



CALCULUS AB
SECTION I, Part A
Time—1 hour
30 Questions

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

1. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
2. The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).

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1. $\frac{d}{dx} \left((x^2 - 11x + 24)^7 \right) =$

(A) $7(2x - 11)^6$

(B) $7(2x - 11)^6 (x^2 - 11x + 24)$

(C) $7(x^2 - 11x + 24)^6$

✓ (D) $7(x^2 - 11x + 24)^6 (2x - 11)$

2. If $\int_1^k \frac{1}{\sqrt{x}} dx = 10$, where k is a constant, what is the value of k ?

(A) $\frac{1}{121}$

✓ (B) 36

(C) 121

(D) 441

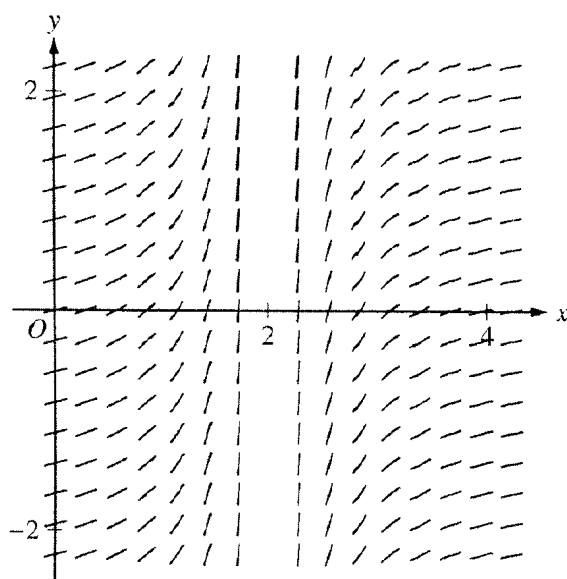
$$x^{\frac{1}{2}} \Big|_1^k = 10$$

$$\sqrt{k} - 1 = 10$$

$$k = 36$$

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5. The figure shown is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{y^2}{(x-2)^2}$

(B) $\frac{dy}{dx} = \frac{1}{(x-2)^3}$

(C) $\frac{dy}{dx} = \frac{1}{(x+2)^2}$

☒ (D) $\frac{dy}{dx} = \frac{1}{(x-2)^2}$

8. $\lim_{h \rightarrow 0} \frac{e^{-(1+h)} - e^{-(1)}}{h}$ is

- (A) $-\frac{1}{e}$ (B) 0 (C) $\frac{1}{e}$ (D) nonexistent

$$f(x) = e^{-x} \quad f'(1) = -e^{-1}$$

9. If $f(x) = \sec x \tan x$, then $f'(x) =$

- (A) $\sec^3 x + \sec x \tan^2 x$
- (B) $2 \sec^2 x \tan x$
- (C) $\sec^3 x \tan x$
- (D) $\sec x$

$$\left(\frac{1}{\cos x}\right)' =$$

$$\begin{array}{cc} u & u' \\ v \cos x & v' - \sin x \end{array}$$

$$0 + \frac{\sin x}{\cos^2 x} = \tan x \sec x.$$

$$\sec x \cdot \tan^2 x + \sec^3 x$$

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11. A cell phone was plugged into a charger. The battery life of the cell phone, in percent of battery life remaining, can be modeled by a differentiable function $P(t)$, where t is the number of minutes after the phone was plugged into the charger and $0 \leq t \leq 20$. Which of the following is the best interpretation of $P'(10) = 1.5$?
- (A) Ten minutes after the phone was plugged into the charger, the battery life was changing at a rate of 1.5 percent per minute.
- (B) Ten minutes after the phone was plugged into the charger, the rate of change in the battery life was changing by 1.5 percent per minute per minute.
- (C) During the first ten minutes that the phone was plugged into the charger, the battery life of the phone was changing at an average rate of 1.5 percent per minute.
- (D) During the first ten minutes that the phone was plugged into the charger, the rate of change in the battery life was changing at an average rate of 1.5 percent per minute per minute.

