



CALCULUS AB
SECTION I, Part A
Time—1 hour
30 Questions

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

1. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
2. The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).

A A A A A A A A A A A A A A A A A A A A A A A A A A A A

- $\frac{d}{dx} \left((x^2 - 11x + 24)^7 \right) =$

- (A) $7(2x - 11)^6$
 (B) $7(2x - 11)^6(x^2 - 11x + 24)$
 (C) $7(x^2 - 11x + 24)^6$
 (D) $7(x^2 - 11x + 24)^6(2x - 11)$

2. If $\int_1^k \frac{1}{\sqrt{x}} dx = 10$, where k is a constant, what is the value of k ?

- (A) $\frac{1}{121}$ (B) 36 (C) 121 (D) 441

3. The graph of the function f is shown on the interval $-3 \leq x \leq 8$. The function g is defined by

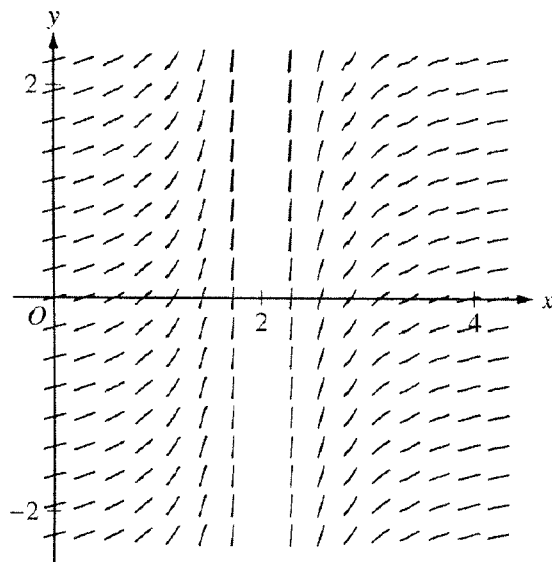
$g(x) = \int_0^x f(t) \, dt$. Which of the following statements is true?

- (A) g is decreasing for $2 < x < 8$.
(B) g is increasing for $1 < x < 6$.
(C) g has a relative maximum at $x = 1$.
(D) g is not differentiable at $x = 2$.

4. If $y = \sin(x^3)$, what is $\frac{d^2y}{dx^2}$?

- (A) $3x^2 \cos(x^3)$
(B) $9x^4 \cos^2(x^3)$
(C) $6x \cos(x^3) - 9x^4 \sin(x^3)$
(D) $-6x \cos(x^3) + 3x^2 \sin(x^3)$

AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA



5. The figure shown is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{y^2}{(x-2)^2}$

(B) $\frac{dy}{dx} = \frac{1}{(x-2)^3}$

(C) $\frac{dy}{dx} = \frac{1}{(x+2)^2}$

(D) $\frac{dy}{dx} = \frac{1}{(x-2)^2}$

6. The function f is defined by $f(x) = \frac{|\sin x|}{x^3 - 1}$. At how many values of x does f have a discontinuity?

- (A) Zero
- (B) One
- (C) Three
- (D) Infinite

7. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 0.3y$ with the initial condition $y(0) = 10$?

- (A) $y = 10e^{0.3x}$ (B) $y = e^{0.3x} + 9$ (C) $y = \sqrt{0.6x + 100}$ (D) $y = 0.15x^2 + 10$

8. $\lim_{h \rightarrow 0} \frac{e^{-(1+h)} - e^{-(1)}}{h}$ is

- (A) $\sec^3 x + \sec x \tan^2 x$
(B) $2 \sec^2 x \tan x$
(C) $\sec^3 x \tan x$
(D) $\sec x$

A A A A A A A A A A A A A A A A A A A A A A A A A A

11. A cell phone was plugged into a charger. The battery life of the cell phone, in percent of battery life remaining, can be modeled by a differentiable function $P(t)$, where t is the number of minutes after the phone was plugged into the charger and $0 \leq t \leq 20$. Which of the following is the best interpretation of $P'(10) = 1.5$?
- (A) Ten minutes after the phone was plugged into the charger, the battery life was changing at a rate of 1.5 percent per minute.
- (B) Ten minutes after the phone was plugged into the charger, the rate of change in the battery life was changing by 1.5 percent per minute per minute.
- (C) During the first ten minutes that the phone was plugged into the charger, the battery life of the phone was changing at an average rate of 1.5 percent per minute.
- (D) During the first ten minutes that the phone was plugged into the charger, the rate of change in the battery life was changing at an average rate of 1.5 percent per minute per minute.

