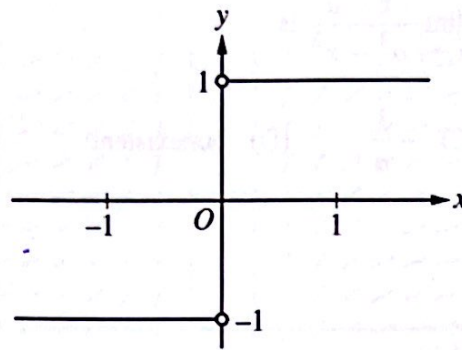


- (A) $7(2x - 11)^6$
 (B) $7(2x - 11)^6(x^2 - 11x + 24)$
 (C) $7(x^2 - 11x + 24)^6$
 (D) $7(x^2 - 11x + 24)^6(2x - 11)$

$$f(x) = \begin{cases} x^3 & \text{for } x < 0 \\ x^2 & \text{for } 0 \leq x \leq 2 \\ 4 & \text{for } x > 2 \end{cases}$$

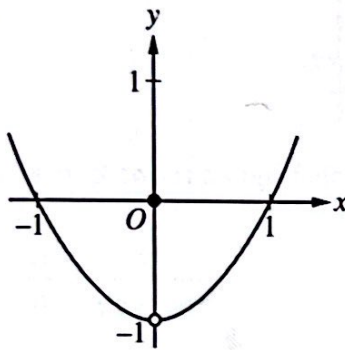
2. If the function f is defined as shown, then f is differentiable for
- (A) all values of x
- (B) all values of x except $x = 0$
- (C) all values of x except $x = 2$
- (D) all values of x except $x = 0$ and $x = 2$



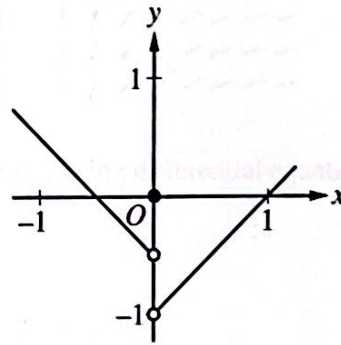
Graph of f'

3. The graph of f' , the derivative of the function f , is shown. Which of the following could be the graph of f ?

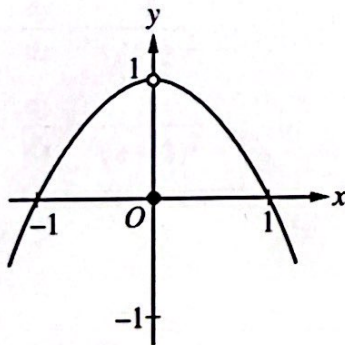
(A)



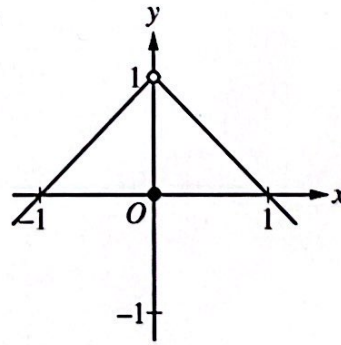
(B)



(C)



(D)



$$(A) \quad \frac{dy}{dx} = \frac{y^2}{(x-2)^2}$$

(B) $\frac{dy}{dx} = \frac{1}{(x-2)^3}$

(C) $\frac{dy}{dx} = \frac{1}{(x+2)^2}$

(D) $\frac{dy}{dx} = \frac{1}{(x-2)^2}$



6. For $x \geq 0$, the function g is continuous and $g(x) > 0$. If the area bounded by the graph of $y = g(x)$ and the x -axis from $x = 0$ to $x = 2$ is 3, what is the average value of g on the interval $[0, 2]$?

- (A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 2 (D) 3

x	0	1	2	3	4	5
$f'(x)$	1	-2	3	0	5	0

7. Let f be a twice-differentiable function. Selected values of f' , the first derivative of f , are given in the table. On which of the following intervals is f guaranteed to attain a local maximum?

- (A) $(0, 1)$ (B) $(1, 2)$ (C) $(2, 4)$ (D) $(3, 5)$



(A) $-\frac{1}{e}$ (B) 0 (C) $\frac{1}{e}$ (D) nonexistent

9. Let f be the function defined by $f(x) = \sin(x^2)$. What are the first three nonzero terms of the Maclaurin series for f' , the derivative of f ?