$$\frac{+4}{4(+3)} = \frac{1}{3}$$

- (A) $\left(-\infty, -\frac{1}{6}\right)$ (B) $\left(-\frac{4}{3}, -\frac{1}{6}\right)$ only (C) $\left(-\frac{1}{6}, 1\right)$ only (D) $\left(-\frac{1}{6}, \infty\right)$

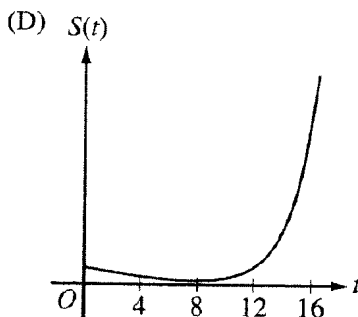
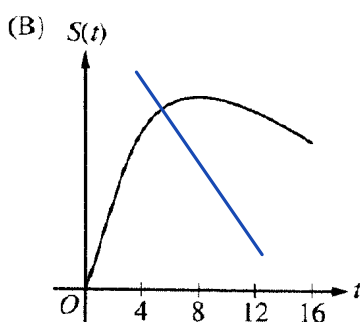
$$\begin{array}{ll} f' < 0 & bx+1 < 0 \\ & x < -1/b \\ f'' > 0 & b \quad x \in \mathbb{R} \end{array}$$

14. Let f be the function defined by $f(x) = k\sqrt{x} + \frac{1}{x}$ for $x > 0$, where k is a constant. If f has a relative extremum at $x = 9$, what is the value of k ?

$$K = \frac{2}{27}$$

- $$y(3) = 9 - 18 + 9 + 4 = 4$$

18. A store sold an item over a 16-month period. The total number of items sold up to time t is modeled by the twice-differentiable function $S(t)$, where t is the number of months since the store started selling the item. For the first 8 months, S' is increasing. After 8 months, S' is decreasing. Which of the following could be the graph of $S(t)$?



(D) $\sin^{-1} x - \frac{3}{2}(1-x)^{2/3} + C$

$$dx = -du$$

$$\sin^{-1} x = \frac{3}{2} u^{\frac{2}{3}} + C.$$

20. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = (1 + y)\cos x$ with initial condition

$$y+1 = 3e^{\sin \frac{\pi}{6}} = 3\sqrt{e} - 1$$

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(A) f is continuous for all x .
 (B) f is discontinuous at $x = -2$ only.
 (C) f is discontinuous at $x = 2$ only.
 (D) f is discontinuous at $x = -2$ and $x = 2$.

$$- \frac{(3+7)}{2} \times 2$$

$$= -10$$



- What is the value of $\int_{-2}^5 (f(x) + 1) \, dx$?

$$-10 + 7$$

(B) -9

✓ (C) -3

(D) 17

(A) $-\frac{3}{5}$ (B) $-\frac{2}{5}$ (C) $\frac{2}{5}$ (D) 1

25. $\int \frac{dx}{x^2 - 6x + 10} = \int \frac{1}{x^2 - 6x + 9 + 1} dx = \int \frac{1}{(x-3)^2 + 1} dx$

- (A) $\ln|x^2 - 6x + 10| + C$
- (B) $\frac{1}{2x-6} \ln|x^2 - 6x + 10| + C$
- (C) $\frac{1}{2} \arctan(x-3) + C$
- (D) $\arctan(x-3) + C$

26. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{2 + \frac{k}{2n}} \cdot \frac{1}{n} \right) =$

(A) $\int_0^{1/2} \frac{1}{2 + \frac{x}{2}} dx$ (B) $\int_0^{1/2} \frac{1}{2+x} dx$ (C) $\int_0^1 \frac{1}{2 + \frac{x}{2}} dx$ (D) $\int_0^1 \frac{1}{2+x} dx$

(A) $\frac{x^5}{2^x}$

(B) $\frac{e^x}{x^{50}}$ X

(C) $\frac{x}{\ln x}$ 

(D) $\frac{x^3 - 4x + 3}{25x^3 + 147x^2 + 98}$ ✓

- $$\int_0^6 \left(\frac{1}{6}x\right)^2 dx + \int_6^{12} \sin\left(\frac{\pi x}{12}\right) dx$$