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14. Let f be the function defined by $f(x) = k\sqrt{x} + \frac{1}{x}$ for $x > 0$, where k is a constant. If f has a relative extremum at $x = 9$, what is the value of k ?

(A) $\frac{2}{3}$ (B) $\frac{2}{27}$ (C) $\frac{2}{81}$ (D) $\frac{2}{243}$

-
15. A particle moves along the y -axis so that its position is given by $y(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 4$ for time $t \geq 0$. What is the position of the particle when it changes from moving down to moving up?

(A) 0 (B) 4 (C) $\frac{14}{3}$ (D) $\frac{16}{3}$



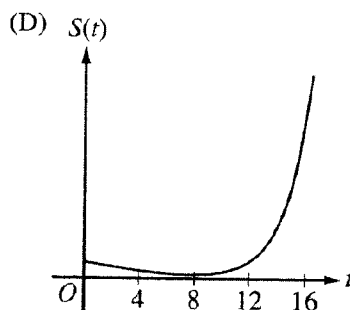
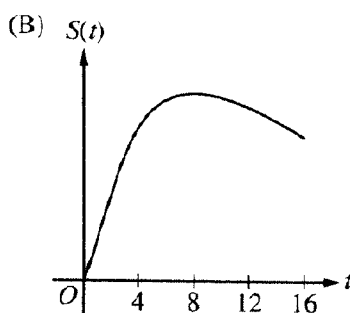
16. What is the slope of the line tangent to the graph of $3x - y^2 - 6y = 11$ at the point where $y = 4$?

- (A) $-\frac{5}{6}$ (B) $-\frac{4}{7}$ (C) $-\frac{3}{8}$ (D) $\frac{3}{14}$

17. $\lim_{x \rightarrow 0} \frac{e^{2x} - \ln(x+1) - \cos(\pi x)}{\sin(\pi x)} =$

- (A) 0 (B) $\frac{1}{\pi}$ (C) $\frac{2}{\pi}$ (D) 1

18. A store sold an item over a 16-month period. The total number of items sold up to time t is modeled by the twice-differentiable function $S(t)$, where t is the number of months since the store started selling the item. For the first 8 months, S' is increasing. After 8 months, S' is decreasing. Which of the following could be the graph of $S(t)$?



19. $\int \left(\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt[3]{1-x}} \right) dx =$

$$(A) \quad \frac{x}{(1-x^2)^{3/2}} + \frac{1}{3(1-x)^{4/3}} + C$$

$$(B) \quad 2\sqrt{1-x^2} + \frac{3}{2}(1-x)^{2/3} + C$$

(C) $\sin^{-1}x + \frac{2}{3}(1-x)^{2/3} + C$

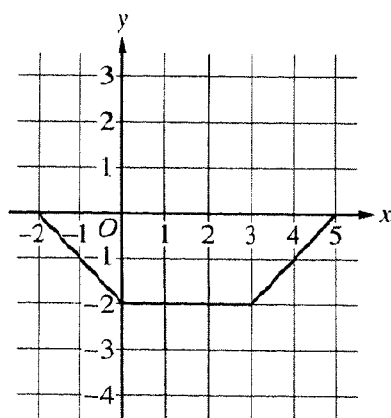
(D) $\sin^{-1} x - \frac{3}{2}(1-x)^{2/3} + C$

20. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = (1 + y)\cos x$ with initial condition $f(0) = 2$. What is the value of $f\left(\frac{\pi}{6}\right)$?

21. The first derivative of the function f is given by $f'(x) = 4x - 8$. At what value of x does f attain an absolute maximum on the closed interval $[0, 3]$?

- (A) 0 (B) $\frac{3}{2}$ (C) 2 (D) 3

- (A) f is continuous for all x .
 (B) f is discontinuous at $x = -2$ only.
 (C) f is discontinuous at $x = 2$ only.
 (D) f is discontinuous at $x = -2$ and $x = 2$.



