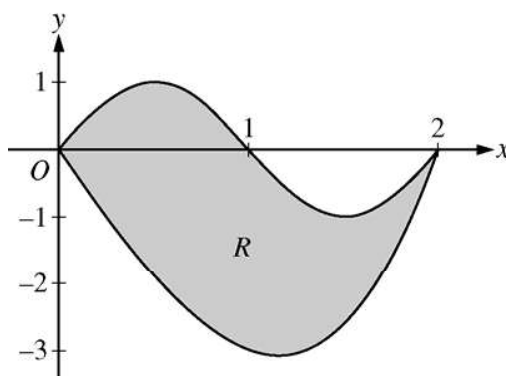


2008 AP calculus BC EXAM 答案

|       |       |
|-------|-------|
| 1-5   | BACDC |
| 6-10  | ADBDB |
| 11-15 | ADBEA |
| 16-20 | DBAAC |
| 21-25 | AEEAB |
| 26-28 | DAD   |
| 76-80 | DDCDE |
| 81-85 | BEBAC |
| 86-90 | CDAED |
| 91-92 | DB    |

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**Question 1**



Let  $R$  be the region bounded by the graphs of  $y = \sin(\pi x)$  and  $y = x^3 - 4x$ , as shown in the figure above.

- Find the area of  $R$ .
- The horizontal line  $y = -2$  splits the region  $R$  into two parts. Write, but do not evaluate, an integral expression for the area of the part of  $R$  that is below this horizontal line.
- The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.
- The region  $R$  models the surface of a small pond. At all points in  $R$  at a distance  $x$  from the  $y$ -axis, the depth of the water is given by  $h(x) = 3 - x$ . Find the volume of water in the pond.

(a)  $\sin(\pi x) = x^3 - 4x$  at  $x = 0$  and  $x = 2$   
 Area =  $\int_0^2 (\sin(\pi x) - (x^3 - 4x)) dx = 4$

3 :  $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b)  $x^3 - 4x = -2$  at  $r = 0.5391889$  and  $s = 1.6751309$   
 The area of the stated region is  $\int_r^s (-2 - (x^3 - 4x)) dx$

2 :  $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \end{cases}$

(c) Volume =  $\int_0^2 (\sin(\pi x) - (x^3 - 4x))^2 dx = 9.978$

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(d) Volume =  $\int_0^2 (3 - x)(\sin(\pi x) - (x^3 - 4x)) dx = 8.369$  or  $8.370$

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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**Question 2**

|                 |     |     |     |     |     |    |   |
|-----------------|-----|-----|-----|-----|-----|----|---|
| $t$ (hours)     | 0   | 1   | 3   | 4   | 7   | 8  | 9 |
| $L(t)$ (people) | 120 | 156 | 176 | 126 | 150 | 80 | 0 |

Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.

- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?

(a)  $L'(5.5) \approx \frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{3} = 8$  people per hour

2 :  $\begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

- (b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4} \left( \frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3) \right) = 155.25 \text{ people}$$

2 :  $\begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$

- (c)  $L$  is differentiable on  $[0, 9]$  so the Mean Value Theorem implies  $L'(t) > 0$  for some  $t$  in  $(1, 3)$  and some  $t$  in  $(4, 7)$ . Similarly,  $L'(t) < 0$  for some  $t$  in  $(3, 4)$  and some  $t$  in  $(7, 8)$ . Then, since  $L'$  is continuous on  $[0, 9]$ , the Intermediate Value Theorem implies that  $L'(t) = 0$  for at least three values of  $t$  in  $[0, 9]$ .

3 :  $\begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

OR

The continuity of  $L$  on  $[1, 4]$  implies that  $L$  attains a maximum value there. Since  $L(3) > L(1)$  and  $L(3) > L(4)$ , this maximum occurs on  $(1, 4)$ . Similarly,  $L$  attains a minimum on  $(3, 7)$  and a maximum on  $(4, 8)$ .  $L$  is differentiable, so  $L'(t) = 0$  at each relative extreme point on  $(0, 9)$ . Therefore  $L'(t) = 0$  for at least three values of  $t$  in  $[0, 9]$ .

OR

3 :  $\begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

[Note: There is a function  $L$  that satisfies the given conditions with  $L'(t) = 0$  for exactly three values of  $t$ .]

(d)  $\int_0^3 r(t) dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

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**Question 3**

| $x$ | $h(x)$ | $h'(x)$         | $h''(x)$         | $h'''(x)$         | $h^{(4)}(x)$      |
|-----|--------|-----------------|------------------|-------------------|-------------------|
| 1   | 11     | 30              | 42               | 99                | 18                |
| 2   | 80     | 128             | $\frac{488}{3}$  | $\frac{448}{3}$   | $\frac{584}{9}$   |
| 3   | 317    | $\frac{753}{2}$ | $\frac{1383}{4}$ | $\frac{3483}{16}$ | $\frac{1125}{16}$ |

Let  $h$  be a function having derivatives of all orders for  $x > 0$ . Selected values of  $h$  and its first four derivatives are indicated in the table above. The function  $h$  and these four derivatives are increasing on the interval  $1 \leq x \leq 3$ .