(A) $-2x^3 + \frac{x^7}{3} - \frac{x^{11}}{60}$

(B) $1 - \frac{x^4}{2} + \frac{x^8}{24}$

(C) $2x - x^5 + \frac{x^9}{12}$

(D) $2x + 2x^3 + x^5$



10. For time $t > 0$, the position of a particle moving along a curve is given by $(x(t), y(t))$, where $x(t)$ and $y(t)$ are differentiable functions of t . An equation of the line tangent to the curve at the point where $t = 1$ is $y = 4 + 3x$.

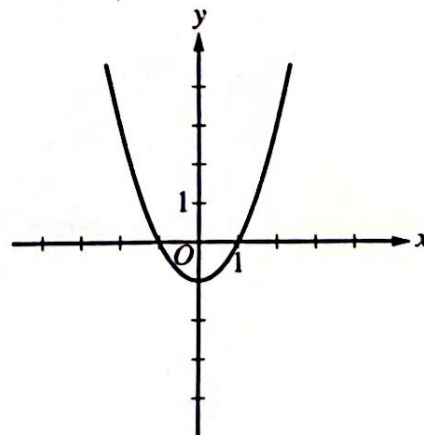
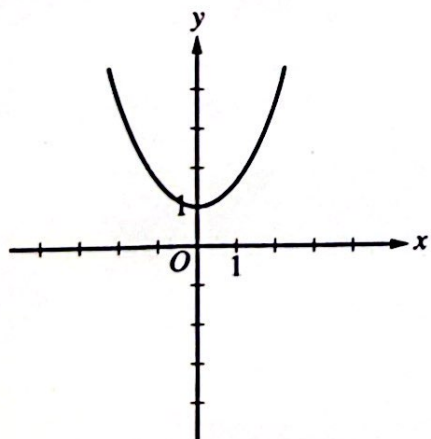
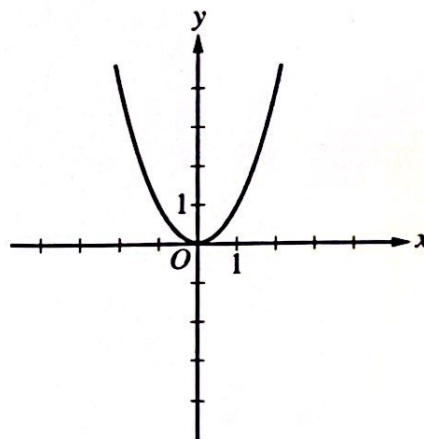
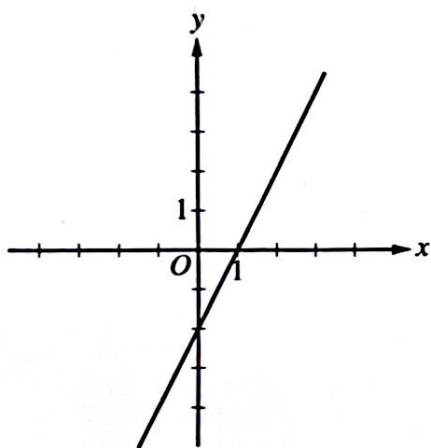
If $x'(t) = 12 \sin\left(\frac{\pi t}{2}\right)$, what is the value of $y'(1)$?

- (A) 3 (B) 4 (C) 36 (D) 40



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18. The figure shows the graph of the function f . If $g(x) = \int_1^x f(t) dt$, which of the following could be the graph of g ?



19. Which of the following is the solution to the differential equation $\frac{dy}{dx} = x(y - 2)$ with the initial condition $y(2) = 3$?

20. $\int_0^{\sqrt{\pi}/2} x \cos(x^2) dx =$

(A) $-\frac{\sqrt{2}}{2}$

(B) $-\frac{\sqrt{2}}{4}$

(C) $\frac{\sqrt{2}}{4}$

(D) $\frac{\sqrt{2}}{2}$



26. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{2 + \frac{k}{2n}} \cdot \frac{1}{n} \right) =$

(A) $\int_0^{1/2} \frac{1}{2 + \frac{x}{2}} dx$

(B) $\int_0^{1/2} \frac{1}{2+x} dx$

(C) $\int_0^1 \frac{1}{2 + \frac{x}{2}} dx$

(D) $\int_0^1 \frac{1}{2+x} dx$

27. Using the comparison test, to which of the following series should the series $\sum_{k=1}^{\infty} \frac{2 + \cos k}{k^3}$ be compared to determine its convergence or divergence?