23. The function f is defined for $-2 \leq x \leq 5$, and the graph of $y = f(x)$ shown consists of three line segments.

(A) -10 (B) -9 (C) -3 (D) 17

$$f(0) = 0 \quad \lim_{x \rightarrow -1} f(x) = \frac{2}{5} \quad \lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x + 1} = -\frac{3}{5}$$

24. The function f satisfies the properties shown. What is the slope of the line tangent to the graph of f at $x = -1$?

(A) $-\frac{3}{5}$ (B) $-\frac{2}{5}$ (C) $\frac{2}{5}$ (D) 1

25. $\int \frac{dx}{x^2 - 6x + 10} =$

- (A) $\ln|x^2 - 6x + 10| + C$
- (B) $\frac{1}{2x-6} \ln|x^2 - 6x + 10| + C$
- (C) $\frac{1}{2} \arctan(x-3) + C$
- (D) $\arctan(x-3) + C$

26. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{2 + \frac{k}{2n}} \cdot \frac{1}{n} \right) =$

$$(A) \int_0^{1/2} \frac{1}{2 + \frac{x}{2}} dx$$

$$(B) \int_0^{1/2} \frac{1}{2+x} dx$$

$$(C) \int_0^1 \frac{1}{2 + \frac{x}{2}} dx$$

(D) $\int_0^1 \frac{1}{2+x} dx$

(A) $\frac{x^5}{2^x}$

(B) $\frac{e^x}{x^{50}}$

(C) $\frac{x}{\ln x}$

(D) $\frac{x^3 - 4x + 3}{25x^3 + 147x^2 + 98}$

- (A) $\int_0^6 \frac{1}{36} x^2 dx + \int_6^{12} \sin\left(\frac{\pi x}{12}\right) dx$
- (B) $\pi \int_0^6 \frac{1}{36} x^2 dx + \pi \int_6^{12} \sin\left(\frac{\pi x}{12}\right) dx$
- (C) $\int_0^1 \left(\sqrt{\sin\left(\frac{\pi x}{12}\right)} - \frac{1}{6} x \right)^2 dx$
- (D) $\pi \int_0^1 \left(\sqrt{\sin\left(\frac{\pi x}{12}\right)} - \frac{1}{6} x \right)^2 dx$

$$f(x) = \begin{cases} x^2 + 3x + 2 & \text{for } x < -1 \\ 1 - |x| & \text{for } -1 \leq x \leq 1 \\ x^2 - 1 & \text{for } x > 1 \end{cases}$$

30. The function f is defined by $f(x) = \frac{2}{x}$ for $x > 0$. For what value of x , if any, is the average rate of change of f over the interval $4 \leq x \leq 16$ equal to the instantaneous rate of change of f at x ?

- END OF PART A**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

B**B****B****B****B****B****B****B****B**

CALCULUS AB
SECTION I, Part B
Time—45 minutes
15 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU FILL IN THE CIRCLES ON THE ANSWER SHEET THAT CORRESPOND TO QUESTIONS NUMBERED 76–90.

YOU MAY NOT RETURN TO QUESTIONS NUMBERED 1–30.

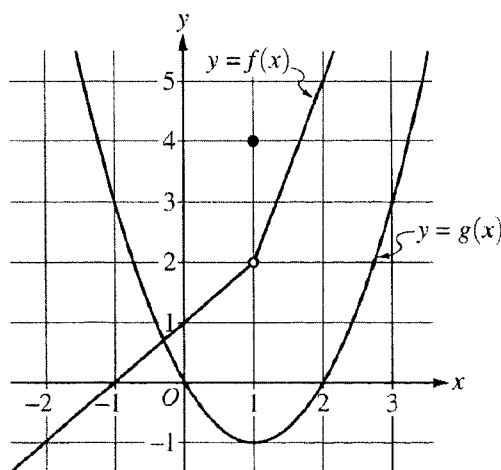
In this exam:

1. The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
3. The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).

B**B****B****B****B****B****B****B****B**

76. A metal rod has a temperature of 75 degrees Fahrenheit ($^{\circ}\text{F}$) at time $t = 0$. The temperature of the rod is increasing at a rate modeled by $47e^{-0.2t}$ degrees Fahrenheit per minute for $0 \leq t \leq 10$ minutes. To the nearest degree, what is the temperature of the rod at time $t = 10$ minutes?