- (a) Write the first-degree Taylor polynomial for  $h$  about  $x = 2$  and use it to approximate  $h(1.9)$ . Is this approximation greater than or less than  $h(1.9)$ ? Explain your reasoning.
- (b) Write the third-degree Taylor polynomial for  $h$  about  $x = 2$  and use it to approximate  $h(1.9)$ .
- (c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for  $h$  about  $x = 2$  approximates  $h(1.9)$  with error less than  $3 \times 10^{-4}$ .

(a)  $P_1(x) = 80 + 128(x - 2)$ , so  $h(1.9) \approx P_1(1.9) = 67.2$

$P_1(1.9) < h(1.9)$  since  $h'$  is increasing on the interval  $1 \leq x \leq 3$ .

4 :  $\begin{cases} 2 : P_1(x) \\ 1 : P_1(1.9) \\ 1 : P_1(1.9) < h(1.9) \text{ with reason} \end{cases}$

(b)  $P_3(x) = 80 + 128(x - 2) + \frac{488}{6}(x - 2)^2 + \frac{448}{18}(x - 2)^3$

$h(1.9) \approx P_3(1.9) = 67.988$

3 :  $\begin{cases} 2 : P_3(x) \\ 1 : P_3(1.9) \end{cases}$

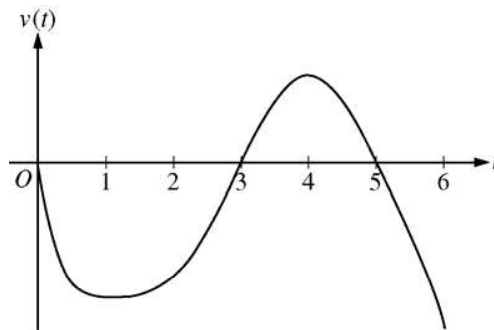
(c) The fourth derivative of  $h$  is increasing on the interval  $1 \leq x \leq 3$ , so  $\max_{1.9 \leq x \leq 2} |h^{(4)}(x)| = \frac{584}{9}$ .

Therefore,  $|h(1.9) - P_3(1.9)| \leq \frac{584}{9} \frac{|1.9 - 2|^4}{4!}$   
 $= 2.7037 \times 10^{-4}$   
 $< 3 \times 10^{-4}$

2 :  $\begin{cases} 1 : \text{form of Lagrange error estimate} \\ 1 : \text{reasoning} \end{cases}$

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**Question 4**



Graph of  $v$

A particle moves along the  $x$ -axis so that its velocity at time  $t$ , for  $0 \leq t \leq 6$ , is given by a differentiable function  $v$  whose graph is shown above. The velocity is 0 at  $t = 0$ ,  $t = 3$ , and  $t = 5$ , and the graph has horizontal tangents at  $t = 1$  and  $t = 4$ . The areas of the regions bounded by the  $t$ -axis and the graph of  $v$  on the intervals  $[0, 3]$ ,  $[3, 5]$ , and  $[5, 6]$  are 8, 3, and 2, respectively. At time  $t = 0$ , the particle is at  $x = -2$ .

- For  $0 \leq t \leq 6$ , find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- For how many values of  $t$ , where  $0 \leq t \leq 6$ , is the particle at  $x = -8$ ? Explain your reasoning.
- On the interval  $2 < t < 3$ , is the speed of the particle increasing or decreasing? Give a reason for your answer.
- During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

- (a) Since  $v(t) < 0$  for  $0 < t < 3$  and  $5 < t < 6$ , and  $v(t) > 0$  for  $3 < t < 5$ , we consider  $t = 3$  and  $t = 6$ .

$$x(3) = -2 + \int_0^3 v(t) \, dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t) \, dt = -2 - 8 + 3 - 2 = -9$$

Therefore, the particle is farthest left at time  $t = 3$  when its position is  $x(3) = -10$ .

- (b) The particle moves continuously and monotonically from  $x(0) = -2$  to  $x(3) = -10$ . Similarly, the particle moves continuously and monotonically from  $x(3) = -10$  to  $x(5) = -7$  and also from  $x(5) = -7$  to  $x(6) = -9$ .

By the Intermediate Value Theorem, there are three values of  $t$  for which the particle is at  $x(t) = -8$ .

- The speed is decreasing on the interval  $2 < t < 3$  since on this interval  $v < 0$  and  $v$  is increasing.
- The acceleration is negative on the intervals  $0 < t < 1$  and  $4 < t < 6$  since velocity is decreasing on these intervals.

$$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t) \, dt \\ 1 : \text{conclusion} \end{cases}$$

$$3 : \begin{cases} 1 : \text{positions at } t = 3, t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$$

1 : answer with reason

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

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**Question 5**

The derivative of a function  $f$  is given by  $f'(x) = (x - 3)e^x$  for  $x > 0$ , and  $f(1) = 7$ .