(A) $\int_0^6 \frac{1}{36} x^2 dx + \int_6^{12} \sin\left(\frac{\pi x}{12}\right) dx$

$$(B) \quad \pi \int_0^6 \frac{1}{36} x^2 dx + \pi \int_6^{12} \sin\left(\frac{\pi x}{12}\right) dx$$

$$(C) \int_0^1 \left(\sqrt{\sin\left(\frac{\pi x}{12}\right)} - \frac{1}{6}x \right)^2 dx$$

$$(D) \quad \pi \int_0^1 \left(\sqrt{\sin\left(\frac{\pi x}{12}\right)} - \frac{1}{6}x \right)^2 dx$$

$$f(x) = \begin{cases} x^2 + 3x + 2 & \text{for } x < -1 \\ 1 - |x| & \text{for } -1 \leq x \leq 1 \\ x^2 - 1 & \text{for } x > 1 \end{cases}$$

- (A) 0 only
(B) 1 only
(C) 0 and 1 only
(D) ☒ -1, 0, and 1

-1. 1 左右与世不符.

- $$\frac{\frac{1}{16} - \frac{1}{4}}{16-4} = -\frac{1}{x^2}$$

$$\frac{1}{16} - \frac{1}{4} = -\frac{12}{x^2}$$

$$\frac{2}{3^2} - \frac{8}{3^2} = -\frac{12}{x^2}$$

$$-\frac{\cancel{x}^1}{32} \times \frac{+1}{\cancel{x}_2^2} = \frac{1}{x^2}$$

$$x^2 = 64 \quad x = 8.$$

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.**

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CALCULUS AB
SECTION I, Part B
Time—45 minutes
15 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU FILL IN THE CIRCLES ON THE ANSWER SHEET THAT CORRESPOND TO QUESTIONS NUMBERED 76–90.

YOU MAY NOT RETURN TO QUESTIONS NUMBERED 1–30.

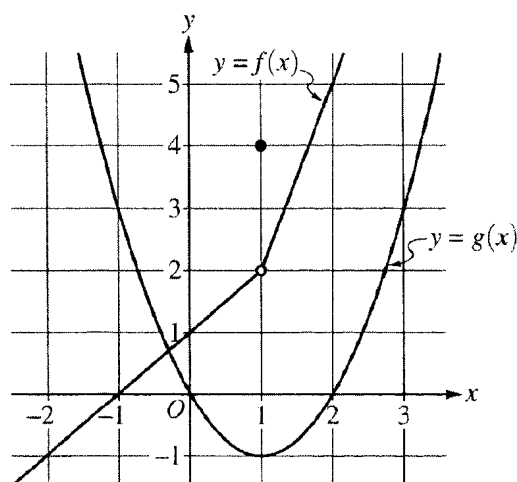
In this exam:

1. The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
3. The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).

B**B****B****B****B****B****B****B****B**

76. A metal rod has a temperature of 75 degrees Fahrenheit ($^{\circ}\text{F}$) at time $t = 0$. The temperature of the rod is increasing at a rate modeled by $47e^{-0.2t}$ degrees Fahrenheit per minute for $0 \leq t \leq 10$ minutes. To the nearest degree, what is the temperature of the rod at time $t = 10$ minutes?

(A) 81°F (B) 203°F (C) 278°F (D) 545°F

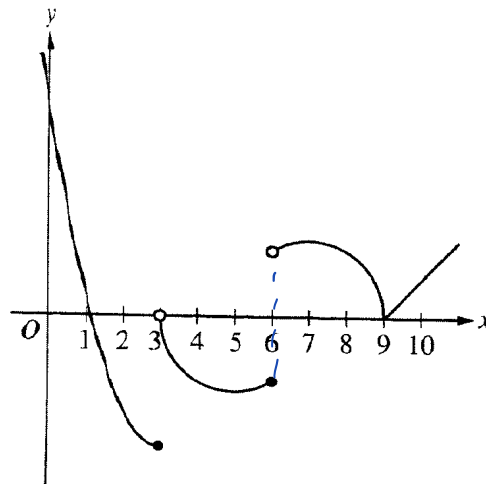


77. The graphs of the functions f and g are shown in the figure. The function f has a removable discontinuity,

and the function g is continuous. What is $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)}$?

