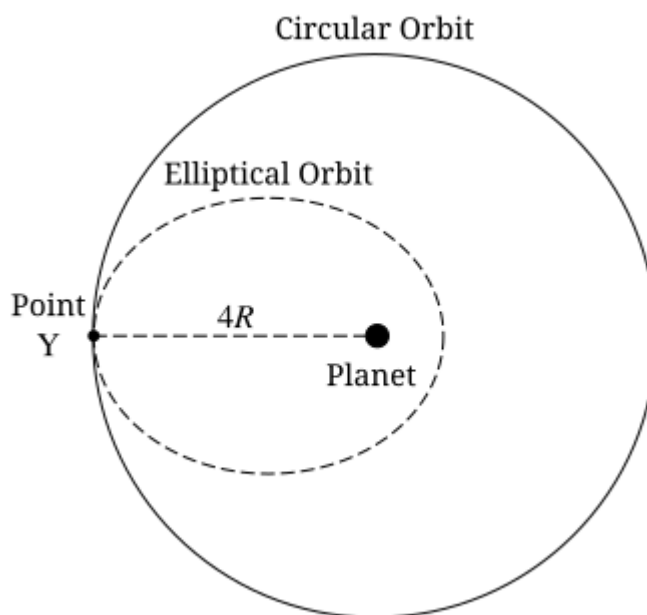


ii. Is Δt_2 greater than, less than, or equal to Δt_1 ?

_____ $\Delta t_2 > \Delta t_1$ _____ $\Delta t_2 < \Delta t_1$ _____ $\Delta t_2 = \Delta t_1$

Justify your answer.

(b) The linear speed of the satellite at points W and Y are v_W and v_Y , respectively. Derive an expression for the linear speed v_W . Express your answer in terms of v_Y , m_0 , M , R , and physical constants, as appropriate.



(c) The satellite's propulsion system exerts a force of magnitude F on the satellite for a short time Δt_0 to change

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the orbit of the satellite at Point Y from the elliptical orbit to the circular orbit shown.

- Derive an expression for the linear speed of the satellite $v_{Y,c}$ at Point Y in the circular orbit in terms of m_0 , M , R , and physical constants, as appropriate.
- Derive an expression for the linear speed of the satellite $v_{Y,e}$ at Point Y in the elliptical orbit in terms of m_0 , M , R , F , Δt_0 , and physical constants, as appropriate.
- The magnitude of the angular momentum of the satellite at Point Y is L_c in the circular orbit and L_e in the elliptical orbit. How does L_c compare with L_e ?

_____ $L_c > L_e$ _____ $L_c = L_e$ _____ $L_c < L_e$

Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



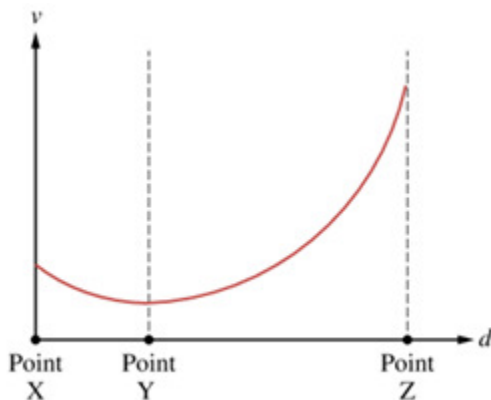
0	1	2	3	4	5	6
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The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a curve that shows that the speed of the satellite only decreases from Point X to Point Y.
- ☐ 1 point is earned for a curve that shows that the speed of the satellite only increases from Point Y to Point Z.
- ☐ 1 point is earned for both curves drawn as concave up.
- ☐ 1 point is earned for selecting $\Delta t_2 < \Delta t_1$ with an attempt at a relevant justification.
- ☐ 1 point is earned for **either** indicating that when the satellite is closer to the planet it has a higher linear velocity (higher kinetic energy) OR for indicating that when the satellite is closer to the planet it has a higher angular velocity.
- ☐ 1 point is earned for indicating that since the satellite travels the same distance at a higher average speed from Point Z to Point X, the satellite will take less time to travel that distance.

Example Response

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Scoring Note: All three points can be earned even if the curve is not continuous across Point Y.

From angular momentum conservation, the average speed would be larger closer to the planet ($v_{xz,avg} < v_{zx,avg}$). Since the distance travelled from Z to X is the same as the distance travelled from X to Z, the time taken will be less.

Alternate Solution

For indicating that the area swept by the satellite travelling from Z to X is smaller than the area swept from X to Z.