- (a) The function  $f$  has a critical point at  $x = 3$ . At this point, does  $f$  have a relative minimum, a relative maximum, or neither? Justify your answer.
- (b) On what intervals, if any, is the graph of  $f$  both decreasing and concave up? Explain your reasoning.
- (c) Find the value of  $f(3)$ .

- (a)  $f'(x) < 0$  for  $0 < x < 3$  and  $f'(x) > 0$  for  $x > 3$

Therefore,  $f$  has a relative minimum at  $x = 3$ .

2 :  $\begin{cases} 1 : \text{minimum at } x = 3 \\ 1 : \text{justification} \end{cases}$

- (b)  $f''(x) = e^x + (x - 3)e^x = (x - 2)e^x$   
 $f''(x) > 0$  for  $x > 2$

$$f'(x) < 0 \text{ for } 0 < x < 3$$

Therefore, the graph of  $f$  is both decreasing and concave up on the interval  $2 < x < 3$ .

3 :  $\begin{cases} 2 : f''(x) \\ 1 : \text{answer with reason} \end{cases}$

- (c)  $f(3) = f(1) + \int_1^3 f'(x) dx = 7 + \int_1^3 (x - 3)e^x dx$

$$u = x - 3 \quad dv = e^x dx$$

$$du = dx \quad v = e^x$$

$$f(3) = 7 + (x - 3)e^x \Big|_1^3 - \int_1^3 e^x dx$$

$$= 7 + \left( (x - 3)e^x - e^x \right) \Big|_1^3$$

$$= 7 + 3e - e^3$$

4 :  $\begin{cases} 1 : \text{uses initial condition} \\ 2 : \text{integration by parts} \\ 1 : \text{answer} \end{cases}$

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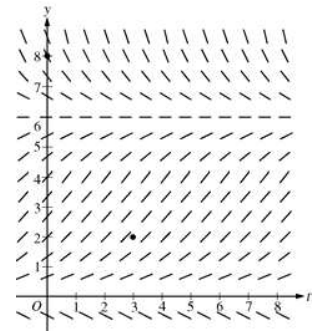
**Question 6**

Consider the logistic differential equation  $\frac{dy}{dt} = \frac{y}{8}(6 - y)$ . Let  $y = f(t)$  be the particular solution to the differential equation with  $f(0) = 8$ .

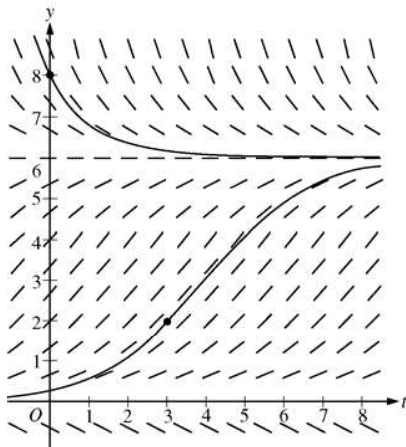
- (a) A slope field for this differential equation is given below. Sketch possible solution curves through the points  $(3, 2)$  and  $(0, 8)$ .

**(Note: Use the axes provided in the exam booklet.)**

- (b) Use Euler's method, starting at  $t = 0$  with two steps of equal size, to approximate  $f(1)$ .
- (c) Write the second-degree Taylor polynomial for  $f$  about  $t = 0$ , and use it to approximate  $f(1)$ .
- (d) What is the range of  $f$  for  $t \geq 0$ ?



(a)



2 :  $\begin{cases} 1 : \text{solution curve through } (0, 8) \\ 1 : \text{solution curve through } (3, 2) \end{cases}$

(b)  $f\left(\frac{1}{2}\right) \approx 8 + (-2)\left(\frac{1}{2}\right) = 7$

$$f(1) \approx 7 + \left(-\frac{7}{8}\right)\left(\frac{1}{2}\right) = \frac{105}{16}$$

2 :  $\begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{approximation of } f(1) \end{cases}$

(c)  $\frac{d^2y}{dt^2} = \frac{1}{8} \frac{dy}{dt} (6 - y) + \frac{y}{8} \left(-\frac{dy}{dt}\right)$

$$f(0) = 8; \quad f'(0) = \frac{dy}{dt}\bigg|_{t=0} = \frac{8}{8}(6 - 8) = -2; \quad \text{and}$$

$$f''(0) = \frac{d^2y}{dt^2}\bigg|_{t=0} = \frac{1}{8}(-2)(-2) + \frac{8}{8}(2) = \frac{5}{2}$$

The second-degree Taylor polynomial for  $f$  about

$$t = 0 \text{ is } P_2(t) = 8 - 2t + \frac{5}{4}t^2.$$

$$f(1) \approx P_2(1) = \frac{29}{4}$$

4 :  $\begin{cases} 2 : \frac{d^2y}{dt^2} \\ 1 : \text{second-degree Taylor polynomial} \\ 1 : \text{approximation of } f(1) \end{cases}$

- (d) The range of  $f$  for  $t \geq 0$  is  $6 < y \leq 8$ .

1 : answer