(A) -4 (B) -2 (C) $-\frac{1}{2}$ (D) Nonexistent

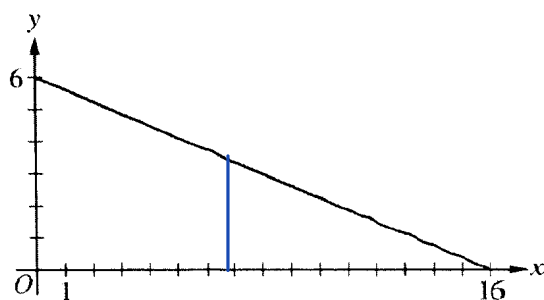
*f is discontinuous at x=1
Not differentiable.*

B**B****B****B****B****B****B****B****B**Graph of f

78. The graph of the function f is shown. In which of the following open intervals (a, b) is there guaranteed to exist a value c such that $f'(c) = \frac{f(b) - f(a)}{b - a}$?

(A) $(2, 4)$ ☒ (B) $(4, 6)$ (C) $(6, 8)$ ☒ (D) $(8, 10)$

f must be a continuous function over the closed interval $[a, b]$ and differentiable on (a, b) .



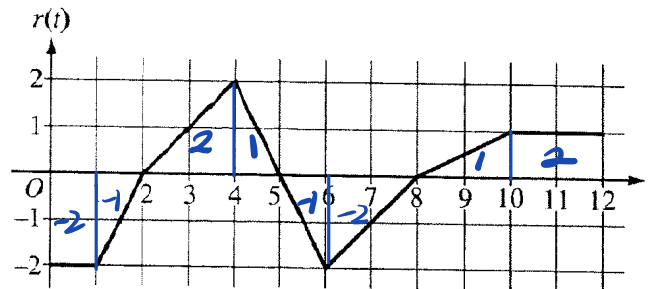
$$r = y/2$$

$$A = \pi \cdot \left(\frac{y}{2}\right)^2$$

79. The base of a solid is a right triangle with legs of length 6 and 16, as shown in the figure. The hypotenuse of the triangle is given by the line $y = -\frac{3}{8}(x - 16)$. Each cross section of the solid perpendicular to the x -axis is a semicircle. What is the volume of the solid?

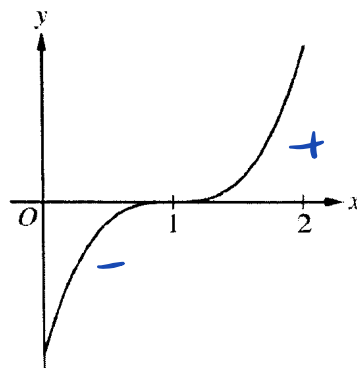
(A) 75.398 ☒ (B) 150.796 (C) 301.593 (D) 603.186

$$\pi \int_0^{16} \left(\frac{-\frac{3}{8}(x-16)}{2} \right)^2 dx \approx 150.796$$

B**B****B****B****B****B****B****B****B**

80. The rate of change of the water level in a lake at time t is modeled by the function r , where $r(t)$ is measured in feet per month and t is measured in months. The graph of r is shown for $0 \leq t \leq 12$. At what times t , for $0 < t \leq 12$, is the water level in the lake equal to the water level at time $t = 0$?

- (A) $t = 5$ only
(B) $t = 5$ and $t = 12$
(C) $t = 2$, $t = 5$, and $t = 8$
(D) $t = 1$ and $t = 6$

B**B****B****B****B****B****B****B****B**Graph of f

81. The graph of the increasing function f is shown in the figure. Which of the following could be a table of values for the function g defined by $g(x) = \int_1^x f(t) dt$?

(A)

x	$g(x)$
0	-16
0.5	-8
1	0
1.5	8
2	16

(B)

x	$g(x)$
0	16
0.5	8
1	0
1.5	8
2	16

$$g(1) = 0$$

$(0, 1) \downarrow$ concave up

$(1, 2) \uparrow$ concave up

slope is increasing

(C)

x	$g(x)$
0	-16
0.5	-1
1	0
1.5	1
2	16

(D)

x	$g(x)$
0	16
0.5	1
1	0
1.5	1
2	16

B**B****B****B****B****B****B****B****B**

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
4	8	-2	-1	7
9	-3	-10	4	5

82. The table gives values of the differentiable functions f and g and their first derivatives f' and g' for selected

values of x . If $h(x) = f(g(x^2))$, what is the value of $h'(3)$?