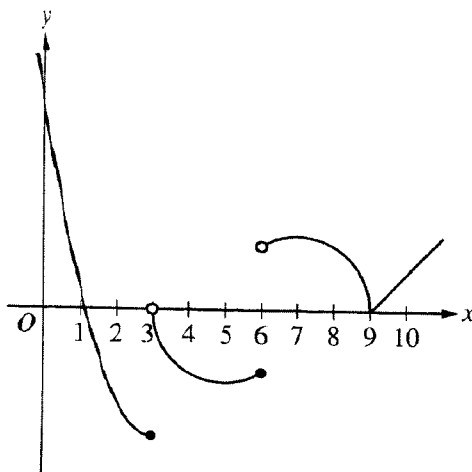
(A) 81°F (B) 203°F (C) 278°F (D) 545°F



77. The graphs of the functions f and g are shown in the figure. The function f has a removable discontinuity,

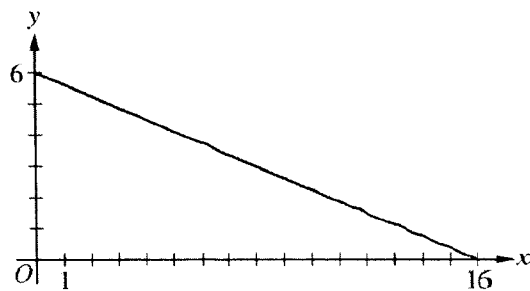
and the function g is continuous. What is $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)}$?

(A) -4 (B) -2 (C) $-\frac{1}{2}$ (D) Nonexistent

B**B****B****B****B****B****B****B****B**Graph of f

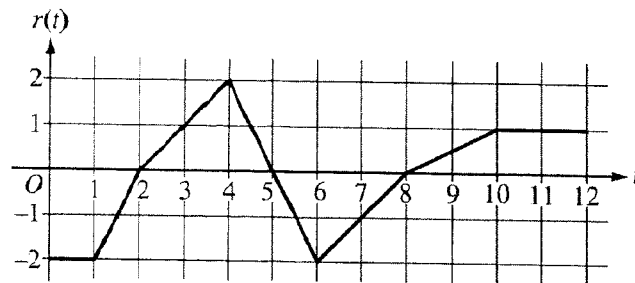
78. The graph of the function f is shown. In which of the following open intervals (a, b) is there guaranteed to exist a value c such that $f'(c) = \frac{f(b) - f(a)}{b - a}$?

(A) $(2, 4)$ (B) $(4, 6)$ (C) $(6, 8)$ (D) $(8, 10)$

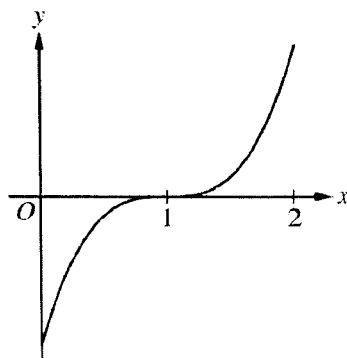
B**B****B****B****B****B****B****B****B**

79. The base of a solid is a right triangle with legs of length 6 and 16, as shown in the figure. The hypotenuse of the triangle is given by the line $y = -\frac{3}{8}(x - 16)$. Each cross section of the solid perpendicular to the x -axis is a semicircle. What is the volume of the solid?

(A) 75.398 (B) 150.796 (C) 301.593 (D) 603.186

B**B****B****B****B****B****B****B****B**

80. The rate of change of the water level in a lake at time t is modeled by the function r , where $r(t)$ is measured in feet per month and t is measured in months. The graph of r is shown for $0 \leq t \leq 12$. At what times t , for $0 < t \leq 12$, is the water level in the lake equal to the water level at time $t = 0$?
- (A) $t = 5$ only
(B) $t = 5$ and $t = 12$
(C) $t = 2$, $t = 5$, and $t = 8$
(D) $t = 1$ and $t = 6$

B**B****B****B****B****B****B****B****B**Graph of f

81. The graph of the increasing function f is shown in the figure. Which of the following could be a table of values for the function g defined by $g(x) = \int_1^x f(t) dt$?