For indicating that since the area swept is smaller from Z to X, the satellite will take less time to travel that distance.

Alternate Example Solution

An object in an elliptical orbit covers equal areas in equal times. Since the area from X to Z is greater than Z to X, Δt_1 is greater than Δt_2 .

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2
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The response accurately includes **both** the following criteria.

- ☐ 1 point is earned for using conservation of angular momentum to equate the angular momentums of the satellite at two points.
OR
For using conservation of mechanical energy to equate the energies of the satellite at two points.
- ☐ 1 point is earned for a correct expression for the speed of the satellite at Point W.

Example Response

$$L = m_0 v_Y (4R) = m_0 v_W R$$

$$\therefore v_W = 4v_Y$$

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OR

$$E_Y = E_W$$

$$\frac{1}{2}m_0v_Y^2 - \frac{GMm_0}{r_Y} = \frac{1}{2}m_0v_W^2 - \frac{GMm_0}{r_W}$$

$$\frac{1}{2}v_Y^2 - \frac{GM}{4R} = \frac{1}{2}v_W^2 - \frac{GM}{R}$$

$$v_Y^2 - \frac{GM}{2R} = v_W^2 - \frac{2GM}{R}$$

$$v_W^2 = v_Y^2 - \frac{GM}{2R} + \frac{2GM}{R} = v_Y^2 + \frac{3GM}{2R}$$

$$\therefore v_W = \sqrt{v_Y^2 + \frac{3GM}{2R}}$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3	4	5	6	7
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The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for equating gravitational force with centripetal force.
- ☐ 1 point is earned for a correct expression for the linear speed of the satellite in the circular orbit.
- ☐ 1 point is earned for equating the impulse delivered to the satellite by the propulsion system to the change of momentum of the satellite.

OR

For using appropriate kinematic relationships with Newton's second law.

- ☐ 1 point is earned for solving for speed in the elliptical orbit and substituting the expression for speed in a circular orbit, consistent with response in (c)(i).
- ☐ 1 point is earned for selecting $L_c > L_e$ with an attempt at a relevant justification.
- ☐ 1 point is earned for indicating that one of the following changes has taken place in the system:
 - that the mass of the satellite and the distance between the satellite and planet are the same at Point Y in both orbits and that the linear speed changes
 - that the rotational inertia of the satellite about the planet is the same at Point Y in both orbits and that the angular speed changes
 - that the propulsion system causes a torque, force, or impulse that changes the speed of the satellite
- ☐ 1 point is earned for stating that the angular or linear speed in the circular orbit is greater than in the elliptical orbit.

Example Response

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$$F_g = F_c$$

$$F_g = m_0 a_c$$

$$\frac{GMm_0}{(4R)^2} = m_0 \frac{v_{Y,c}^2}{4R}$$

$$F_g = m_0 a_c$$

$$\frac{GMm_0}{(4R)^2} = m_0 \frac{v_{Y,c}^2}{4R}$$

$$\therefore v_{Y,c} = \sqrt{\frac{GM}{4R}}$$

$$F\Delta t_0 = \Delta p$$

$$F\Delta t_0 = m_0(v_{Y,c} - v_{Y,e})$$

OR

$$v = v_0 + at$$

$$a = \frac{F_{net}}{m}$$

$$v_{Y,c} = v_{Y,e} + a\Delta t_0$$

$$a = \frac{F}{m_0}$$

$$v_{Y,e} = v_{Y,c} - \frac{F}{m_0}\Delta t_0$$

$$\Delta p = F\Delta t_0$$

$$\rightarrow F\Delta t_0 = m_0(v_{Y,c} - v_{Y,e})$$

$$v_{Y,e} = v_{Y,c} - \frac{F\Delta t_0}{m_0}$$

$$\therefore v_{Y,e} = \sqrt{\frac{GM}{4R}} - \frac{F\Delta t_0}{m_0}$$

OR

$$v = v_0 + at$$

$$a = \frac{F_{net}}{m}$$

$$v_{Y,c} = v_{Y,e} + a\Delta t_0$$

$$a = \frac{F}{m_0}$$

$$v_{Y,e} = v_{Y,c} - \frac{F}{m_0}\Delta t_0$$

$$\therefore v_{Y,e} = \sqrt{\frac{GM}{4R}} - \frac{F\Delta t_0}{m_0}$$

The mass and radial distance of the satellite to the planet are the same for the different orbits at Point Y, but the thruster increases the linear speed for the circular orbit and so the angular momentum for the circular orbit is greater.