(A) -120

(B) -60

(C) -12

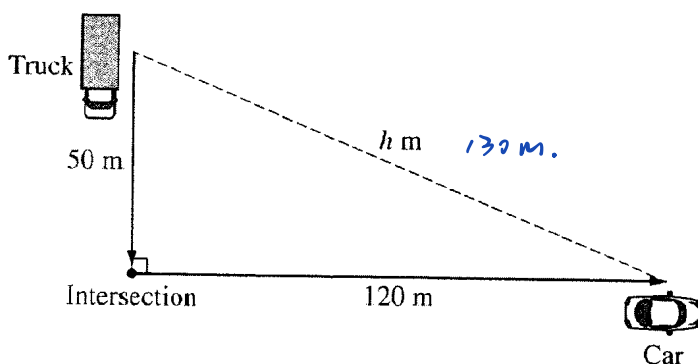
(D) -10

$$h'(x) = f'(g(x^2)) \cdot g'(x^2) \cdot 2x$$

$$h'(3) = f'(g(9)) \cdot g'(9) \cdot 6$$

$$= f'(4) \cdot 5 \cdot 6$$

$$= (-2)(30) = -60$$



83. A truck is traveling south toward the intersection of two roads, while a car is traveling east away from the intersection, as shown in the figure. When the truck is 50 meters from the intersection, it is traveling toward the intersection at 15 meters per second. At that moment, the car is 120 meters from the intersection and traveling away from the intersection at 25 meters per second. At what rate, in meters per second, is the distance, h meters, between the two vehicles changing at that moment?

(A) 1.308

(B) 10.000

(C) 17.308

(D) 28.846

$$\frac{dx}{dt} = 25 \text{ m/s.}$$

$$\frac{dy}{dt} = -15 \text{ m/s. 注意运动方向!}$$

$$x^2 + y^2 = z^2 \quad (-)$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$$

$$240 \times 25 - 2 \times 50 \times 15 = 260 \times \frac{dz}{dt}$$

$$\frac{dz}{dt} \approx 17.3077$$

B

B

B

B

B

B

B

B

B

84. The function f is continuous for all x , and $\int_1^2 f(3x - 2) dx = a$. Which of the following is equal to

$$\int_1^4 f(u) du?$$

(A) $\frac{a}{3}$

(B) a

(C) $2a$

(D) $3a$

$$u = 3x - 2$$

$$du = 3dx$$

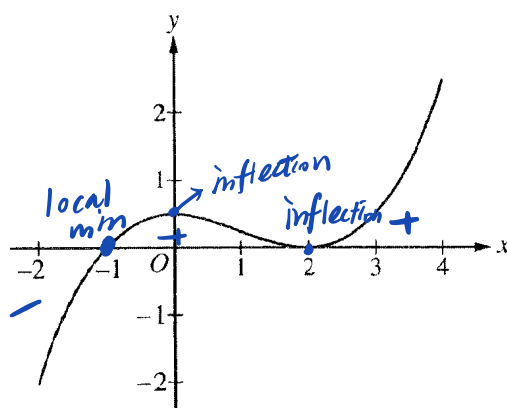
$$dx = \frac{1}{3}du$$

$$1 < x < 2$$

$$1 < u < 4$$

$$\frac{1}{3} \int_1^4 f(u) du = a.$$

$$\int_1^4 f(u) du = 3a.$$



Graph of f'

85. The graph of f' , the derivative of the function f , is shown for $-2 < x < 4$. Which of the following statements about f must be true for $-2 < x < 4$?

(A) f has only one relative extremum, and the graph of f has only one point of inflection.

(B) f has only one relative extremum, and the graph of f has two points of inflection.