(A) $\sum_{k=1}^{\infty} \frac{3}{k}$

(B) $\sum_{k=1}^{\infty} \frac{2}{k^3}$

(C) $\sum_{k=1}^{\infty} \frac{3}{k^3}$

(D) $\sum_{k=1}^{\infty} \frac{1}{k^4}$





28. Let g be the function defined by $g(x) = e^{x^3}$. Using the Maclaurin series for e^x , what is $g^{(21)}(0)$, the twenty-first derivative of g at $x = 0$?

- (A) $\frac{1}{21!}$ (B) $\frac{1}{7!}$ (C) 1 (D) $\frac{21!}{7!}$



29. If f is the function defined by $f(x) = \int_5^{x^3} \tan(t) \, dt$, then $f'(x) =$

- (A) $\sec^2(x^3)$ (B) $\tan(x^3)$ (C) $3x^2 \sec^2(x^3)$ (D) $3x^2 \tan(x^3)$

30. $\int \frac{1}{x^2 - 4x + 3} \, dx =$

- (A) $\ln|x^2 - 4x + 3| + C$
(B) $\ln|x - 3| - \ln|x - 1| + C$
(C) $\frac{1}{2} \ln|x - 3| - \frac{1}{2} \ln|x - 1| + C$
(D) $2 \ln|x - 3| - 2 \ln|x - 1| + C$

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.



PLACE SEAL HERE

NO TEST MATERIAL ON THIS PAGE



PART B STARTS ON PAGE 26.



B**B****B****B****B****B****B****B****B**

CALCULUS BC
SECTION I, Part B
Time—45 minutes
15 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU FILL IN THE CIRCLES ON THE ANSWER SHEET THAT CORRESPOND TO QUESTIONS NUMBERED 76–90.

YOU MAY NOT RETURN TO QUESTIONS NUMBERED 1–30.

In this exam:

1. The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
3. The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).



B**B****B****B****B****B****B****B****B**

76. In a community of 6000 people, the number of people informed about an upcoming event after t weeks is modeled by a function $P(t)$ that satisfies a logistic differential equation. When 1000 people are informed of the event, the rate of growth of informed people is 100 people per week. If all people in the community eventually become informed of the event, which of the following differential equations describes the situation?

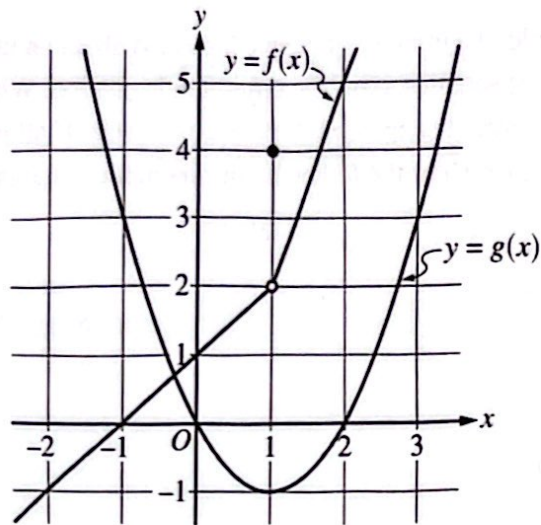
(A) $\frac{dP}{dt} = 0.12t \left(1 - \frac{t}{1000}\right)$

(B) $\frac{dP}{dt} = 0.12t \left(1 - \frac{t}{6000}\right)$

(C) $\frac{dP}{dt} = 0.12P \left(1 - \frac{P}{5000}\right)$

(D) $\frac{dP}{dt} = 0.12P \left(1 - \frac{P}{6000}\right)$



B**B****B****B****B****B****B****B****B**

77. The graphs of the functions f and g are shown in the figure. The function f has a removable discontinuity,

and the function g is continuous. What is $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)}$?

- (A) -4 (B) -2 (C) $-\frac{1}{2}$ (D) Nonexistent



B**B****B****B****B****B****B****B****B**

78. Let f be the function with first derivative given by $f'(x) = x^2 \sin(e^x)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

- (A) One (B) Two (C) Three (D) Four



B

B

B

B

B

B

B

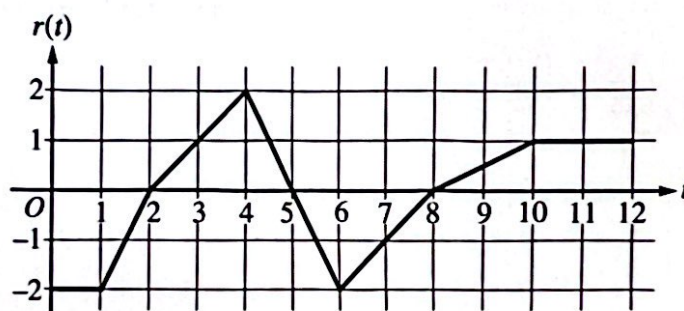
B

B

x	$h(x)$	$h'(x)$
2	-3	5
4	7	-1

79. The table gives values of a function h and its derivative at selected values of x . If f is the function defined by $f(x) = h(2x) + 3$, what is the value of $f'(2)$?