(A)

x	$g(x)$
0	-16
0.5	-8
1	0
1.5	8
2	16

(B)

x	$g(x)$
0	16
0.5	8
1	0
1.5	8
2	16

(C)

x	$g(x)$
0	-16
0.5	-1
1	0
1.5	1
2	16

(D)

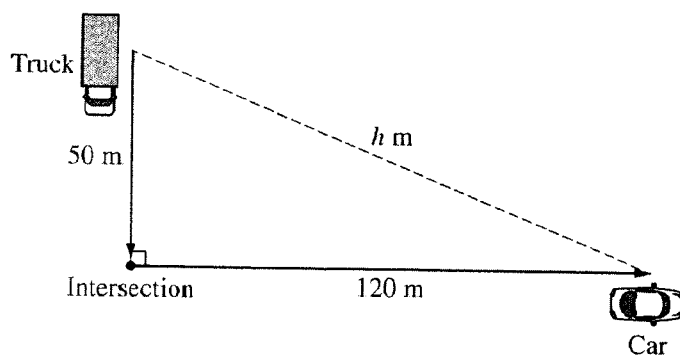
x	$g(x)$
0	16
0.5	1
1	0
1.5	1
2	16

B**B****B****B****B****B****B****B****B**

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
4	8	-2	-1	7
9	-3	-10	4	5

82. The table gives values of the differentiable functions f and g and their first derivatives f' and g' for selected values of x . If $h(x) = f(g(x^2))$, what is the value of $h'(3)$?

(A) -120 (B) -60 (C) -12 (D) -10



83. A truck is traveling south toward the intersection of two roads, while a car is traveling east away from the intersection, as shown in the figure. When the truck is 50 meters from the intersection, it is traveling toward the intersection at 15 meters per second. At that moment, the car is 120 meters from the intersection and traveling away from the intersection at 25 meters per second. At what rate, in meters per second, is the distance, h meters, between the two vehicles changing at that moment?

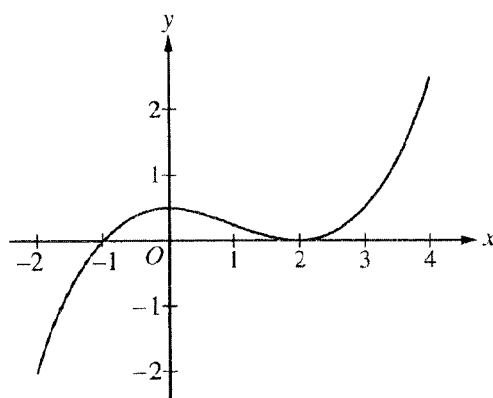
(A) 1.308 (B) 10.000 (C) 17.308 (D) 28.846

B**B****B****B****B****B****B****B****B**

84. The function f is continuous for all x , and $\int_1^2 f(3x - 2) \, dx = a$. Which of the following is equal to

$$\int_1^4 f(u) \, du?$$

- (A) $\frac{a}{3}$ (B) a (C) $2a$ (D) $3a$



Graph of f'

85. The graph of f' , the derivative of the function f , is shown for $-2 < x < 4$. Which of the following statements about f must be true for $-2 < x < 4$?

- (A) f has only one relative extremum, and the graph of f has only one point of inflection.
(B) f has only one relative extremum, and the graph of f has two points of inflection.
(C) f has two relative extrema, and the graph of f has only one point of inflection.
(D) f has two relative extrema, and the graph of f has two points of inflection.

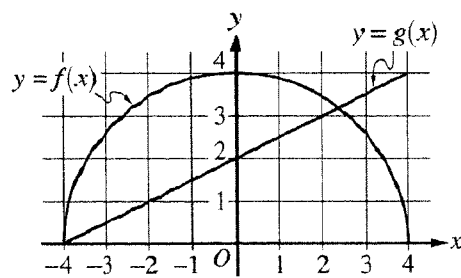
B **B** **B** **B** **B** **B** **B** **B**

86. The function f has first derivative f' defined by $f'(x) = x^2 \tan(1 - 2x)$ for $-0.25 < x < 0.75$. At which of the following values of x does the graph of f have a point of inflection?

- (A) $x = 0.324$ only
- (B) $x = 0.5$ only
- (C) $x = 0$ and $x = 0.324$
- (D) $x = 0$ and $x = 0.5$

87. The function f models the amount of time $f(v)$, in hours, that it takes to ride a bicycle along a straight path from point A to point B , where v is the average velocity of the bicycle in miles per hour (mph). What are the units for $f'(v)$?

- (A) mph
- (B) hours
- (C) mph per hour
- (D) hours per mph

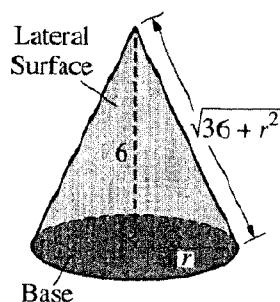
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88. The functions f and g are defined for $-4 \leq x \leq 4$. The graph of $y = g(x)$ is a line segment and the graph of

$y = f(x)$ is a semicircle, as shown in the figure. The function h is defined by $h(x) = \frac{f(x)}{g(x)}$. What is the

instantaneous rate of change of h with respect to x at $x = 0$?