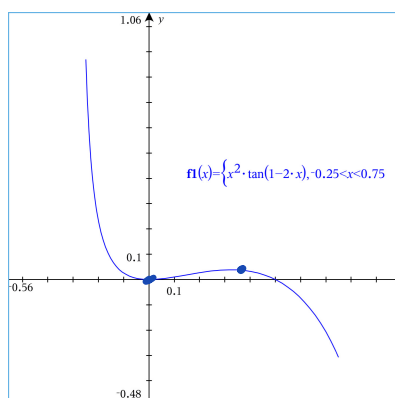
(C) f has two relative extrema, and the graph of f has only one point of inflection.

(D) f has two relative extrema, and the graph of f has two points of inflection.

B **B** **B** **B** **B** **B** **B** **B**

86. The function f has first derivative f' defined by $f'(x) = x^2 \tan(1 - 2x)$ for $-0.25 < x < 0.75$. At which of the following values of x does the graph of f have a point of inflection?

- (A) $x = 0.324$ only
 (B) $x = 0.5$ only
 (C) $x = 0$ and $x = 0.324$
 (D) $x = 0$ and $x = 0.5$



87. The function f models the amount of time $f(v)$, in hours, that it takes to ride a bicycle along a straight path from point A to point B , where v is the average velocity of the bicycle in miles per hour (mph). What are the units for $f'(v)$?

- (A) mph (B) hours (C) mph per hour (D) hours per mph

$$\frac{h}{m/h}$$

B

B

B

B

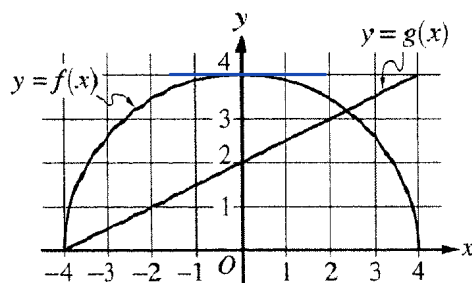
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$$g' = \frac{4}{8} = \frac{1}{2}$$

$$f(0) = 4$$

$$g(0) = 2$$

88. The functions f and g are defined for $-4 \leq x \leq 4$. The graph of $y = g(x)$ is a line segment and the graph of

$y = f(x)$ is a semicircle, as shown in the figure. The function h is defined by $h(x) = \frac{f(x)}{g(x)}$. What is the

instantaneous rate of change of h with respect to x at $x = 0$?



(A) $-\frac{1}{2}$

(B) 0

(C) $\frac{1}{2}$

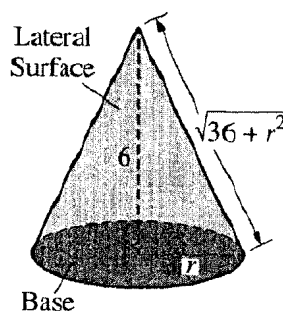
(D) 2

$$\frac{f'(0)g(0) - g'(0)f(0)}{(g(0))^2}$$

$$= \frac{0 - \frac{1}{2} \cdot 4}{4}$$

$$= -\frac{1}{2}$$

B B B B B B B B



89. The right circular cone shown has a radius of r meters and a constant height of 6 meters. The lateral surface area of the cone, in square meters, is given by $S(r) = \pi\sqrt{36r^2 + r^4}$. At the instant when the radius is 0.75 meter, the radius is increasing at the rate of 1.15 meters per minute. At what rate is the lateral surface area of the cone changing, in square meters per minute, at that instant?

- (A) 14.247
(B) 16.384
(C) 19.288
(D) 22.182

$$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} \bigg|_{r=0.75}$$

$$= 22.1818$$

90. The velocity of a particle moving along the x -axis is given by $v(t) = 4\sqrt{t+1} \cos t$ for time $t \geq 0$. At time $t = 0$, the particle is at position $x = 3$. What is the position of the particle at time $t = 2$?