(A) -2 (B) -1 (C) 1 (D) 10



80. The rate of change of the water level in a lake at time t is modeled by the function r , where $r(t)$ is measured in feet per month and t is measured in months. The graph of r is shown for $0 \leq t \leq 12$. At what times t , for $0 < t \leq 12$, is the water level in the lake equal to the water level at time $t = 0$?

(A) $t = 5$ only
(B) $t = 5$ and $t = 12$
(C) $t = 2$, $t = 5$, and $t = 8$
(D) $t = 1$ and $t = 6$



B**B****B****B****B****B****B****B****B**

81. What is the length of the curve defined by the parametric equations $x(t) = t^3 + t$ and $y(t) = t^2 - 6t + 1$ from $t = 0$ to $t = 4$?

- (A) 78.063 (B) 72.217 (C) 68.469 (D) 49.041
-

82. The velocity of a particle moving along the x -axis is given by $v(t) = t^2 - 9 - 4\sin(t^2 - 9)$ for $t \geq 0$. At what time t , for $0 \leq t \leq 3$, is the particle farthest to the left?

- (A) 1.183 (B) 2.009 (C) 2.554 (D) 2.772



B**B****B****B****B****B****B****B****B**

83. Bacteria are small, single-celled organisms. The weight of a population of bacteria, in grams, is modeled by the function $w(t) = 0.1(t^2 + 3)e^{-0.2t}$, where t is measured in hours since the beginning of an experiment. What is the average weight of the population, in grams, during the first 3 hours of the experiment?

- (A) 0.120 (B) 0.418 (C) 0.479 (D) 1.255
-

84. What is the area of the region bounded by the polar curve $r = 0.45\theta + 0.2\cos(0.5\theta)$ for $0 \leq \theta \leq \pi$ and the x -axis?

- (A) 1.283 (B) 1.310 (C) 2.567 (D) 2.621



85. Let f be a function such that $f(1) = 2$ and the n th derivative of f at $x = 1$ is given by $f^{(n)}(1) = \frac{-n}{(2n-1)^2}$.

What is the estimate for $f(2)$ using the third-degree Taylor polynomial for f about $x = 1$?

- (A) $-\frac{8}{9} - \frac{24}{25}$
(B) $-\frac{4}{9} - \frac{4}{25}$
(C) $1 - \frac{2}{9} - \frac{3}{25}$
(D) $1 - \frac{1}{9} - \frac{1}{50}$
-

86. The function f has first derivative f' defined by $f'(x) = x^2 \tan(1 - 2x)$ for $-0.25 < x < 0.75$. At which of the following values of x does the graph of f have a point of inflection?

- (A) $x = 0.324$ only
(B) $x = 0.5$ only
(C) $x = 0$ and $x = 0.324$
(D) $x = 0$ and $x = 0.5$



B

B

B

B

B

B

B

B

B

87. At time $t \geq 0$, a particle moving in the xy -plane has a velocity vector given by $\langle \sin(2t), 2 + \cos(3t) \rangle$. What is the total distance the particle travels from time $t = 0$ to time $t = 1$?

(A) 1.655

(B) 2.226

(C) 2.432

(D) 5.259

n	2	3	4	5
a_n	0.2	0.1835	0.1715	0.1622

88. The series $\sum_{n=1}^{\infty} (-1)^n a_n$ converges to S , and $a_n > a_{n+1} > 0$ for all $n \geq 1$. Selected values of a_n are shown in

the table. What is the smallest value of k for which the alternating series error bound guarantees that

$$\left| S - \sum_{n=1}^k (-1)^n a_n \right| < 0.1667 ?$$

(A) 2

(B) 3

(C) 4

(D) 5



89. Which of the following power series has an interval of convergence of $0 < x \leq 2$?

(A) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$

(B) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+2}}{(n+1)(n+2)}$

(C) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+1)^{n+1}}{n+1}$

(D) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+1)^{n+2}}{(n+1)(n+2)}$



B**B****B****B****B****B****B****B****B**

90. Which of the following limits of a Riemann sum gives the value of the definite integral $\int_{-2}^3 (x^2 + 4x) dx$?

- (A) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{3}{n} \left(-2 + \left(\frac{3}{n} k \right)^2 + 4 \left(\frac{3}{n} k \right) \right)$
- (B) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{3}{n} \left(\left(-2 + \frac{3}{n} k \right)^2 + 4 \left(-2 + \frac{3}{n} k \right) \right)$
- (C) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{5}{n} \left(-2 + \left(\frac{5}{n} k \right)^2 + 4 \left(\frac{5}{n} k \right) \right)$
- (D) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{5}{n} \left(\left(-2 + \frac{5}{n} k \right)^2 + 4 \left(-2 + \frac{5}{n} k \right) \right)$



CALCULUS BC

SECTION II, Part A

Time—30 minutes

2 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.