- (A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 2

B**B****B****B****B****B****B****B****B**

89. The right circular cone shown has a radius of r meters and a constant height of 6 meters. The lateral surface area of the cone, in square meters, is given by $S(r) = \pi\sqrt{36r^2 + r^4}$. At the instant when the radius is 0.75 meter, the radius is increasing at the rate of 1.15 meters per minute. At what rate is the lateral surface area of the cone changing, in square meters per minute, at that instant?

- (A) 14.247
(B) 16.384
(C) 19.288
(D) 22.182

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90. The velocity of a particle moving along the x -axis is given by $v(t) = 4\sqrt{t+1} \cos t$ for time $t \geq 0$. At time $t = 0$, the particle is at position $x = 3$. What is the position of the particle at time $t = 2$?

- (A) -1.431 (B) 0.569 (C) 4.365 (D) 7.365

CALCULUS AB

SECTION II, Part A

Time—30 minutes

2 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. A 1000-gallon tank is used to collect rainwater. The tank is empty at time $t = 0$. Rainwater enters the tank at a

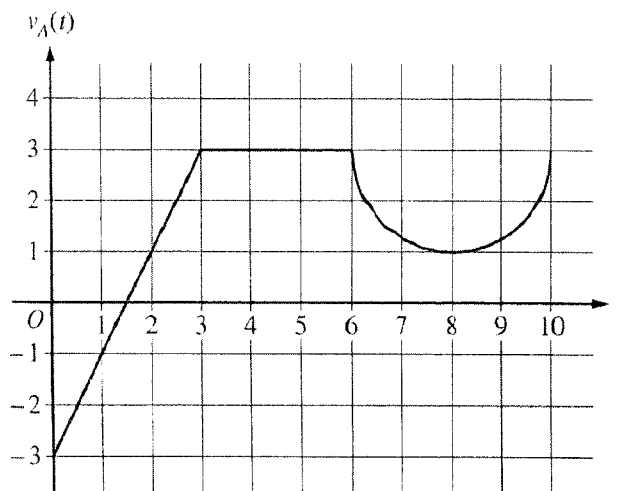
rate modeled by the piecewise-defined function r defined by $r(t) = \begin{cases} 200e^{-0.2t} & \text{for } 0 \leq t \leq 2 \\ 115 + 25 \sin(0.2t^2) & \text{for } 2 < t \leq 5 \end{cases}$,

where $r(t)$ is measured in gallons per hour and t is measured in hours.

(Note: Your calculator should be in radian mode.)

- (a) Find $r'(4)$. Indicate units of measure.
- (b) Explain why $\lim_{t \rightarrow 2} r(t)$ does not exist. Show the calculations that lead to your answer.
- (c) Using the given model, find the amount of rainwater, in gallons, that enters the tank over the time interval $0 \leq t \leq 2$. Show the setup for your calculations.
- (d) From time $t = 0$ to $t = 5$, rainwater enters the tank at a rate modeled by the piecewise-defined function r . From time $t = 2$ to $t = 5$, a pump removes rainwater from the tank at a constant rate of 100 gallons per hour. Find the total amount of rainwater, in gallons, in the tank at time $t = 5$. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Graph of v_A

2. Particle A moves along the x -axis such that, for time $t \geq 0$, its velocity is modeled by the function $v_A(t)$. The graph of $v_A(t)$ for $0 \leq t \leq 10$ is shown and consists of two line segments and a semicircle.
- (a) Find the value of $v_A'(1)$ or explain why the value does not exist. Find the value of $v_A'(3)$ or explain why the value does not exist.
- (b) Interpret the meaning of $\int_0^{10} v_A(t) dt$ in the context of the problem. Evaluate $\int_0^6 v_A(t) dt$ and $\int_6^{10} v_A(t) dt$.
Show the work that leads to your answers.
- (c) Particle B is also moving along the x -axis such that, for $0 \leq t \leq 10$, its velocity is given by $v_B(t) = (3 - t)\ln(15 - 1.45t)$. Find the average velocity of particle B over the time interval $0 \leq t \leq 4$.
Show the setup for your calculations.
- (d) Find the total distance traveled by particle B , defined in part (c), from time $t = 4$ to time $t = 10$. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

CALCULUS AB

SECTION II, Part B

Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

3. The number of walleye, a type of fish, in a lake at time t years after 2015 can be modeled by the function W

that satisfies the differential equation $\frac{dW}{dt} = W \cdot e^{-t}$, where $W(t)$ is measured in number of walleye and

$W(1) = 2000$. It can be shown that $\frac{d^2W}{dt^2} < 0$ for $t > 0$.