- (A) -1.431 (B) 0.569 (C) 4.365 (D) 7.365

$$3 + \int_0^2 v(t) dt$$

$$\approx 7.36478$$

net change

① a. $r'(4) = -39,93177 \text{ gallon/h.}$

b. $\lim_{t \rightarrow 2^-} = 200 e^{-0.21t}$
 $= 131.409364$

$\lim_{t \rightarrow 2^+} = 115 + 25 \sin(0.12t^2)$
 $= 132.9339.$

$\lim_{t \rightarrow 2^-} \neq \lim_{t \rightarrow 2^+}$ DO not exist

c. $\int_0^2 r(t)$
 $= 326.6220764$

d. $\int_0^2 200 e^{-0.21t} dt - \int_2^5 100 dt$
 $= 26.62207637$

②

$$a. \quad v'_A(1) = \frac{f(3) - f(0)}{3 - 0} = \frac{3 + 3}{3} = 2.$$

$$v'_A(3) = \text{DNE} \quad \text{拐点}$$

b. $\int_0^{10} v_A(t) dt$ is antiderivative particle A's velocity from 10 to 0

$$\int_0^6 v_A(t) dt = 3 \times 3 = 9.$$

$$\int_6^{10} v_A(t) dt = 3 \times 4 - (2\pi) = 12 - 2\pi$$

$$c. \quad \frac{1}{4} \int_0^4 v_B(t) dt$$

$$= \frac{10158049651}{4} = 2,645,124,128.$$

$$d. \quad \int_4^{10} |v_B(t)| dt$$

$$= 27.88775349$$

③

a. point = (1, 2000)

$$\text{slope} = 2000 \cdot e^{-1} = \frac{2000}{e}$$

$$w - 2000 = \frac{2000}{e} (t - 1)$$

$$w = \frac{2000}{e} (t - 1) + 2000$$

$$w(2) = \frac{2000}{e} + 2000$$

b. $w'(2) = -e^{-2} < 0$.

w concave down. overestimate

c. $\frac{dw}{dt} = w \cdot e^{-t}$

$$\int \frac{dw}{w} = \int e^{-t} dt$$

$$\ln|w| = -e^{-t} + C$$

$$\ln|2000| = -e^{-1} + C$$

$$C = \ln|2000| + e^{-1}$$

$$\ln|w| = -e^{-t} + \ln|2000| + e^{-1}$$

$$w = 2000 e^{e^{-1} - e^{-t}}$$

d. $\lim_{t \rightarrow \infty} = \frac{11000}{2 + 3e^{-\infty}} = \frac{11000}{2} = 5500$

④

$$a. \frac{f(12) - f(3)}{12 - 3} = \frac{15 + 10}{9} = \frac{24}{9}$$

$$b. m(3) = f'(f(3)) = -10$$

$$m(12) = f'(f(12)) = 36$$

$$m(3) < m(x) < m(12)$$

because of IVT, x is between the close interval $[3, 12]$.

$$c. \int_3^{15} f''(x)^2 dx$$

$$f''(5)^2 \times 2 + f''(12)^2 \times 7 + f''(15)^2 \times 3$$

$$= 2 + 28 + 108 = 138$$

$$d. g(5) = \int_{-2}^{15} f'(t) dt + 1$$

$$= 1 + 36 - (-10) = 53$$

$$g'(5) = \frac{d}{dx} \int_{-2}^{3x} f'(x) dt$$

$$= 3 \cdot f(x)$$

$$= -3$$

⑤

a. $f(x) = 0$

$$x = \ln 2$$

$$AR = \int_0^{\ln 2} 2 - e^x \, dx$$

b. $g(1) = \frac{1}{\sqrt{3}}$ $R = \int_0^{\frac{1}{\sqrt{3}}} \pi \cdot 1^2 \, dy + \int_{\frac{1}{\sqrt{3}}}^1 \frac{1-y^2}{2y^2} \, dy$

c. $\int_0^1 \left(\frac{1}{\sqrt{2x+1}} \right)^2 \, dx$

⑥

a point (3, 1)

$$\text{slope} = \frac{3 \times 2 \times 1}{3 \times 1 - 9} = \frac{6}{3-9} = -1$$

$$y - 1 = -(x - 3)$$

$$y = -2.9 + 3 + 1 = 1.1$$

$$b. \frac{2\left(\frac{dx}{dx} \cdot y + \frac{dy}{dx} \cdot x\right)(3y^2 - x^2) - (6y \cdot \frac{dy}{dx} - 2x)(2xy)}{(3y^2 - x^2)^2}$$

$$c. i. \frac{dy}{dx} = \frac{0}{12-0} = 0.$$

$$ii. f''(2) < 0 \quad \text{relative maximum}$$