1. A 1000-gallon tank is used to collect rainwater. The tank is empty at time $t = 0$. Rainwater enters the tank at a

rate modeled by the piecewise-defined function r defined by $r(t) = \begin{cases} 200e^{-0.21t} & \text{for } 0 \leq t \leq 2 \\ 115 + 25 \sin(0.2t^2) & \text{for } 2 < t \leq 5 \end{cases}$,

where $r(t)$ is measured in gallons per hour and t is measured in hours.

(Note: Your calculator should be in radian mode.)

- (a) Find $r'(4)$. Indicate units of measure.
- (b) Explain why $\lim_{t \rightarrow 2} r(t)$ does not exist. Show the calculations that lead to your answer.
- (c) Using the given model, find the amount of rainwater, in gallons, that enters the tank over the time interval $0 \leq t \leq 2$. Show the setup for your calculations.
- (d) From time $t = 0$ to $t = 5$, rainwater enters the tank at a rate modeled by the piecewise-defined function r . From time $t = 2$ to $t = 5$, a pump removes rainwater from the tank at a constant rate of 100 gallons per hour. Find the total amount of rainwater, in gallons, in the tank at time $t = 5$. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



2. A particle moving along a curve in the xy -plane is at position $(x(t), y(t))$ at time $t \geq 0$. The particle moves in such a way that $x'(t) = 3 \cos(\sqrt{1+t^2})$ and $y'(t) = -5 + 4\sqrt{2 + \sin t}$. At time $t = 0$, the particle is at the point $(0, 3)$.

(Note: Your calculator should be in radian mode.)

- (a) Find the acceleration vector of the particle at time $t = 5$. Show the setup for your calculations.
- (b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.
- (c) For $0 \leq t \leq 4$, find the time t when the particle's vertical distance above the line $y = 0$ is greatest. Justify your answer.
- (d) For $0 < t < 4$, determine the time t when the line tangent to the path of the particle is vertical. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



END OF PART A

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.
DO NOT GO TO PART B UNTIL YOU ARE TOLD TO DO SO.**



NO TEST MATERIAL ON THIS PAGE



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CALCULUS BC

SECTION II, Part B

Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.



CALCULUS BC

SECTION II, Part B

Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.



Answer QUESTION 4 parts (a) and (b) on this page.

x	-2	3	5	12	15
$f(x)$	-10	-2	-1	15	36
$f'(x)$	12	5	0	9	2
$f''(x)$	-9	-5	-1	2	-6

Response for question 4(a)

Response for question 4(b)



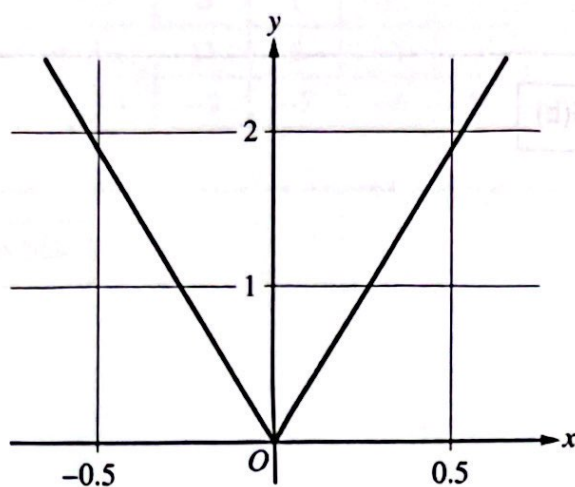
Answer QUESTION 5 part (a) on this page.

x	0	1	2	4
$f(x)$	11	6	-1	3
$f'(x)$	-2	-7	-6	14

Response for question 5(a)



Answer QUESTION 6 parts (a) and (b) on this page.



Graph of $y = |f^{(5)}(x)|$

Response for question 6(a)

Response for question 6(b)



DCBBD
BDABC
BDBAC
BDDAC
DDCAA
CBCDC
DBCAB
BCBBD
CBCAD