- (a) Find the slope of the line tangent to the graph of W at $t = 1$. Approximate $W(2)$, the number of walleye in the lake in 2017, using the line tangent to the graph of W at $t = 1$.
- (b) Is the approximation found in part (a) an overestimate or an underestimate for $W(2)$, the number of walleye in the lake in 2017? Give a reason for your answer.

- (c) Use separation of variables to find $y = W(t)$, the particular solution to the differential equation

$$\frac{dW}{dt} = W \cdot e^{-t} \text{ with initial condition } W(1) = 2000.$$

- (d) The number of walleye in a different lake at time t years after 2015 can be modeled by the function N , defined by $N(t) = \frac{11,000}{2 + 3e^{-t}}$. Approximate the number of walleye in the lake as time approaches infinity.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

x	-2	3	5	12	15
$f(x)$	-10	-2	-1	15	36
$f'(x)$	12	5	0	9	2
$f''(x)$	-9	-5	-1	2	-6

4. Let f be a twice-differentiable function. Selected values of f and its first two derivatives are shown in the table.
- (a) Find the average rate of change of f' over the interval $3 \leq x \leq 12$.
- (b) Let $m(x) = f(f'(x))$. Explain why there must be at least one value of x , for $3 \leq x \leq 12$, such that $m(x) = 0$.
- (c) Use a right Riemann sum with the three subintervals $[3, 5]$, $[5, 12]$, and $[12, 15]$ and data from the table to approximate the value of $\int_3^{15} (f''(x))^2 dx$.
- (d) Let $g(x) = 7 + \int_{-2}^{3x} f'(t) dt$. Find $g(5)$ and $g'(5)$. Show the work that leads to your answers.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

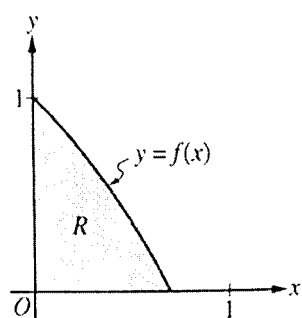


Figure 1

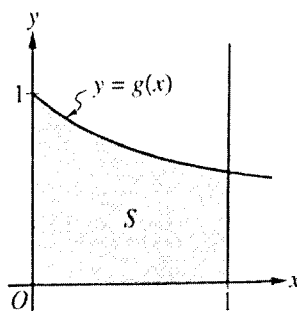


Figure 2

5. Let R be the region in the first quadrant bounded by the graph of $f(x) = 2 - e^x$, the x -axis, and the y -axis, as shown in Figure 1. Let S be the region in the first quadrant bounded by the graph of $g(x) = \frac{1}{\sqrt{2x+1}}$, the x -axis, the y -axis, and the vertical line $x = 1$, as shown in Figure 2.
- Find the area of region R . Show the work that leads to your answer.
 - Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when region R is revolved about the y -axis.
 - Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis is a square. Find the volume of the solid. Show the work that leads to your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

6. Consider the curve given by the equation $x^2y - y^3 = 8$. It can be shown that $\frac{dy}{dx} = \frac{2xy}{3y^2 - x^2}$.

(a) There is a point on the curve near $(3, 1)$ with x -coordinate 2.9. Use the line tangent to the curve at $(3, 1)$ to approximate the y -coordinate of this point.

(b) Find $\frac{d^2y}{dx^2}$ in terms of x , y , and $\frac{dy}{dx}$.

(c) The portion of the curve where $-1 < x < 1$ is the graph of a continuous function f , and the point $(0, -2)$ is on the graph of f .

I. Verify that f has a critical point at $(0, -2)$.

II. It can be shown that $f''(x) = -\frac{1}{3}$ at $(0, -2)$. Determine whether f has a relative minimum or a relative maximum at $(0, -2)$. Give a reason for your answer.

(d) For the curve given by the equation $x^3 + y^5 = \sin(y^2) + 7$, find $\frac{dy}{dx}$ in terms of x and y . Show the work that leads to your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.